Bias Of Some Commonly Used Estimates Of Autocorrelation

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Abstract Bias in the Autocorrelation coefficient estimators of Durbin-Watson (DW), Prais-Winston (PW), Kmenta-Gilbert (KG) and newly developed Pade Approximation Method (PAM) in a linear model with AR (1) errors have been studied. It is shown by Monte Carlo experiments that the absolute bias in the estimation of AR(1) coefficient in errors was minimum by PAM than the remaining estimators. The Pade approximation estimator is better than DW, PW and KG for medium and high order Autocorrelation both positive and negative. The AR coefficient by PW, KG and DW showed a negative bias at all levels of autocorrelation while in PAM it was negative for $\varnothing > 0$ but a positive bias for $\varnothing < 0$.

1. INTRODUCTION

In many circumstance the assumption of a serially independent disturbance term may not be plausible in the linear regression model

$$Y = X\beta + U,\ E(UU') = \sigma^2 I \qquad (1)$$

This simply means that the disturbance terms have non-zero covariance i.e. $E(u_t u_{t+s}) \neq 0$ for all t and s>0. The simplest alternative is that the disturbance team has a first order autoregressive structure defined by

$$u_t = \varnothing u_{t+s} + e_t \qquad 0<\varnothing<1 \quad (2)$$

where $E(e_t) = 0$ and $E(e^2_t) = \sigma^2_e$. Under these circumstances

$$E(uu') = \phi_e^2 \Omega(\phi)$$

and the assumption of a serially independent disturbance term is not satisfied.

Because autocorrelated disturbances result in serious problems in linear regression analysis, several tests have been proposed to detect the presence of autocorrelated disturbance structure. Of the many of these tests the most frequently used is the one by Durbin-Watson (1950, 1951).

Error estimates in the linear regression analysis of time series data are virtually always based on asymptotic formulae. Usually one has a situation where large sample variances can be computed. While the asymptotic behaviour of the variance has been quite extensively studied but much less work has been done on the bias.

When the form of error structure is included in the regression model the whole function becomes nonlinear and an iterative procedure for solving this equation needs closer estimates of all parameters. In linear regression model with AR(1) errors Park and Mitchell (1980) showed in a study that Prais-Winston (PW) estimators (1954) performs considerably better than the Cochran-Orcutt (1949) estimator and the full maximum likelihood estimator (Beach and Mackinnon 1978). Dagenais (1994) suggests through large sample approximations as well as Monte Carlo simulations that when there are error in variables among the independent variables the regression models with autocorrelated error may yield estimators with marked biases and large RMSE. Kwok and Veall (1988) stated that the negative bias of the estimate for $\varnothing$ is a serious problem in the Hypothesis testing. Hashimoto (1989)considers small sample properties of estimators for an autocorrelation coefficient and of test statistic based on PW estimators in a linear model with AR(1) errors. He proposed two kind of bias-corrected PW estimators and showed by Monte Carlo experiments that one of the modified estimators performs remarkably better in hypothesis testing than the original

PW estimator especially in highly autocorrelated cases.

The purpose of this paper is to study through simulation the sample bias of $\varnothing$ estimated by Pade Approximation method (PAM) introduced by Kumar (1986) and compare it with the PW, KG and DW estimators. Recently Srivastava (1994) used this method to estimate the autocorrelation of errors in linear and non-linear regression models. Kumar and Srivastava (1995) have shown the successful use of PAM in specifying the ARMA(p, q) error structure to the residuals of linear model. The use of PAM was found to be simple and satisfactory in estimating the ARMA structure of errors. An added advantage of this method is that it gives the error structure and the estimates of its coefficients simultaneously.

The estimators of PW, KG and DW has been given in section 2. The steps in performing the simulation is given in section 3. The results are discussed in section 4 and finally in section 5 the conclusion have been drawn.

2. OTHER ESTIMATORS

The DW test is convenient to use and has been shown generally to be at least as powerful as the proposed alternatives for testing the presence of autocorrelated disturbances (L'Esperance and Taylor 1975; Abrahamse and Koerts 1969). The statistic for testing autocorrelation by DW test is well known (Durbin-Watson 1950, 1951). Henceforth, it will be denoted by Φ DW.

Kmenta-Gilbert (1986) statistic is

$$\phi_{kg} = \frac{N_i \sum u_i u_{i-1}}{(N-1)\sum u_i^2}$$

where N is the length of the series.

The Prais-Winston estimator used is

$$\phi_{pw} = \frac{\sum u_i u_{i-1}}{\sum u_i^2}$$

In the PAM we rearrange the whole series into columns according to the lag order we want to fit. Solve this by OLS to obtain the coefficients whose number is equal to the lag order undertaken.. While in other estimators (considered here) are calculated as per the given functional relationships.

Evidently there is vast difference in the method of estimation of $\varnothing$ by the PAM and in the remaining ones ,i.e, PW, KG &DW. Incedently the estimate of $\varnothing$ in the AR(1) model is negative if the first coefficient value of the OLS solution of a very high lag order fitting of residuals.

3. SIMULATION

The properties of four kind of estimators of $\varnothing$, i.e., PW,KG, DW and PAM have been examined here. The performance of these estimators is assessed by the bias based on Monte Carlo simulation. The model used in our experiments is the two variable regression model with AR(1) errors, which is similar to one used in Beach and Mackinnon (1978).

The experiments are distinguished on the basis of the value of AR(1) coefficient in the specification of the parameter of the error structure and sample size. Each of these specification are examined for two sample size N=50 and N=100. For a given set of parameters and sample size, the experiment consisted of 200 independent replications. To reduce the dependence of the results for the autocorrelated errors on initial values, the process was iterated for 100 observations before beginning each sample.

The steps for the analysis is as follows;

I. Data is generated to fit $Y=50+2.4X+u_t$ where $u_t = \varnothing u_{t-1} + e_t$ for $\varnothing = \pm0.3, \pm0.5, \pm0.7$ and ±0.9 and X integer.

II. Data in Step I is fitted by OLS and residuals are recorded.

III. The estimated of $\varnothing$ have been obtained for all the four estimators using the formulae discussed above.

4. RESULTS AND DISCUSSION

Table1 gives the estimates $\hat{\phi}$ of $\varnothing$ along with their standard errors by each of the method of estimation. Table2 gives the bias in the estimation of autocorrelation coefficient by different methods. The bias has been obtained by $\text{Bias} = (\hat{\phi} - \phi_0)$

The results can be viewed from three angles viz.; for negative autocorrelation, for positive autocorrelation and for increase in the sample size. The major findings of this experiment are given as below.

There is a phenomenal rise in the bias of autocorrelation for $\phi > 0$ than for $\phi < 0$.

4.1 Results for N = 50

The PAM showed least deviation from the true value of AR(1) coefficient for higher order autocorrelation, $\phi \geq -0.9$. In terms of bias the PAM is a better estimator of ϕ than DW if the autocorrelation was more than -0.5. For the low order negative autocorrelation, i.e. $\phi \leq -0.5$ the performance of PW, KG and DW estimators was better than the PAM estimator.

The performance of PW, KG and DW estimators was better than PAM for the low order positive autocorrelation i.e. for $\phi = 0.5$, however, for the same value of autocorrelation the DW estimator was found better than the PAM. The PAM is the best among the four estimators for high value of autocorrelation, i.e. $\phi \geq 0.7$. Although for high positive autocorrelation the DW estimator was inferior to PAM but it was better than the PW and KG estimators. The PW estimator was found better than the KG if ϕ was greater than zero.

4.2 Results for N = 100

PAM is the best estimator with minimum bias for $\phi = -0.9$. It is still a better estimator of ϕ than the DW and estimator for $\phi = -0.5$ and -0.7. PW and KG are better estimators than PAM and DW estimators for the autocorrelation coefficient -0.7 to -0.3.

In case the autocorrelation is of low order and positive i.e. $\phi = 0.3$ then the DW estimator showed minimum bias hence it can be regarded as the best estimator under this situation. However, for the same value of autocorrelation, ($\phi = 0.3$), the PAM estimator was better than the PW and KG estimators. For autocorrelation greater than or equal to 0.5 ($\phi \geq 0.5$) the bias was least by PAM estimator followed by DW estimator. Thus, we can conclude that the PAM is a better estimator than PW, KG and DW for positive autocorrelation of medium and high order. DW estimator, for $\phi > 0$ shows a better performance than the PW and KG estimators. Also on comparing the performance of PW and KG estimators under the circumstances that $\phi > 0$ it was found that the PW estimator performs better than the KG.

4.3 Other Findings

There is a marked reduction in bias from the series of size N = 50 to N = 100 for all values of autocorrelation. However, the rate of reduction was maximum for the Pade approximation estimator. The absolute bias was higher for positive autocorrelation as compared to negative autocorrelation in both the groups, N = 50 and 100. Quantitatively, the bias show a J type picture for PW, KG and DW estimators when plotted form -.9 to +.9 in both the experimental groups. However, in the case of PAM there was a monotic rise in bias as ϕ_0 varied from -.9 to +.9. It is interesting to note that the standard error of ϕ did not vary much when it was estimated by PW, KG and DW methods at all levels of autocorrelation while it was higher in the case of PAM estimator than the remaining ones.

5. CONCLUSION

As time series often covers only a short period and have some trends in its residuals, it is the matter of prime importance to develop more accurate method of estimation of ϕ. We have treated PW, KG, DW and PAM estimates of AR(1) coefficient using the simulation method. A common feature of all these estimators is that they can be used as initial estimates for more elaborate procedures like maximum likelihood or generalized least square when they are used to solve the final regression model in which the error structure is also incorporated.

We examined these estimators for series of two different sizes N =50 and N =100 and found that the performance of PAM is considerably better for the higher order autocorrelations both positive and negative. The bias in autocorrelation clearly diminishes with increasing n. The ϕ was almost uniformly under estimated by PW, KG and

DW in the whole range i.e. -0.9 to + 0.9. While in the PAM it was overestimated for $\varnothing < 0$ and under estimated for $\varnothing > 0$.

If the series was of size N= 100 then the PAM was the best method specially for positive autocorrelation more than 0.5. For the low order negative autocorrelation (i.e. $\varnothing$ = -0.3, -0.5) the performance of PW estimator was best. However, if the autocorrelation was positive and small ($\varnothing$ = 0.3, 0.5) then the DW was the best estimator.

The PW, KG, and DW estimators are relatively more stable than Pade approximation estimator. However, in the iterative procedures and short range forecasting the property of stability is of minor importance. Since this is for the first time such estimation study was carried out, the theoretical reasons for such a variation in PAM to the DW, PW and KG is yet to be established.

6. Reference

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Table 1: Estimates of ∅.

∅	PW	KG	DW	PAM
N=50				
-.9	-.8678±.0697	-.8497±.0756	-.8305±.0761	-.9159±.1778
-.7	-.6853±.0960	-.6746±.0989	-.6541±.1008	-.7366±.1784
-.5	-.5019±.1141	-.4939±.1135	-.4730±.1148	-.5464±.1778
-.3	-.3182±.1258	-.3125±.1229	-.2916±.1228	-.3547±.1777
.3	.2318±.1364	.2283±.1341	.2508±.1280	.2201±.1781
.5	.4128±.1327	.4048±.1324	.4281±.1246	.4121±.1775
.7	.5871±.1262	.5727±.1275	.5991±.1194	.6039±.1761
.9	.7427±.1166	.7134±.1178	.7523±.1107	.7841±.1808
N=100				
-.9	-.8823±.0485	-.8741±.0508	-.8641±.0506	-.9119±.1122
-.7	-.6857±.0730	-.6784±.0748	-.6691±.0743	-.7170±.1134
-.5	-.4928±.0874	-.4877±.0878	-.4781±.0881	-.5202±.1135
-.3	-.3015±.0955	-.2983±.0949	-.2886±.0954	-.3229±.1131
.3	.2682±.0949	.2652±.0942	.2761±.0942	.2695±.1117
.5	.4582±.0900	.4527±.0898	.4646±.0886	.4666±.1116
.7	.6482±.0811	.6395±.0826	.6531±.0793	.6630±.1124
.9	.8327±.0653	.8174±.0688	.8369±.0650	.8567±.1150

Table 2

Bias in the estimation of ∅ by different methods

	m	PW	KG	DW	PAM
N=50					
-.9	0.0322	0.0503	0.0695		0.0159
-.7	0.0147	0.0254	0.0459		0.0366
-.5	0.0019	0.0061	0.0270		0.0464
-.3	0.0182	0.0125	0.0084		0.0547
.3	0.0682	0.0710	0.0505		0.0799
.5	0.0972	0.0952	0.0719		0.0879
.7	0.1129	0.1273	0.1009		0.0961
.9	0.1573	0.1866	0.1477		0.1159
N=100					
-.9	0.0177	0.0259	0.0359		0.0119
-.7	0.0143	0.0216	0.0309		0.0170
-.5	0.0072	0.0123	0.0219		0.0202
-.3	0.0015	0.0017	0.0114		0.0229
.3	0.0318	0.0348	0.0239		0.0305
.5	0.0418	0.0473	0.0353		0.0334
.7	0.0518	0.0605	0.0469		0.0370
.9	0.0677	0.0826	0.0631		0.0433