Scale Dependence in Terrain Analysis

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Abstract Topographic attributes computed from Digital Elevation Models are dependent on the resolution of the elevation data from which they are computed. A regular rectangular grid is not an ideal representation of topographic surfaces for the study of scale effects. Spectral and wavelet techniques are obvious alternatives but have several deficiencies, particularly in their use of oscillatory basis functions. The positive wavelet representation has very attractive properties of localisation and feature representation. Preliminary application to 1 dimensional topographic data (profiles) yields useful results including the identification of changes in topographic structure with scale. Extension to 2 dimensional analysis will allow quantification of characteristic shapes, scales and orientations in the landscape.

1. INTRODUCTION

Topographic analysis (or terrain analysis) is the quantitative analysis of topographic surfaces with the aim of studying surface and near-surface processes. A number of topographic attributes can be calculated, including specific catchment area, slope, aspect and plan curvature (contour curvature). Slope and specific catchment area are key variables in hydrology and are used to predict spatial patterns of soil water content and erosion (Beven and Kirkby, 1979; Moore et al., 1991; Moore and Wilson, 1992; Moore et al., 1993d; Moore, 1995; Wilson and Gallant, 1996). Solar radiation estimation is based on slope and aspect, modified by topographic shadowing (Moore et al., 1993c; Gallant and Wilson, 1996). The spatial distribution of soil physical and chemical properties can be modelled within uniform geological settings using a combination of topographic attributes (Moore et al., 1993a; Gessler et al., 1995). Vegetation distribution, which responds to water, light and nutrient availability, can be modelled using combinations of topographic attributes which capture much of the landscape-scale variability of these parameters (Moore et al., 1993c). In short, topographic analysis provides the basis for a wide range of landscape-scale environmental models which are used to address both research and management questions.

It is now widely recognised that topographic analysis results are sensitive to the resolution of the source data. As the spacing of elevation samples increases, fine-scale features are lost and the surface becomes more generalised. This affects all topographic attributes but in varying ways. The resolution-dependence of slope and specific catchment area have been the most intensively studied because of their regular application in hydrological modelling (Moore et al., 1993b; Zhang and Montgomery, 1994; Quinn et al., 1995). In these and similar studies the primary question to be addressed has been "What grid resolution should I use for a particular modelling exercise?", and some useful answers have emerged. But it is worth taking a step back and asking why topographic attributes are scale dependent and what that can tell us about the topographic surface. What are the characteristics of topographic surfaces which induce the observed scale dependence of topographic attributes? Is the scale dependence consistent across scales? Do topographic features have similar shapes at different scales? Is there a different kind of organisation of the landscape at different scales? In this paper we will explore some approaches to studying scale dependence of topographic surfaces.

2. SCALE ANALYSIS OF SURFACE TOPOGRAPHY

Is grid resolution an adequate representation of scale? When we subsample an elevation grid to obtain another grid at coarser resolution, we are not only losing fine scale features of the surface (the intended change) but also changing the number of square cells into which the surface is divided. This is of particular importance when studying specific catchment area which is generally computed by accumulating cell areas from adjacent cells, and this network of connections is changed when the grid resolution is changed. If we wish to study the scale properties of a topographic surface, rather than the effect of grid resolution, it would be best to use a method which does not confuse scale effects with grid effects.

A useful representation of a topographic surface for the analysis of scale effects would be one which allows analysis of the size and shape of features at different scales, and facilitates a parameterisation of the surface that can be used to compare or define regions. It would be nice if the representation also allowed such operations as removal of fine scale features (for generalisation) and addition of finer scale features (for local improvement of a broad-scale data set, for example).

2.1 Spectral Analysis

Is there a natural method for examining the characteristics of a surface at various scales? Spectral analysis using Fourier transforms is an established method which estimates the variance (or signal power) at spatial wavelengths ranging from the size of the sample down to twice the sampling interval (grid resolution).

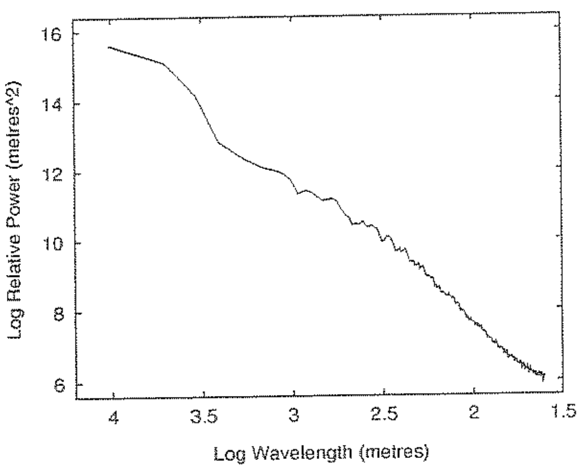

Figure 1: Spectral analysis of Brindabella 20 m DEM

Figure 1 shows the spectral analysis of a 100km^2 area of the Brindabella range on the western border of the Australian Capital Territory. The DEM is at 20 m resolution and was derived using ANUDEM (Hutchinson, 1989) from contours and spot heights digitised manually from 1:25000 scale topographic maps. The elevation grid was windowed using a circular cosine bell to reduce spectral leakage and extended with zeros to make the number of points in both x and y directions a power of 2. The data was then transformed using a Fast Fourier Transform and reduced to a one-dimensional spectrum by averaging all directions for a given wavelength.

The spectrum shows four distinct regions. At the broadest scales, on the left of the plot, there is a peak indicating substantial variation at the scale of the whole data set. At wavelengths ranging from 2500 m down to 250 m (log wavelength from 3.4 to 2.4), there is an approximately linear section in the log-log plot, indicating a power law relationship between power and wavelength with an exponent of -3. At a wavelength of about 250 m (log wavelength 2.4) a somewhat steeper linear section commences with power law exponent of -5. Finally, a short curved section is apparent at the shortest wavelengths which is attributable to aliasing and is therefore excluded from any further analysis.

This analysis suggests that in this landscape there is a change of surface morphology or texture at wavelengths of 2500 m and 250 m. The change in texture at 250 m wavelength could be due to smoothing of the surface by interpolation from contours to the grid as suggested by Polidori[1991], but another DEM produced using TIN methods from 1:10000 scale contour data in the same area shows similar behaviour. The alternative hypothesis is that the change in morphology is a real feature of the land surface. The most obvious feature of the landscape at that scale is the dissection of the surface into ridges and valleys, which would imply an average hillslope length of 125 m, since a wavelength of 250 m contains a complete cycle or two adjacent hillslopes. This hillslope length is consistent with observations in the area. The authors do not have an interpretation of the change at 2500 m wavelengths at this stage, and the 250 m change is subject to further investigation using high-resolution GPS survey data of the area.

In spite of the ability of spectral analysis to extract information about changes in surface morphology at various scales, there are at least two significant difficulties with applying spectral analysis to topographic data. The first is that the Fourier transform assumes stationarity of the signal — the mean, variance and higher order moments should all be independent of location. This is clearly not true for topographic data, and in fact the spatial variation of such parameters is of considerable interest. Another way of stating this problem is to note that the Fourier transform uses a single sinusoidal function to describe all the variation at a particular wavelength, which means that the fine scale detail at one location is assumed to be related to fine scale location at a distant location. This is clearly not appropriate. This property of Fourier transforms also means it is impossible to localise features within a sample since the analysis treats the sample as one unit and produces results which describe the whole sample. The second difficulty with Fourier transforms is that they use oscillatory waveforms (sine functions) as the basis functions into which the sample data is decomposed. If the sine functions are not a good representation of the fundamental shapes occuring in the landscape then good localisation in scale (wavelength) is not possible. Non-sinusoidal shapes produce harmonics at shorter wavelengths which can overwhelm the contribution of smaller-amplitude features at those wavelengths. This also implies that a single feature in the landscape (if such a thing exists) is represented in the Fourier transform by a substantial number of sinusoidal components, and conversely a single component in the Fourier transform contains contributions from a number of surface features. The Fourier transform is thus not a particularly suitable representation for the analysis of scale effects in topographic data.

2.2 Wavelet Analysis

If we wish to overcome the first problem of non-stationarity we might use a small window which is moved across the data set to perform a spectral analysis at each window location. This is practical but frequency resolution must be traded off against location resolution, since a large window is needed to obtain good resolution in frequency (or wavelength) while a small window is needed to obtain good resolution in location. A more practical alternative is the wavelet transform which offers the best of both worlds by effectively using a window length that changes with wavelength. Short wavelengths use small windows thus giving good location resolution for fine-scale phenomena, while long wavelengths use large windows giving good wavelength resolution for broad-scale phenomena (Chui, 1992). Wavelet analysis uses a single basis function that is localised in both position and frequency and can translated and dilated to cover the entire position-frequency plane. This contrasts with Fourier analysis which uses many basis functions each at a different frequency. Wavelet analysis could produce a power spectrum at every point in the data set, which overcomes the non-stationary/non-local problem of the Fourier transform but results in a very large data set which then has to be processed further to obtain interpretable results. Furthermore, wavelet analysis still does not address the second problem of using oscillatory basis functions. Although there is a wide choice of basis functions, the wavelet transform requires the basis function to have zero mean which implies a degree of oscillation.

2.3 Positive Wavelets

Positive wavelet analysis (Watson and Jones, 1993) is similar to wavelet analysis in that it uses a single function which is translated and dilated, but it uses a positive pulse as its basis instead of an oscillatory function. Because the positive pulse does not have zero mean it is not possible to reconstruct the original data from the wavelet coefficients in the normal way. However, by using a process of correlation detection, individual features in the form of translated and dilated copies of the basis function can be extracted from sample data. The algorithm as used here finds the scale and location giving the largest correlation value

$$T(y,L) = L^{-1/2} \int_{-\infty}^{\infty} g(x) F\left(\frac{x-y}{L}\right) dx \qquad (1)$$

which is the real wavelet transform of the function $g(x)$ using the wavelet $F(x)$, with y the location and L the scale (or dilation). The amplitude of the wavelet at that location and scale is just

$$m_i = C L_i^{-1/2} T(y_i, L_i) \qquad (2)$$

where y_i and L_i are the location and scale of the detected wavelet. C is a constant dependent on the wavelet shape

$$C = \frac{1}{\int_{-\infty}^{\infty} F^2(x)dx} \qquad (3)$$

The detected wavelet $m_i F(\frac{x-y_i}{L_i})$ is subtracted from the original function $g(x)$ and the process repeated. This iteration finds the largest magnitude features first (which in topographic data tend to also be at large (i.e. broad) scales) followed by progressively smaller features. This leads to a relatively sparse representation of the sample being analysed provided the shape of the positive wavelet is a good match to the shape of the features in the sample. It is likely that some of the small scale features will be corrections for errors in the large scale components rather than real features in the data and these can be eliminated by global minimisation after detection of the features (Watson and Jones, 1993), but this step has not been included in the analysis in this paper. The analysis procedure readily generalises to 2 dimensions with the number of parameters increasing to 5 for elliptical elements (location in 2 dimensions, scale in 2 directions and orientation), but the results reported here are only for 1 dimensional data (profiles).

The basis function used in this analysis is the sixth-order polynomial:

$$F(x) = \begin{cases} (1-4x^2)^3 & |x| < 1/2 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

and is shown in Figure 2. This form was chosen because it has continuous first and second derivatives at the boundaries. A cubic B-spline, possibly with variable knots, would also be a suitable choice. End effects in equation (1) are avoided by reflecting the data at each end which works well for a symmetrical wavelet. The mean is removed before analysis. For each iteration, the (y, L) plane is scanned for the largest correlation value $T(y, L)$,

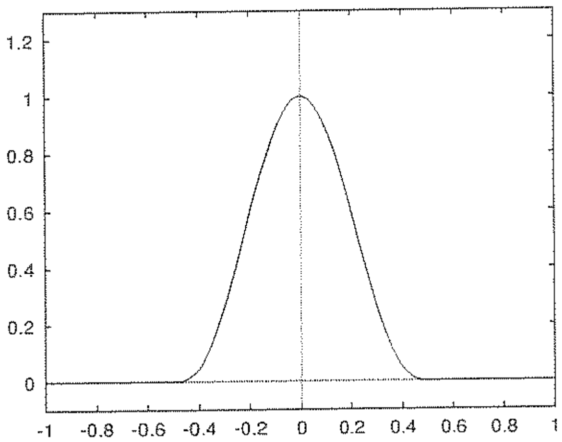

Figure 2: The unit positive wavelet $F(x)$.

followed by a local maximisation algorithm to determine the optimum y and L values; the continuity of derivatives of $F(x)$ helps maintain stability of the maximisation algorithm. The scan uses linear steps in y and logarithmic steps in L.

3. RESULTS

Figure 3(a) shows the profile being analysed and the first 3 components detected using the iterative procedure. The close fit of the positive wavelet to the large central feature is an encouraging indication that the choice of wavelet is reasonable. Figure 3(b) shows the next three components detected after those of figure 3(a), and figure 3(c) shows the representation using the 6 components. Figure 3(d) shows the fit using 20 components. As these plots show, the method progressively adds more "bumps" to the fitted profile to bring it closer to the data. It is important to note that the detection process does not seek to minimize the difference between the data and the positive wavelet representation at each step: the new wavelet is only subtracted from the data after its parameters are determined, so that the next wavelet can be detected. A total of 75 wavelets were used to decompose the profile (which itself contains 291 points). The amplitude of the 75th component was less than 1.5m, and there is no visible difference between the original data and the wavelet representation using all 75 components.

Once the profile has been decomposed into positive wavelet components, the parameters of those components can be analysed to obtain information about the structure of the surface at different scales.

Figure 4 shows the height (absolute value of amplitude) and scale of all 75 wavelets, with the first 6 components labelled to correspond with Figure 3. Positive and negative components are shown separately and the curve at the lower left of the plot represents a correlation T of 1.8 which is the correlation of the 75th component. Components to the left of this curve have been ignored in this representation. It is interesting to note that at scales between about 200 and 1000 m the positive components (hills) closely follow a straight line on the log-log plot, again indicating a

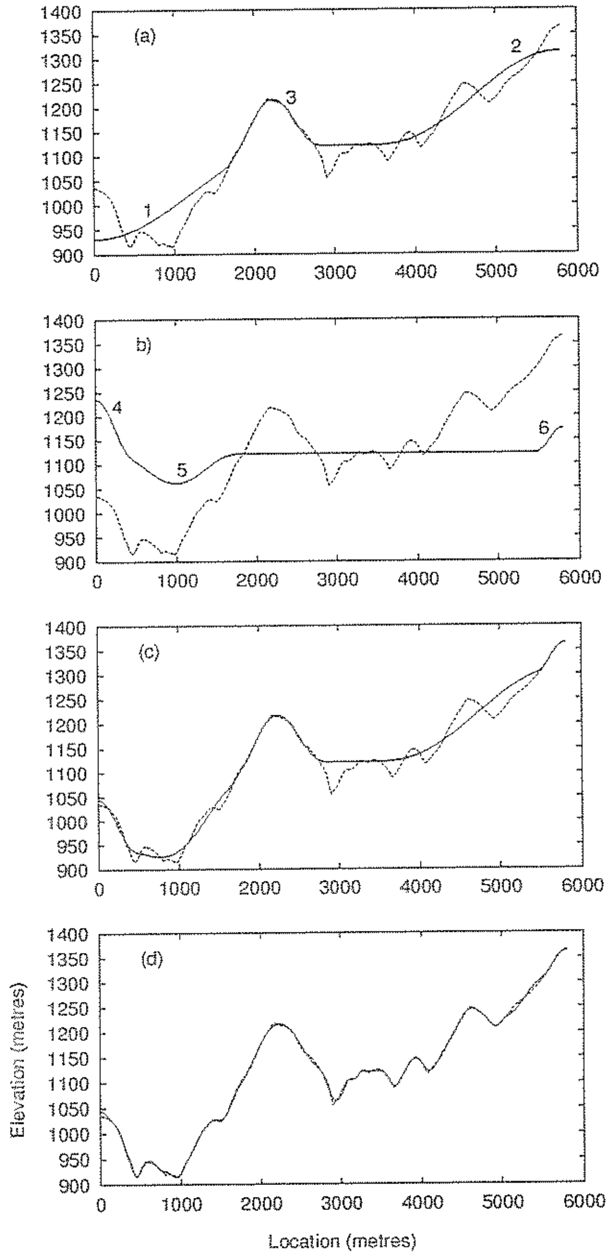

Figure 3: Representation of a profile from Brindabella 20 m DEM using positive wavelets. The original profile is shown dashed and the positive wavelet representation is the solid line. (a) Representation using the first three components; (b) the next three components; (c) the first 6, (a and b combined); and (d) the first 20 components.

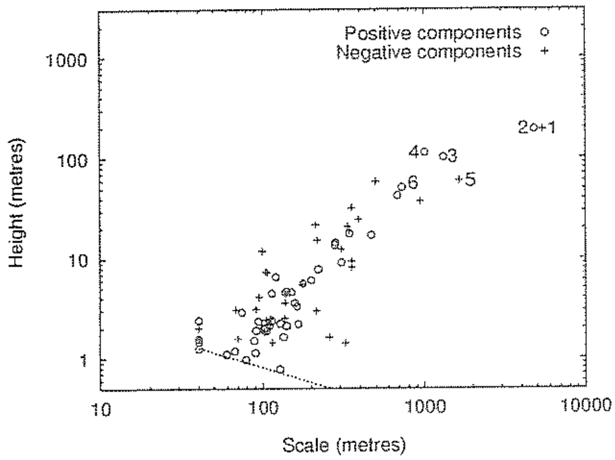

Figure 4: The height (amplitude) and scale of the 75 wavelet components used to represent the Brindabella 20 m DEM profile.

power-law relationship with an exponent of 1.5. The negative components are much more scattered but follow a similar trend. This indicates that the hills tend to become more peaked as scale increases within this range. The exponent of 1.5 is exactly half the magnitude of the exponent obtained from the power spectrum but the opposite sign. This is entirely consistent given that the power spectrum is measuring the square of the amplitude, and the independent variable is frequency which is the reciprocal of scale. The two analysis techniques appear to be detecting similar relationships over similar ranges of scales but the positive wavelet analysis shows that this relationship is much stronger for positive components than negative components.

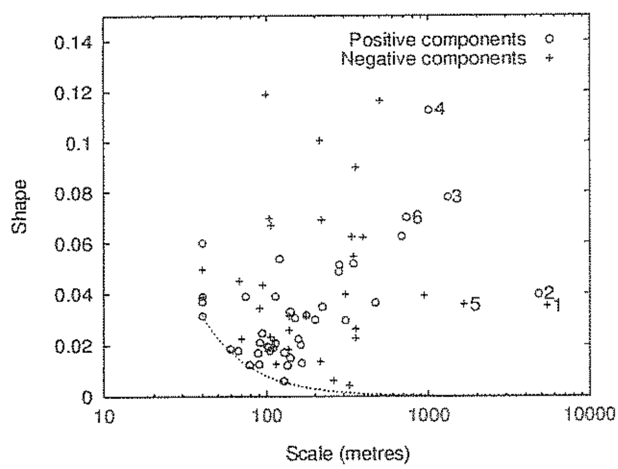

Figure 5: Shape (height divided by scale) of the 75 wavelet components.

Figure 5 shows component shape for all 75 wavelet components, where shape is defined as the height divided by the width (scale). A large value represents a very peaked feature, while a small value indicates a fairly flat feature. The positive and negative components (hills and valleys) are again plotted separately. Several interesting features are apparent:

1. For scales less than 600 m, the largest shape values are associated with negative components (valleys), while for scales greater than 600 m the largest shape values are associated with positive components (hills). Presumably this reflects the incised nature of the landscape, with the sharpest valleys at smaller scales than the most peaked hills. The significance of the 600 m threshold is not clear at this stage.

2. The maximum shape values of about 0.12 correspond to maximum slopes of 41%. These slopes are associated with features at scales ranging from 100 to 1000 m. The maximum slope associated with the broadest scale features is 14%.

3. For both negative and positive components, the shape values are largest in the intermediate range of scales, and smaller at the finer and coarser scales. (The lack of small shape values at small scales is due to the truncation of the representation based on correlation values as shown by the curve).

These results indicate that this decomposition is able to provide useful quantitative information about the topographic surface. In answer to two of the questions posed at the beginning of the paper, this decomposition of one admittedly small data set indicates that there are indeed different shaped features at different scales, and scale dependence does not appear to be consistent across scales since the relationships emerging from the analysis apply only for limited ranges of scales. The extension of the method to 2 dimensions will allow analysis of the shapes and orientation of elliptical features over a range of scales. The method could also be extended to use a library of different shaped positive wavelets, such as triangular and step profiles.

4. CONCLUSIONS

The positive wavelet decomposition presented here appears to satisfy all of the requirements stated earlier for scale-based analysis. It directly represents the size and shape of features and clearly permits removal of fine scale features to generalise the surface and addition of fine scale features for local improvement of broad scale data. The simple analysis of the wavelet parameters shown here indicates that they have considerable potential for quantifying the geometric properties of features in the landscape, and the way those properties vary with scale. The extension to two dimensional analysis will allow a more complete characterisation of topographic structure and (hopefully) lead to a better understanding of scale-related phenomena including the scale dependence of topographic attributes.

5. REFERENCES

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