Optimization using GIS: Case of Power-Line Routing

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Abstract Topography related optimization problems abound in reality. Linear/non-linear programming problems of Urban/transportation/Environmental Engineering, vehicle routing problem, facilities location/allocation problem and minimum cost flow problem are some examples of them. Considerable work can be sighted on modeling aspects and algorithms. However practical applications are limited to small simple cases. Reasons for this status are (a) inability to express topographic data in a mathematical form easily and (b) inability to manage large volumes of topographic data that is required during solving these problems. Many large terrain based optimization problems can be modeled and solved by combined approach of GIS and OR techniques and is explained in this paper by means of solving power-line routing problem. First, various topographic parameters, that influence cost, were assembled and digitized into a GIS software system. Cost function in the form of matrix (grid) was calculated using the digitized topographic data and was further approximated to improve the smoothness and continuity using the visual perspective of GIS. Two models were developed for the power-line routing; a non-linear programming model (NLP) and a calculation intensive minimum cost network flow model (network model). Using the approximated cost function, NLP model was solved to obtain an approximate solution. Using the approximate solution as the basis, the domain was decomposed into a smaller size, and network model was solved to obtain optimum solution.

1. INTRODUCTION

Many optimization problems that deal with the parameters of the earth's topography can be sighted in the fields of Urban engineering, Environmental engineering and Transportation planning and management. Some examples are optimum placement of airport (considering earth-fill work), placement of factory for minimum pollution, road network planning and traffic planning. In the field of Operations Research (OR), many models and algorithms exist to tackle these problems. A grasp of the extent of work can be obtained from Avijit Ghosh et.al.,[1987], G.L. Nemhauser et.al.,[1989]. Inspite of models and algorithms, solving these problems are quite difficult because of the distributed and arbitrary nature of the topographic parameters.

Geographic Information System (GIS) offers ability to develop topographic databases and offers computational basis to model and view data efficiently. A simple exposition of GIS can be obtained from C. Dana Tomlin[1990]. So far, GIS has been largely applied for comprehensive data representation in engineering works. Shuming Bao et.al.,[1995] have used GIS for regional analysis, Seetharam et.al.,[1993] and ARC/INFO Map book[1993] describe various applications of GIS in development projects.

Not many literature could be sighted were topographic optimization problems had been solved using GIS. Path finding algorithm has been applied using GIS for optimum forest road network design, Agung Setyabudi[1994]. Spanning tree, path finding algorithms have been included in the recent release of ARC/INFO GIS software, Network analysis module[1992].

Combined approach of GIS and OR offers potential to solve many topography related problems. This is illustrated in this paper by means of routing "High Voltage Power Line" over a domain of 40km x 40km on a test site in Saitama Prefecture, Japan.

1.1 GIS Advantages in Optimization

The salient features of any topography related optimization problems are:

a) Raw data that make up objective function exists as many huge matrices and not as a mathematical expression. Raw data can be converted into a mathematical form through curve fitting, but requires immense time and effort.

b) Large volumes of various kinds of data such as land-use, vegetation, road proximity and elevation needs to be manipulated consistently to calculate Objective and constraint function

An alternative to the curve fitting process is proposed by using GIS techniques as follows:

a) Comprehend mathematical properties, such as continuity and smoothness qualitatively using visual aspect available in any GIS software.

b) Manipulate the data quickly and efficiently to improve the mathematical properties without loosing the data's inherent characteristics. The interpolation techniques of GIS provide a convenient way to refine the topographic data.

Also, most algorithms of NLP problems find only locally minimum and not globally minimum solution. It is difficult to understand if a solution obtained is local or global due to lack of visual perspective and limitation of mathematical means. *GIS complement the deficiencies of NLP algorithms by providing visual perspective of objective and constraints that can be used to differentiate the global from local solutions.*

Further, by using GIS methodology, it is possible to handle many spatial data together in an automated and correlated fashion. Large volumes of data can be overlaid and mathematically manipulated. Spatial data can be easily converted to matrix form at different spatial units (greater than initial input resolution). GIS systems can identify spatial adjacency. Spatial queries such as maximums in the neighborhood, minimum in the neighborhood and path length are easy to evaluate.

2. ROUTING PROBLEM AND MODELS

2.1 Power-Line Routing

Figure-1 shows power-line with independent and dependent routing variable that influence route selection. Many towers need to be built between start and end points and cable has to be laid over these towers. The objective of the Power-line Routing problem is to seek locations of towers over the terrain such that the overall cost of tower construction and cable laying will be minimum.

Parameters influencing cost can be classified into two categories: topography related such as land elevation, vegetation distribution, urbanization, road & rail network, etc., and tower related such as tower height, tower foundation requirements due to route curvature, etc.

From Literature survey we found that, Kazuo Miyagawa et.al.,[1992] has describe various constraints and practical issues in power-line routing. Yoshio Saito[1992] has studied using GIS for power-line routing, even though the model does not consider any constraints.

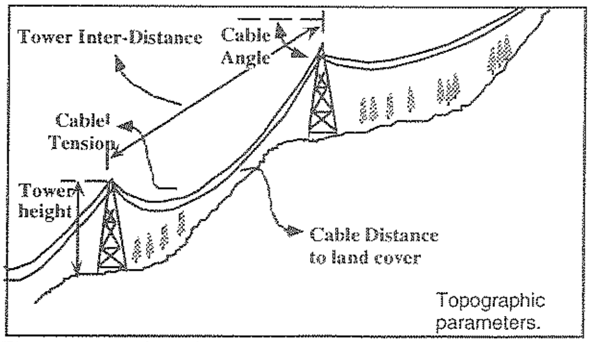

Figure-1: High Voltage Power Transmission Towers and Cables.

Cable weight acts downwards and is countered by tensioning the cable. Tower foundation must be strong to counter the weight and tension on the cable. Two important constraints restrict tower locations namely, cable tension (t) and cable height from land (h). These constraints can be expressed as

$$t = f(X_i - X_{i-1},\ w,\ z_{i-1}+th_{i-1},\ z_i+th_i) \qquad \leq TS \qquad \text{(I)}$$

$$h = g(X_i - X_{i-1},\ z_{i-1}+th_{i-1},\ z_i+th_i) \qquad \geq 10\text{m} \qquad \text{(II)}$$

Where $X_i - X_{i-1}$ = tower inter-distance; $X_i = (x_i, y_i, z_i)$, w = weight per unit length; z_i, z_{i-1} = elevation of location i,i-1; th_i, th_{i-1} = tower heights at i, i-1, TS = tensile strength of cable. Minimum allowed distance from cable to land is assumed as 10 meters. Tower height th, was considered constant at 70.3m. Horizontal cable tension and weight per unit length of cable were also considered constant.

In practice, eqs - (I) and (II) are approximated into an empirical form as

$$\text{tower inter-distance} = | X_i - X_{i-1} | \leq D_{max} \text{ and} \qquad \text{(III)}$$

$$\text{height difference} = | z_i - z_{i-1} | \leq H_{max} \qquad \text{(IV)}$$

Where D_{max} and H_{max} are empirical constants. The constraint $|X_i - X_{i-1}|$ in the model is actually $\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}$

2.2 NLP Model

Let $X_i = (x_i, y_i, z_i(x_i, y_i))$ be a continuous real variable representing the location of the tower "i" over the terrain. Figure-2 expresses the power-line routing problem as mathematical model.

The objective function of power-line routing problem is the sum of two functions, T(X) and C(X), representing the cost due to topography and cost due to tower at the point X respectively. In detail,

T(X) = cost due to land purchase + land clearing + land access + foundation cost due to land gradient.

= function of land-use, vegetation cover, road proximity, elevation & gradient

C(X) = foundation cost due to tower height + foundation cost due to route curvature + route length cost.

= function of only inter-tower distance since tower height is considered constant and route curvature effect ignored.

As both functions are of arbitrary nature, one can assume that they are non-linear. The overall problem can be considered as constrained Non-Linear optimization problem of moderately large size.

2.3 Network Model Description

The whole terrain was considered as a mesh of size (m x n). It was assumed that towers can only be placed at grid nodes (center point of each cell). With these presumptions, it is possible to model the power-line routing problem as a single commodity minimum cost network flow model.

Let N represent the set of nodes in the mesh (= nxm nodes). Let A represent the undirected arcs that can be made by connecting each node with the remaining nodes in the mesh. Each pair of nodes, i, j can have an arc (i,j). Thus the total number of arcs for a mesh of (m x n) is $(n^2m^2-nm)/2$. Thus nodes N and arcs A form a network G=(N,A)

Let us define z_i and C_i to represent the elevation and cost of tower construction at node i respectively. Let d_{ij} be the length of the arc (i,j). "*s*" the source node, "*t*" the sink node and "K" the cost per unit length of cable. Also let

$$X_{ij} = \begin{cases} 1 & \text{if arc (i,j) is selected} \\ 0 & \text{if otherwise} \end{cases}$$

The power-line routing problem can be modeled as shown in figure-3.

The computation time of the network problem expands exponentially with the increase in number of arcs and is often very difficult to solve. However, by using GIS, it is possible to visualize the possible path of optimum route and select a smaller domain, thus reducing the problem size. This approach was used in our case study

3. CASE STUDY

Above procedures were applied to a test site in Saitama Prefecture, Japan. The area of the test site was 40kmx40km. Elevation & road network details were obtained from "Kokudo GSIJ". Land-use data was obtained through Remote Sensing. Non-Linear Programming computer program code "CFSQP" from University of Marylands was used for this study. General purpose OR software CPLEX was used for the Network Model.

The problem specification we studied is summarized in table-1.

Table-1: Specification of the Power-line Routing Problem.

Description	**Value**	**Comments**
Computation Domain	40x40 km^2	
Topograhic Parameters (Elevation, gradient, land-use, etc.,)		influences cost (topography related)
Tower Height (other heights are 54.3, 66.3, 82.3m)	70.3m	influences cost (tower related, constant)
Route Curvature		influences cost (tower related, *not considered*)
Tower Inter-Distance	≤ 700 m	D_{max} constraint
Height Difference	≤ 70 m	H_{max} constraint

Network model size is dependent on the domain size and the mesh resolution. For, mesh cell size of 20 meter, Network model size is $4x10^6$ nodes and $8x10^{12}$ arcs which is extremely large.

3.1 Data Preparation

For the purpose of data preparation, GRID module of GIS software, ARC/INFO was used. GRID, represents topographic data in a raster format, convenient for manipulation as matrices. Schematic diagram of the constituents of objective function is as shown in figure-4. Data required for the creation of objective and constraint function was prepared using the GIS tools by the following procedure:

1. Primary topographic data of the target region -- contour (elevation) map and land-use map were digitized into computer at 10m surface resolution. Road, river network was also digitized into separate coverages. Digitized map is called a grid or coverage depending on the data-type.
2. From the contour grid, surface gradient and surface aspect were derived mathematically.
3. From the Land-use grid and relative cost index provided in table-2, graded land cost grid was created. If absolute cost information is available, it can be used instead of relative cost index.
4. In a similar way, gradient coefficient grid was derived from gradient grid and land access coefficient was derived from elevation and road coverages
5. Final relative cost matrix was derived by multiplying land-cost grid with gradient coefficient and land access coefficient.

Table-2: Relative cost and relative importance of each variable in the model.

Ground variable	**Relative cost index**			**Relative importance**
	Low	**Medium**	**High**	
Land-Use	Open, industrial land,	Low density residential land.	Prime land, high density land.	Very important. provides land purchase cost
Gradient (%)	0~25	25~45	>45	Very important It affects cost & feasibility.
Elevation (m) Road Proximity (m)	∓ 200 & <500	∓200~600 & < 1.5km	> ∓ 600 & >1.5km	Important. It affects cost

Final cost matrix is in ARC/INFO grid format. Using GIS tools data was visualized with colors from green to red representing low to high costs. Black was used to represent regions where there are no data.

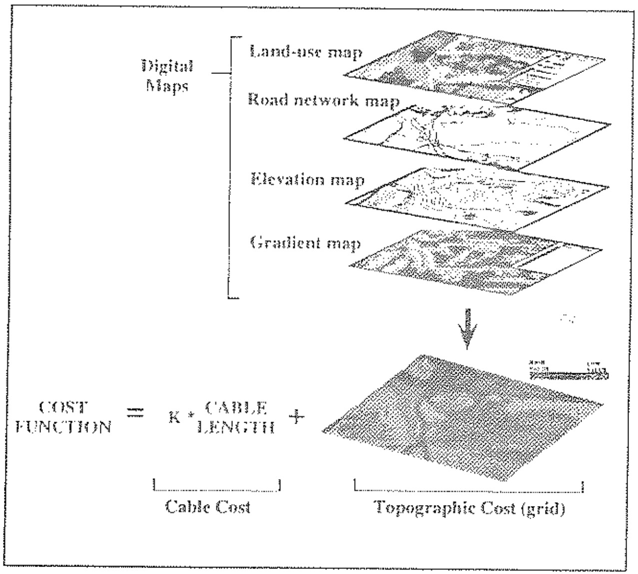

Figure 4: Schematic diagram of objective function

3.1.1 Further Data Preparation for NLP model

Using green to red color ramp, the cost grid was visualized in 2 and 3 dimensions. Each grid cell is assigned a color between green and red depending on its value, red being high and green being low. Predominately green areas are lost cost regions and predominately red are high cost regions. Mathematical properties were checked and improved as follows:

Continuity: If there are black color regions visible, then the data is not continuos. There are many reasons for discountinuity; input data is not available at that point or it is a restricted zone where tower construction is not possible or represents a region where tower construction is impossible such as a road or a building. Discontinuity problems can be easily overcome by assigning a high cost to the no-data region and finally verifying that tower locations are not in any of these regions.

Smoothness & Differentiability: If there are sharp color changes in the neighborhood of a grid cell, one can assume that the function is not smooth. As GIS understand spatial adjacency and offer practical methods to interpolate topographic data, function's smoothness can be improved by smoothening the data using the GIS tools.

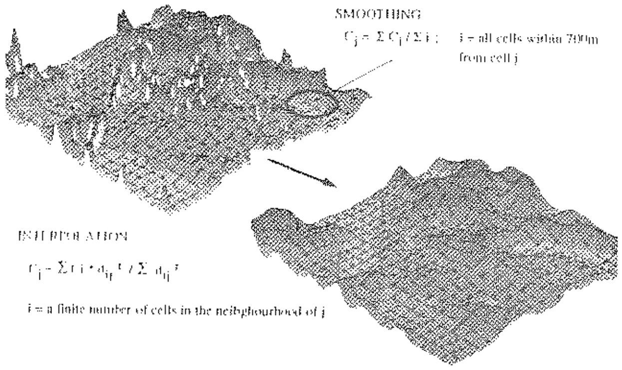

Figure 5: Smoothing and Interpolation

3.2 Computation

Final refined cost grid was the basis for the function T(X). Value of T(X) was obtained through bi-linear interpolation. Similarly, elevation grid forms the basis for z. It is possible to directly interface an external OR software to ARC/INFO if ARC/INFO's interface libraries are present. In such a case, OR software can directly read ARC/INFO's data and optimal route outputs can be directly displayed on screen. If ARC/INFO's interface libraries are not available, ARC/INFO's grid data has to be exported into ASCII format file which in turn was used as input in OR software and visa versa.

Through visual perception of cost and elevation grid, few interesting paths where identified where the final optimum solution is likely to fall. Arbitrary route was chosen along each of the interesting paths and was considered as the starting X required while solving NLP model. Gradient of cost function required for solving NLP model was obtained through finite difference method. Local optimum solutions where obtained for each of the interesting paths and the path with the lowest objective value were considered as the most likely path for optimum route.

Region around the most likely path was selected using the ARC/INFO tools and was used as the domain for the network model, thus reducing the problem size. Further, actual cost (with out smoothing) function can be used with network model and minimum cost network flow algorithm provides globally optimum solution. Thus, solution obtained using network model is more reliable than NLP model and was considered the optimum solution. Results of NLP and network model solutions are summarized in figure-6.

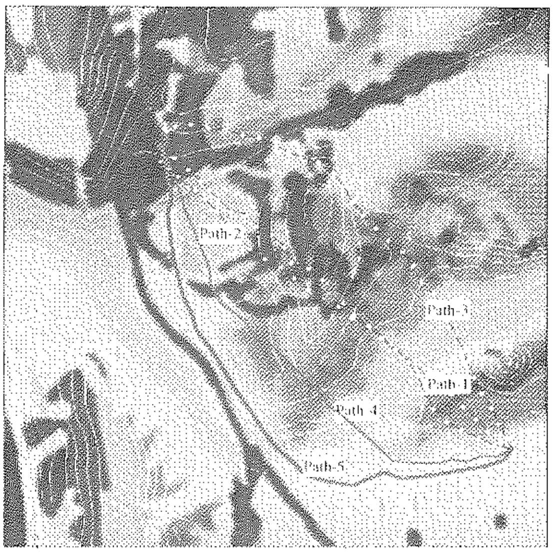

Figure-6: cost grid overlaid with local optimal routes and the final optimal route obtained through network model.

Table-3: Cost matrix with summary of results along the interesting paths and optimum solution. White regions indicate low cost and black high cost. Paths 1~4 are NLP local optimum solutions. Path-4 is the best local optimum solution. Path-5 is the solution obtained using network model.

Path ID	Total Length (KM)	Relative Objective Value
path-1	18.5	27,031
path-2	20.2	24,043
path-3	20.7	25,691
path-4	21.6	22,218
path-5	22.6	19,411

4. CONCLUSION

Continuous linear/non-linear programming models rely on the expression of objective and constraints in mathematical forms that are twice continuously differentiable. Terrain parameters are complex and there are no easy methods of approximating them into mathematical forms.

Using GIS spatial surfaces can be visualized and expressed in a near mathematical form. Mathematical requirements such as continuity, smoothness can be checked qualitatively. Visualizing the functions allow ability to understand probable areas of optimum solution thus reducing the need for twice differentiable functions. Processed data can be used with NLP solver, or any other optimization codes, to calculate optimum solutions. Network models can also be solved in similar fashion.

In the case of power-line routing, continuity and smoothness of elevation and cost function directly influence solvability NLP model. Smoothing reduces the accuracy of the cost function and NLP algorithm itself can yield only locally optimum solution. Thus output o NLP model is approximate and only provides the region of most probable region. By using network model in the most probable region, exact route could be obtained.

This study was undertaken mainly to establish feasibility of the approach and to show that combined approach of GIS and Operations Research can be used effectively for terrain based optimization problems.

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Minimize				*Minimize* sum of
$\sum_{i=1}^{p} T(X_i) + \sum_{i=1}^{p} C(X_i)$				Cost Due to Topography + Cost Due to Towers
Subject to				***Constraints***
$\lvert X_i - X_{i-1} \rvert$	$\leq$	700	$i=2,3,\ldots p$	Tower Inter-Distance ≤ 700 m
$\lvert X_i - X_{i-1} \rvert$	$\geq$	0	$i=2,3,\ldots p$	Tower Inter-Distance ≥ 0 m
$z_i - z_{i-1}$	$\leq$	70	$i=2,3,\ldots p$	Cable height less than 70m
X_1	$=$	A		Starting Point
X_n	$=$	B		Destination point
p	$\geq$	2		Number of towers ≥ 2
$X_i = (x_i, y_i, z_i)$	ε	R^3	$i=1,2,\ldots p$	Distance vector; namely x,y,z co-ordinates

Figure-2: Mathematical representation NLP model for Power-line routing.

Minimize				*Minimize* sum of
$\sum_{i}\sum_{j} C_i{*}X_{ij} + \sum_{i}\sum_{j} K{*}d_{ij}{*}X_{ij}$				Cost Due to Topography + Cost Due to Cable
Subject to				***Constraints***
$\sum_{\forall j:(i,j)\,\xi\,A} X_{ij} - \sum_{\forall j:(j,i)\,\xi\,A} X_{ji}$	$=$	1	if i=s	Essential condition for arcs to be connected to form a route
	$=$	0	if i≠ s,t for all i ε N	
	$=$	-1	if i=t	
$d_{ij}{*}X_{ij}$	$\leq$	700	for all (i,j) ε A	Tower Inter-Distance ≤ 700 m
$\lvert z_i - z_j \rvert {*}X_{ij}$	$\leq$	70	for all (i,j) ε A	Cable height less than 70m
X_{ij}	ε	(0,1)	for all (i,j) ε A	0-1 integer variable

Figure-5: Minimum cost network flow model of Power-line routing.