Dynamical models for ecological/environmental modelling and planning

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Abstract Forest growth and yield prediction models developed through ordinary least squares regression account for most of the empirically based and process (physiological) models. More recently use of non-linear minimisation techniques have contributed greatly to developing growth and yield models with stable parameters. However, heteroscedasticity problems are difficult to deal with, especially with non-linear least squares regression models or linear least squares regression models developed from data covering different geographical areas. It is of great importance that models have good statistical properties, be simple to implement, quick to validate, be stable and parametrically efficient. Dynamical models provide a panacea in this regard and excellent qualities for control designs (optimisation strategies) that are the end-point of management decision support systems. They account for some time dependent non-linearities as linear trends with good statistical properties. This paper compares a multiple regression model, a non-linear continuous-time model and a discrete-time dynamical model developed from the same data.

1. INTRODUCTION

In forest planning, growth models have been developed to aid decision-making. Planning is the problem of determining an optimal procedure for attaining a set of objectives [Luenberger, 1969]. The traditional approach in forestry has been to separately identify different forest growth variables, then measure and model them. These variables are commonly mortality, basal area, height, tree diameter distribution and volume. Decision support systems are then developed by integrating the variable models in some sequential way and these systems are used to simulate forest management strategies. Expert knowledge is used to suggest possible strategies to meet management objectives. The decision support system simulates the various strategies, providing the basis for comparison and elimination of the less likely options.

Optimisation techniques can also be used to determine an optimal strategy to maximise management objectives. Because of the multistage decision-making nature of forest planning problems, dynamic programming has always been found appealing. In forestry, however, dynamic programming has been used sparingly. Arimizu [1958] used it to regulate intermediate cutting with the objective of producing a maximum harvest volume. Hool [1965], using simply a 'cut' or 'do not cut' strategy, applied a dynamic programming model. Later he introduced a Markov chain approach to production control using a dynamic programming model [Hool, 1966]. Amidon and Akin [1968] compared traditional marginal analysis with dynamic programming for determining optimal growing stock and found the latter to be more flexible and convenient. Other authors have illustrated the feasibility of dynamic programming for deriving optimal cutting schedules for timber stands [Risvand 1969; Kilkki and Vaisanen 1969; Schreuder, 1971]. Many researchers in forest management studied dynamic programming to support the sequential decision making required for decisions about the thinning regime and rotation of even-aged stands [Brodie and Kao, 1979; Chen et. al., 1980; Martin and Ek, 1981; Haight et. al., 1985].

Unfortunately many of the above papers are difficult to follow because explicit derivation of the solution procedures is lacking [Chen et al., 1980]. An additional shortcoming of several of the papers is the absence of suitable forest growth models - that is, ones directly related to the decision variable. These two factors, plus the unfamiliarity of most readers with the special conditions which must be met for a problem to be solved as a dynamic programming problem, account for the limited application of dynamic programming in forestry [Chen et al., 1980]. Chikumbo and Mareels [1995a] have demonstrated the application of the maximum principle (a solution technique similar to dynamic programming) to determine optimum thinning strategies for *Pinus patula* Schl. et Cham., in South Africa by using appropiate decision stand variable-models (dynamical models). In addition to their suitability in a multistage optimisation formulation, the dynamical models were easy to test, parametrically efficient and easy to understand.

Despite the simplicity of these dynamical models, how reliable are they in predicting growth? Lack of comparable data and conventional models predicting the same response variable has prevented the comparison of models developed from different approaches. Conventional models developed for forest management can be categorised into two broad classes: mechanistic and empirical models. The mechanistic models are based on an understanding of the physical or physiological functioning of the system. Empirical models describe the trends and relationships with other variables. In this paper three models for basal area prediction, that is, dynamical, mechanistic and empirical, were compared. Details of the models are found in sub-section 3.

To carry out a consistent test which would determine the best model in terms of predicting basal area, data not used in model development were used to cross validate the three models. Cross validation checks the performance of a model against a fresh data set that has not been used for model development. It is a sensible way of comparing different models obtained from different methods.

2. DATA

The data used for this model development and cross validation were obtained from *P. patula* correlated curve trend (CCT) spacing trials in Nelshoogte, (South Africa). The CCT experiment was established in 1937 with four replications (*A*, *B*, *C* and *D*) of each of the 16 spacing treatments. Eight nominal stand densities (plots 1-8), were established at 2965 stems per hectare (stems/ha). The plots were 'thinned in advance of competition' such that the initial densities ranged from 124-2965 stems/ha. Plots 9-16 were thinned to investigate the various degrees of suppression and release. The thinning plots are shown in Table 1.

Table 1. Treatment specifications (Schedule of stem number reduction)

Plot	Age (years)											
	0	1.67	3.50	4.00	5.00	6.00	7.00	8.00	10.67	15.17	19.25	23.33
1	2965											
2	2965	1483										
3	2965	1483	988									
4	2965	1483	988	741								
5	2965	1483	988	741	494							
6	2965	1483	988	741	494	371						
7	2965	1483	988	741	494	371	247					
8	2965	1483	988	741	494	371	247	124				
9	2965								1976	988	494	247
10	2965					1976			988	494	247	
11	2965	1976				988			494	247		
12	2965	988				494			247			
13	2965								988			
14	2965								494			
15	2965								494			
16	2965	988							494			

3. MODELS

The three types of models compared were as follows:
(a) A linear time invariant first order dynamical model which had parameters that were density dependent was developed from all the *A* replicates of plots 1-8 and the remainder of the replicates used for cross validation. The model performed well [Chikumbo and Mareels, 1995b]. The correlation function of the residuals and the cross-correlation of the residuals and input variable to the model (commonly known as a correlogram) indicated a model that was representative of the observed basal area trend within 99% confidence. The model was as follows:

$$BA(t) = a(x)BA(t-1) + b(x) \quad (1)$$

where
BA = stand basal area (m^2/ha)

x = stand density (stems/ha)
t = time(years)

$$a(x) = 0.93 + 0.01\frac{x}{1000} - 0.047\left(\frac{x}{1000}\right)^2 + 0.01\left(\frac{x}{1000}\right)^3$$

$$b(x) = 2.32 + 4.24\frac{x}{1000} - 0.354\left(\frac{x}{1000}\right)^2$$

(b) The classical mechanistic growth model structure, commonly known as the Chapman-Richard's generalised von Bertalanffy's model, was fitted using all the replicates from plots 1-8. The basal area function was developed to account for the changes in the planting densities and was fitted with a coefficient of determination of 0.9567 and a mean squared error of 3.82m^2 [Harrison et. al., 1994]. The mean squared error of an estimate, $\hat{y}$ is $E(\hat{y}-y)^2$, or the expected value of its squared deviation from the parameter y. For the unbiased estimator the mean squared error is equal to the variance of the estimator. However, a mean squared error close to zero is highly desirable because it is more likely to produce accurate estimates for y [Devore, 1991]. The model was as follows:

$$BA = 57.1609[1 - e^{-(0.0044x^{0.4077})t}]^{1.95} \qquad (2)$$

(c) The multiple regression model based on the empirical evidence of the CCT trial (plots 1-8), fitted well with a coefficient of determination of 0.987 and a mean squared error of 12% [Harrison et. al., 1994]. Its difference form (for simulation purposes) was as follows:

$$\begin{aligned} ln(BA_2) &= ln(BA_1) - 34.0846\left(\frac{1}{t_2} - \frac{1}{t_1}\right) \\ &\ldots + 0.1717(ln\, x_2 - ln\, x_1) + 0.518(ln\, HD_2 - ln\, HD_1) \\ &\ldots + 2.9567\left(\frac{ln\, x_2}{t_2} - \frac{ln\, x_1}{t_1}\right) + 4.3352\left(\frac{ln\, HD_2}{t_2} - \frac{ln\, HD_1}{t_1}\right) \end{aligned} \qquad (3)$$

where
ln = natural logarithm

$$x_2 = x_0 - (x_0 - x_\infty)[1 - e^{-0.0001598t^{2.0175+0.0001588t_0}}]$$

x_∞ = asymptotic density (fixed at 90 trees/ha)

$$HD_2 = HD_1\left[\frac{1 - e^{-0.0482t_2}}{1 - e^{-0.0482t_1}}\right]^{0.9446}$$

4. RESULTS

Mean squared error was used to compare the three models against fresh data from plots 9-16. The mean squared error is a reliable statistic for comparing any two data sets since it takes into account their individual distributions. Any prolonged period of basal area measurements, between any two successive thinnings from a replicate in plots 9-16, were used for cross validating the three models. The results are shown in Table 2, with model (1) showing the best results because of the low mean squared errors. Model (2) had the lowest mean squared error in three cases and a mean squared error that exceeded the variance of the observed data in two cases. Model (3) was not reliable and in six cases out of nine had a mean squared error that exceeded the variance of the observed data.

Table 2. Calculated mean squared errors from cross validation data of the three models. The lowest mean squared value for each test is shown in bold print and any that is greater than the variance of the observed data is shown in italics.

PLOT REPLICATES	AGE RANGE	INITIAL DENSITY (stems/ha)	VARIANCE	MEAN SQUARED ERRORS		
				model (1)	model (2)	model (3)
9D	24-36	247	29.42	**0.55**	*163.99*	*45.28*
11B	16-36	247	43.01	**1.49**	*47.32*	*48.55*
12C	24-34	222	13.33	3.87	**1.87**	9.77
13C	11-29	988	122.87	**8.75**	61.72	*136.27*
14B	13-29	482	75.85	**0.68**	52.76	*92.52*
15B	11-36	494	139.19	**0.46**	32.11	*201.79*
15D	24-36	482	29.07	13.07	**5.52**	*37.11*
16B	3-10	988	139.3	**1.32**	4.96	36.54
16B	11-36	494	106.12	2.42	**1.98**	55.76

5. DISCUSSION

Model (1) had good results because the modelling structure is based on regressing against the previous values of the response variable and can therefore map a first order non-linear trend. The modelling approach is based on system theory and thus concentrates on modelling trends without a full comprehension of the processes influencing the behaviour of the system.

Model (2) is developed from the general form:

$$y = y_0(1 - e^{-bt})^c \qquad (4)$$

which is a continuous-time model. Ratkowsky [1983] demonstrated that the least squares estimates of the parameters b and c in equation (4) undergo considerable variation making it hard to estimate them. This instability led Ratkowsky [1983] to recommend 'reparameterisation' where one of the offending (sensitive) parameters is expressed as a function, only of the parameters of the other models, without the expression containing the explanatory variables, or the error term. Maximum likelihood estimation, that does not assume any underlying distribution of residuals, can be used for specifying the loss function or the objective function to maximise [Press et. al., 1992]. Non-linear least squares routines [Press et. al., 1992] such as the Levenberg-Marquardt method (also called Marquardt method) can be employed for the gradient search, because of its increased robustness and iterative efficiency. However, data acquisition in forestry is in discrete-time and it is better to stick to discrete-time model development. Model (4) can be discretised to a state space representation which would help to fix the unstable parameters and estimate them more easily. The discretised model can be represented as follows:

$$z(t+1) = Az(t) + B \qquad (5)$$
$$y^m = Cz(t)$$

where
A and B matrices depend on c in a non-linear fashion.
z = state.

In principle m can take any values but can be easily approximated as a rational number.

Model (3) is an empirical function that is reflecting physiological processes. Its under-performance can be attributed to the fact that the explanatory variables have to be estimated from other models and this may be more difficult than estimating basal area itself. Thus a statistically significant equation (3) was estimated but the function did not prove to have good predictive properties. The multiple regression approach compounds the estimation error of the response variable [Pindyck and Rubinfeld, 1981]. In particular, model (3) required the prediction of mortality which is a daunting task, because of its sporadic nature. *P. patula* in particular is a non-differentiating species and mortality is therefore difficult to model [Oliver and Larson, 1990].

A simple model such as (1), which is fairly robust and responds to thinning, would be easy in terms of mathematical tractability, to control in an optimisation formulation [Chikumbo and Mareels, 1995a] and so attractive for use in a decision support system.

6. CONCLUSION

Efficiency of dynamical models in predicting basal area has been demonstrated over other forestry conventional models. Dynamical models for other growth variables are being developed but comparison with the conventional models may be a problem due to the unavailability of fresh data sets for cross validation. Multiple regression models need to be applied appropriately, that is, in representing and understanding the mechanics and processes at work in any system. For management purposes where prediction and optimisation play a major role, simple models are desirable provided this is not at the cost of accuracy.

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