A Smart Market for Ground Water using the Eigenmodel Approach

M. Plagmann[1] **and J.F. Raffensperger**[1]

[1] Dept. of Management, University of Canterbury, Christchurch, New Zealand
Email: john.raffensperger@canterbury.ac.nz

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EXTENDED ABSTRACT

We propose a smart market system for ground water, based on an analytical hydrology model and a linear program (LP). A smart market is an auction-based market that is cleared with the assistance of computer models, which manage complexities that an ordinary market or auction cannot. In this case, the models address the environmental externalities of water use.

This revolutionary technology provides an impressive list of advantages. It provides a venue for trading (a single web site), so users need not search for trading partners. The computer models vet all transactions simultaneously to ensure satisfaction of all relevant environmental constraints such as stream flows. Trading can be done on a weekly, daily, or even hourly basis. Price data is immediately available, and is highly detailed temporally and spatially. Thus, the smart market eliminates transaction costs with water trading, while imposing strict environmental constraints. Perhaps most spectacularly, the system eliminates the market requirement for physical control of water. The proposed system is not a “free market,” but rather a highly controlled auction, in which trading is easy, but constrained to be within the bounds of sustainability.

From a legal point of view, privatisation is not needed. What is traded is the administrative right to use a quantity of water at a specific location. This approach fits in very well with New Zealand’s existing legal framework. The market would be a spot market, in which users can lease the right to take additional water for a fixed period of time in addition to their existing consented rate, or to hire out some of their consented rate. Trade is not one-to-one, because water use has many externalities. Rather, users trade with the market via the smart auction.

The key to the system is the combination of hydrological simulation and a linear program (LP). The hydrology model predicts the behaviour of the aquifer. The LP maximizes the economic value of pumped water, subject to constraints on flows in lowland streams. Contributions are (1) a computationally efficient smart market for ground water, and (2) empirical results on the time horizon required for the smart market.

The smart market is developed as a hydrological optimisation. Most hydrological optimisations use MODFLOW to develop a response matrix, which is then used in the LP. Generation of the response matrix is computationally intensive, and can require hours or even days to compute. Recently, a few researchers showed that a simpler approach based on eigenmodels can produce the response matrix much more easily.

Based on our earlier work using MODFLOW, we developed an improved model for a smart market using the eigenmodel approach, and tested this on real data. We confirmed the ease of computation, and showed that this could be the foundation of a smart market for ground water. The constraints in the LP correspond to minimum flow constraints at lowland streams. These constraints serve to guarantee the sustainability of the water market. We used uniform bids, which serve to demonstrate spatial and temporal pricing implications of the environmental constraints. We simulated weekly trading for two years, a wet year and a dry year. While our system has not been implemented with real users (which is likely to require years of political work), we have used real hydrology and environmental constraints from New Zealand’s Selwyn District.

We found that prices for ground water tend to be higher for wells that are nearest to the environmental control points. We also found that the environment flows must be simulated for over a year, beyond the end of the annual market period. This is due to the delayed effect of abstraction on groundwater. To prevent giving users the false impression that they can take water without effect near the end of the year, the LP must contain sustainability constraints into the next year.

1. INTRODUCTION

It has long been recognized that water should be properly priced, and that the best way to price water is through market mechanisms (Hirshleifer et al. 1960). However, experience to date with water markets has been mixed. Markets have worked at best with modest effectiveness, and only where water is controlled, as shown by extensive experience in Chile, Texas, and Australia. (Bauer 2004; Rosegrant et al. 2000; Griffin and Characklis 2002; Bjornlund 2003). The problems with operating water markets continue despite the amount of research and policy attention given to problems of water.

This paper describes one possible approach for setting up a sustainable spot market for ground water. The market is a "smart market," assisted by two computer models, an analytical model of the relevant hydrology, and a linear program (LP) for the economics. Smart markets are computer-assisted auctions, where the computer manages complexities that an ordinary auction cannot (McCabe et al. 1991). Smart markets have been developed for surface water (Murphy et al. 2000), radio spectrum rights (Chakravorti et al., 1995), natural gas (McCabe et al., 1994), electricity (Hogan, Read & Ring, 1996; Alvey et al., 1998), and course registration (Graves et al., 1993).

The market is cleared by the LP. The objective coefficients are users' bids as entered through a web page, and constraints are (1) balance of flow at each well, and (2) required environmental flows. Constraint coefficients are developed from an eigenmodel of the catchment. Our smart market goes beyond groundwater simulations, and uses hydrological optimization. Hydrological optimization seeks to find optimal solutions to water management problems. Ahlfeld, Barlow & Mulligan (2005) developed optimization packages that use the simulation software MODFLOW (Harbaugh 2005). For this ground water smart market, the problem is to maximize the economic value of the water taken (as specified by users' bids), subject to the requirement that flows at environmental control points are above the minimum required. An environmental control point is a location at which a relevant environmental measure (such as stream flow) is taken. This paper extends Raffensperger & Milke (2005), which gives an overview of the smart market for ground water, and Raffensperger (2007), which describes in detail how MODFLOW can be used to find the response matrix, and proposes a plausible deterministic LP formulation for a smart market for ground water.

The use of MODFLOW makes the development of the market LP extremely time consuming. Wells must be simulated one at a time for a unit change in abstraction, with all other wells held constant, to find the change in head at each time period and each environmental control point. A simulation for one well could take a few minutes to an hour, and the market could encompass hundreds of wells. Furthermore, the work requires detailed three-dimensional modelling of the catchment, with detailed data about permeability, land contours, historical head levels, well locations and depths, as well as an experienced hydrologist who can operate the software.

The contributions in this paper are to apply the eigenmodel method to a smart market model, and empirical results regarding the length of the auction planning horizon. We describe the use of an analytical hydrology model, called an eigenmodel, in place of MODFLOW. Compared to MODFLOW, an analytical model has shorter calculation time; results are obtained in seconds rather than hours, and the programming effort is considerably reduced. While an analytical model is usually not intended to model intricate details of the aquifer under investigation, in contrast to MODFLOW. Rather, an analytical model can model the behaviour of the aquifer as a whole.

We used the eigenmodel approach, first described by Sahuquillo (1983). The model assumes that the aquifer exhibits a linear response to stress. The details are described nicely by Pulido-Velazquez et al (2005) and Bidwell (2005); the latter coined the term "eigenmodel" and showed that the Canterbury Plains ground water behaves linearly. Our analytical hydrology model (Selwyn AHM) simulates the hydrology of Selwyn District aquifer, to use as input for the market model. (Selwyn District is in the centre of NZ's South Island.) The eigenmodel was easy to set up, much easier than if we had used MODFLOW. The model can consider forecasted rain, and is easily extendible to include surface water. We found that the market horizon length is important, due to the relatively long-term effects of abstraction.

2. THE SELWYN ANALYTICAL HYDROLOGY MODEL, SELWYN AHM

2.1. Derivation of the model

Given historical rainfall and well heads, we wish to (1) develop an equation of well head as a function of rainfall, and then (2) calculate the effect of a marginal change in each well head on each of a set of environmental control points.

The eigenmodel is based on an autoregressive integrated moving average (ARIMA) model, which estimates the head at a given well and time period, given historical rainfall. The model treats the aquifer as a box of water with a no-flow zone at the mountains, and a flow zone at the coast, and assumes a constant aquifer-wide transmissivity and storage capacity. The aquifer has localized differences in transmissivity and storage, but its behaviour overall can be described with good accuracy by assuming a constant transmissivity and specific storage. Denote $h_{i,t}$ = estimated head at well i at time t, h_0 is base head; R_t = recharge at time t. Then $h_{it} = h_0 + \sum_{u=t-2}^{t} R_u b_u + \sum_{u=t-4}^{t} h_{iu} a_{t-u}$. Coefficients b_u and a_{t-u} depend on constants found by a nonlinear solver, minimizing total squared error to actual head. The number of terms for recharge and previous heads (in this case, 3 and 5 respectively) is chosen to select a level of admissible error. Often, one term is sufficient for an error of 1% of pump rate. We used 10 terms, as calculation time is short. Coefficients depend on the aquifer, but are otherwise independent of the well location. We assume the aquifer is uniformly recharged over its whole surface area, so rainfall R_t is independent of well location. This is reasonable, as our model considers a week at a time. Thus, we can predict the head at each well, in each week, as a function of rainfall.

Similarly, we can use an ARIMA model to predict the low-flow base level of a given stream, as a function of the head in a given well. Combining equations for well head as a function of recharge, and stream level as a function of well head, we can calculate the change in the low-flow base level of the stream, given an abstraction (a negative recharge) at the well. Of course, all of these are time dependent. Selwyn AHM thus produces a response matrix F, where coefficient $f_{i,j,t}$ is the change in head at a control point j, in t periods after a unit abstraction at location i. The coefficients $f_{i,j,t}$ are necessary in our market LP.

The aquifer is assumed to be 45 km long. Its boundaries are the Rakaia River, the Waimakariri River, the foothills, and the coast. The standard eigenmodel treats wells as on a straight line on one side of the lowland stream control points. The aquifer is treated as two-dimensional, as its width is not considered; a well's effect on the coast is affected only by its distance to the coast perpendicularly. Ground water effects are superimposable and linear, so we introduced an effective distance from each well to each lowland stream. This effective distance controls the magnitude and timing of each well's effect on each control point. The use of this effective distance is reasonable if aquifer flow is sufficiently slow, so the cone of stress from each well is a circle, or at least an ellipse with low eccentricity. In a fast flow regime, the effective distance between control points and wells is longer for wells off-axis than for wells on axis. We assume a slow flow regime, due to the low transmissivity of the Canterbury Plains aquifer.

Selwyn AHM includes the 25 largest wells by volume in Selwyn District. The wells were chosen from a pool of about 1500 consents. These 25 consents account for about 15% of the total water consents issued in Selwyn District. Most consents are for irrigation, some are for domestic and stock supply, and three are public water supplies for the townships of Rolleston (well 7) and Leeston (wells 24 and 25). Each well is modelled as a point exerting an external stress on the aquifer.

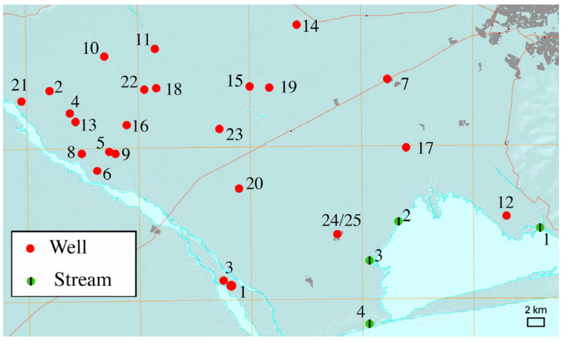


Figure 1. Map of wells and control points at streams. Map taken from NZTopoOnline.

The model has four lowland streams, Halswell River, Irwell River, Birdlings Brook River, and Waikewai Creek, used as environmental constraints. The constraints were associated with the mouth of each stream. Figure 1 shows the positions of the wells and environmental constraints. The streams were treated as fully penetrating. We assumed that the hydraulic connection between the aquifer and the stream is not restricted by semi-pervious layers and therefore the transmissivity constant assumed for the whole aquifer holds for this region too. This is conservative, as the model is likely to suggest that streams are more depleted than they actually would be, because the streams are only partially penetrating. A partially penetrating stream could be added, if its penetration coupling were known. Determination of the sensitivity of these assumptions would require considerable field work, which is beyond the scope of this paper.

2.2. Behaviour of Selwyn Aquifer

Figure 2 shows the impact that one day of pumping at a well has on the depletion at four different control points. A well that is close to a lowland stream has a relatively immediate and strong impact on the stream flow, while wells further away have less impact, though their impact can last for a longer time. The peak impact is lower with increasing well distance.

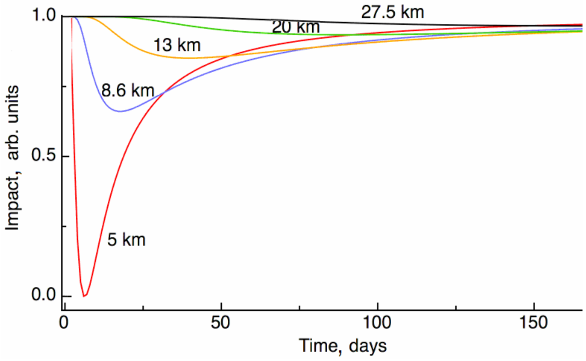


Figure 2. Impact of abstraction (peak normalized) on control points at five different distances.

The lowland streams are near the coast. Most wells are at a medium to long distance from the lowland streams. Some of their depletion impact will be carried into the wet season, when the aquifer is recharged. Each distant well has a small impact, but the cumulative effects contribute to stream depletion. The depletion timing impacts the market. The market model must account for the different impacts of different wells, and for the associated economic externalities.

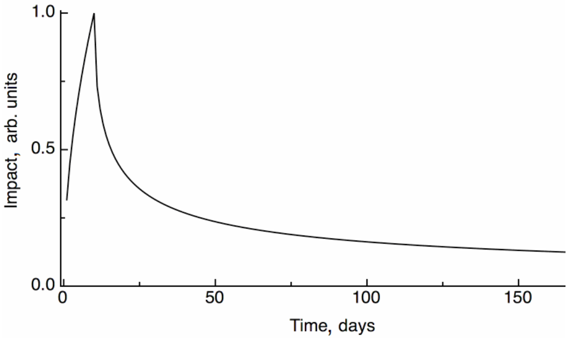


Figure 3. Impact of 1 day of rain on 1 control point.

The market model takes into account recharge from rain. Figure 3 shows the impact of one day rain on stream flow. Recharge is distributed over the whole aquifer, so the effect on stream flow has a short peak and a long decay. Similar to the abstraction dynamics shown in Figure 2, the peak impact is delayed by about 10 days following the rain. The impact then tapers off rapidly, but a low long term effect is observable for some time. Thus, Selwyn AHM calculates depletion at the mouth of each stream (the control point) in each future period, as a function of the abstraction at each well. It was programmed in Mathematica (Wolfram 2003). Inputs include the aquifer constants of transmissivity, specific storage and aquifer length, the positions of the wells and the control points. The model operates on a time interval which can be set to days, weeks, or months. Pumping or recharge is assumed to be a constant stress over this interval. The programme saves the calculated response matrix to a file, which is then used as data in the LP. The aquifer dynamics are thus inter-dependent. Farmers are not buying water from their neighbours. Rather, they are buying it from the environment and everyone in the catchment, because their pumping has an effect everywhere. In order to market to be perceived as fair, the market model must address this externality.

3. A DETERMINISTIC LP FOR A GROUND WATER SMART MARKET

The LP for the smart market is taken from Raffensperger & Read (2007). The LP was developed in the open source FLOPC++ code, and solved with the open source Coin-OR CLP package. Solution times are extremely short. For the bid data, we assumed market participants have the same price sensitivity as a percentage of their consents. Each trader's marginal value curve is approximated by five piecewise linear tranches:

Price	\$1.00	\$1.25	\$1.50	\$1.60	\$2.00	\$5.00
Quantity	200%	183%	150%	50%	33%	17%

Thus, for price ≤ \$1/Ml, we assume a user would buy an additional volume equal to his consent; for more than \$5/Ml, we assume a user would sell all their consent. We assumed identical water use profiles. In a real auction, each participant would set their own prices and volumes, and would trade according to their individual needs, crops, risk averseness, and future plans. These individual behaviours were outside the scope of our project. The market model allows trades in the future, over the full planning horizon *T*. At each auction, users may buy or sell consent in the firm spot market for the current period, but may also adjust their positions for future periods. This would mitigate users' risk by giving somewhat greater security of future supply, such as to finish a crop at season's end, or to sell if drought is of such severity that a crop would be lost. We assume that winter rains recharge the aquifer. Any deficit or surplus flow at the environmental constraints will be carried into the next trading period. Thus, a wet year will lower prices, and/or allow greater abstraction in the current year, and the next. A deficit will have the opposite effect as environmental constraints will be tighter.

3.1. Notation for the LP

Indices: *b*, bid, *b*=1,…, 5.
i, *j*, *k*, location, with *i*, *j*, *k* =1,…, *N*.
u, *t*=1,…, *T*, time periods. Period 1 is the present.

Parameters (UPPER CASE): N = # of wells,
C_{it} = initial quantity position of user i for period t.
As in hydrological convention, $C_{it} \leq 0$.
F_{ijt} = draw down response at control point i due to pumping at well j at period t,

LH_{it} = lower limit on head, control point i, time t.
UH_{it} = upper limit on head, control point i, time t.
P_{itb} = reservation price for bid b at well i, period t.

Q_{itb} = the bound on marginal bid b for water at well i, period t. If $Q_{itb} < 0$, then the offer is to sell. If $Q_{itb} > 0$, then the bid is to buy.

Decision variables (lower case):
bid_{itb} = amount of bid b accepted for well i, time t;
p_{it} = market price/ unit of water at well i, period t. This is the dual price on constraint 0 below;
$q_{it} \leq 0$, abstraction rate at well i, period t.

3.2. Model HydrologyNZ

The model is named in honour of the NZ Hydrological Society, which gave early funding for this research.

(1) Maximize consumer + producer surplus.
Maximize $\sum^{n}_{j=1}\sum^{T}_{t=1}\sum^{5}_{b=1} P_{itb}bid_{itb}$, subject to

(2) Bids are bounded by their marginal quantities.
Sell (buy) bids are negative (positive).
Sell bids: $Q_{itb} \leq bid_{itb} \leq 0$ for $Q_{itb} < 0$;
buy bids: $0 \leq bid_{itb} \leq Q_{itb}$ for $Q_{itb} > 0$, for bids b=1,…, 5, wells i=1,…, N, and periods t=1,…, T.

(3) Well abstraction (≤ 0) equals the consent (≤ 0) minus cleared sell/buy amounts. The dual variable p_{it} is the marginal value to the economy for another unit of water at well i, period t.
$q_{it} + \sum^{5}_{b=1} bid_{itb} = C_{it}$, for all wells i=1,…, N, and periods t=1,…, T. (Dual variable p_{it}.)

(4) Bounds on heads relative to the natural head. These bounds are the sustainable limits to extraction. Users' pump capacities are not included; users account for capacity in their bids.
$LH_{it} \leq \sum^{n}_{j=1} \sum^{t}_{u=1} F_{i,j,t-u+1}\, q_{ju} \leq UH_{it}$, for all control points i=1,…, N, and periods t=1,…, T.

4. RESULTS

4.1. Impact on time horizon

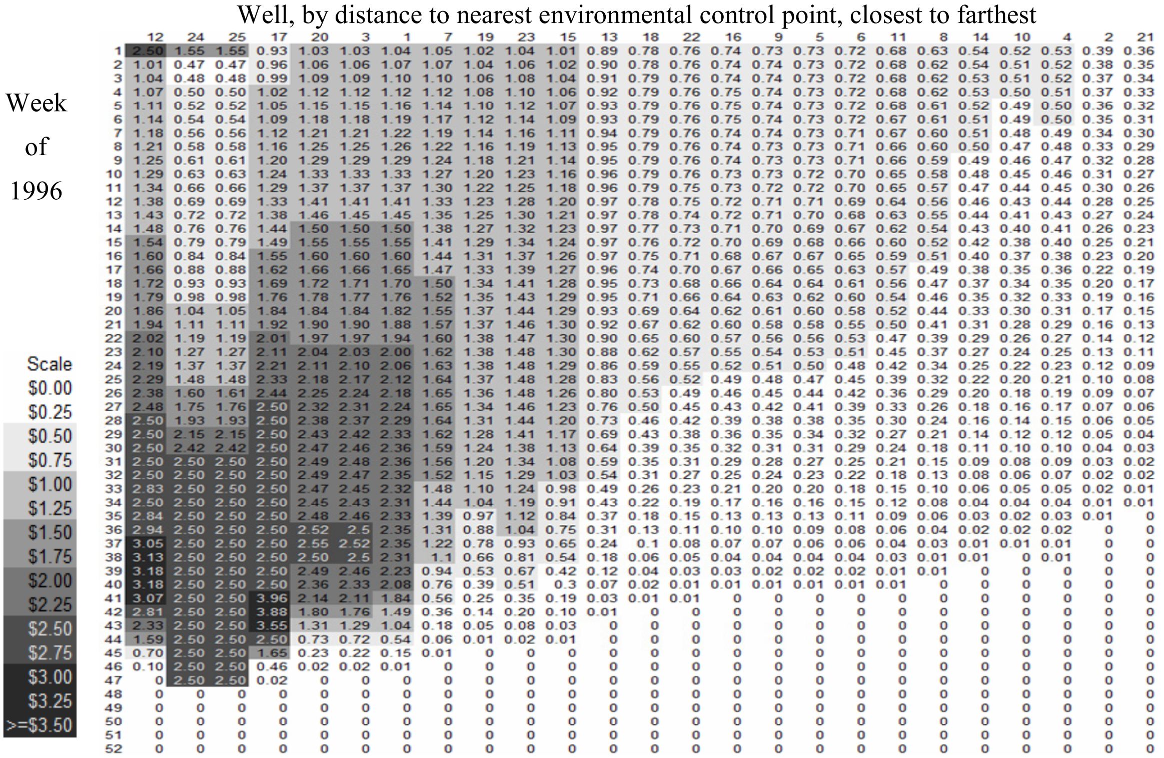
In the first runs of the model, we found that the length of the trading and the modelling periods were critical. We initially guessed that a 10-week trading horizon would be appropriate, but this short horizon allowed users to take as much water as they wished in the final weeks. The reason is due to the delay between abstraction and effect on the streams, and this delay is most pronounced for the wells that are relatively far away from the control points. The hydrology tends to diffuse impact of wells far from the streams, while impacts from wells close to the streams are almost immediate, as Figure 2 showed. Consider the effect of abstraction from a well that is close (e.g., 5 km) and from one that is further away (e.g., 23 km). While the near well shows immediate impact, the distant well has effect only after 10 periods, and the rate of change is slow. This delay would allow the distant well to buy water 10 weeks prior to the end of the trading period with no environmental impact. The distant well could buy an unrestricted volume, and the price would not reflect the impact on the environment. Figure 2 suggests that the peak impact typically occurs in much less than 10 weeks, but that is only from the abstraction in the first week of trading; the abstraction in the last week of trading would be virtually unconstrained. We therefore included environmental constraints for periods beyond the trading period, to ensure the true impact on the environment is taken into account. The Selwyn model has environmental constraints associated with the recharge period during the winter month, spanning 52 weeks.

4.2. Price dependence on weather, location

The different precipitation recharge and the different timing of those effects (as in Figure 2) results in dynamic prices, and different quantities that can be traded. The prices depend on the recharge, the environmental constraints, the distance of each well to the lowland streams, all the inter-related timings, and the bids. We ran market simulations with precipitation data for 1996, a wet year, and 1998, a dry year. For 1996, Figure 4 gives a table of prices. Shading indicates magnitude, and wells are ordered by distance to the closest control point. The figure shows that wells closest to the streams are most price sensitive, as they have immediate impact on lowland stream flow, e.g., Leeston wells 24 and 25. Well 12 has the highest price of $3.96/unit in week 41. For 1998, Figure 5 similarly gives a table of prices. This was a relatively dry year. Prices went higher than for the 1996 simulation. Well 12 had the highest price of $6.39 in week 45. As before, wells closer to the environmental control points are more price sensitive.

5. CONCLUSIONS

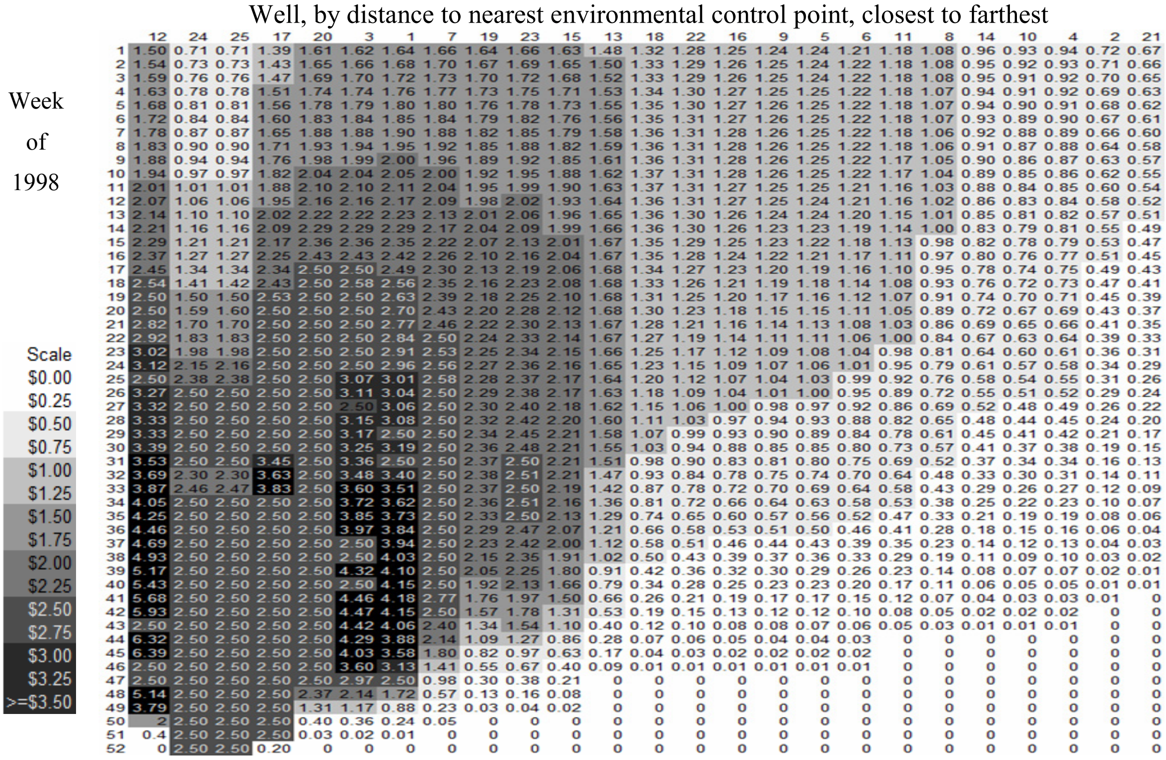
A smart market for ground water could be developed with the eigenmodel approach. It is simpler than the widely used MODFLOW method to hydrological optimisation. This dramatically reduces the computational effort required to operate a valid market. While our model includes only the externalities of flow depletion in lowland streams, additional constraints may be added easily. We further showed that the time horizon required for such a market depends heavily on the hydrology. For Selwyn District, the time horizon must be at least a year, to prevent end-of-horizon problems which would falsely imply that users could take a large amount of water without environmental impact.



Well, by distance to nearest environmental control point, closest to farthest

Week of 1996	12	24	25	17	20	3	1	7	19	23	15	13	18	22	16	9	5	6	11	8	14	10	4	2	21
1	2.50	1.55	1.55	0.93	1.03	1.03	1.04	1.05	1.02	1.04	1.01	0.89	0.78	0.76	0.74	0.73	0.73	0.72	0.68	0.63	0.54	0.52	0.53	0.39	0.36
2	1.01	0.47	0.47	0.96	1.06	1.06	1.07	1.07	1.04	1.06	1.02	0.90	0.78	0.76	0.74	0.74	0.73	0.72	0.68	0.62	0.54	0.51	0.52	0.38	0.35
3	1.04	0.48	0.48	0.99	1.09	1.09	1.10	1.10	1.06	1.08	1.04	0.91	0.79	0.76	0.74	0.74	0.73	0.72	0.68	0.62	0.53	0.51	0.52	0.37	0.34
4	1.07	0.50	0.50	1.02	1.12	1.12	1.12	1.12	1.08	1.10	1.06	0.92	0.79	0.76	0.75	0.74	0.73	0.72	0.68	0.62	0.53	0.50	0.51	0.37	0.33
5	1.11	0.52	0.52	1.05	1.15	1.15	1.16	1.14	1.10	1.12	1.07	0.93	0.79	0.76	0.75	0.74	0.73	0.72	0.68	0.61	0.52	0.49	0.50	0.36	0.32
6	1.14	0.54	0.54	1.09	1.18	1.18	1.19	1.17	1.12	1.14	1.09	0.93	0.79	0.76	0.75	0.74	0.73	0.72	0.67	0.61	0.51	0.49	0.50	0.35	0.31
7	1.18	0.56	0.56	1.12	1.21	1.21	1.22	1.19	1.14	1.16	1.11	0.94	0.79	0.76	0.74	0.74	0.73	0.71	0.67	0.60	0.51	0.48	0.49	0.34	0.30
8	1.21	0.58	0.58	1.16	1.25	1.25	1.26	1.22	1.16	1.19	1.13	0.95	0.79	0.76	0.74	0.73	0.73	0.71	0.66	0.60	0.50	0.47	0.48	0.33	0.29
9	1.25	0.61	0.61	1.20	1.29	1.29	1.29	1.24	1.18	1.21	1.14	0.95	0.79	0.76	0.74	0.73	0.73	0.71	0.66	0.59	0.49	0.46	0.47	0.32	0.28
10	1.29	0.63	0.63	1.24	1.33	1.33	1.33	1.27	1.20	1.23	1.16	0.96	0.79	0.76	0.73	0.73	0.72	0.70	0.65	0.58	0.48	0.45	0.46	0.31	0.27
11	1.34	0.66	0.66	1.29	1.37	1.37	1.37	1.30	1.22	1.25	1.18	0.96	0.79	0.75	0.73	0.72	0.72	0.70	0.65	0.57	0.47	0.44	0.45	0.30	0.26
12	1.38	0.69	0.69	1.33	1.41	1.41	1.41	1.33	1.23	1.28	1.20	0.97	0.78	0.75	0.72	0.71	0.71	0.69	0.64	0.56	0.46	0.43	0.44	0.28	0.25
13	1.43	0.72	0.72	1.38	1.46	1.45	1.45	1.35	1.25	1.30	1.21	0.97	0.78	0.74	0.72	0.71	0.70	0.68	0.63	0.55	0.44	0.41	0.43	0.27	0.24
14	1.48	0.76	0.76	1.44	1.50	1.50	1.50	1.38	1.27	1.32	1.23	0.97	0.77	0.73	0.71	0.70	0.69	0.67	0.62	0.54	0.43	0.40	0.41	0.26	0.23
15	1.54	0.79	0.79	1.49	1.55	1.55	1.55	1.41	1.29	1.34	1.24	0.97	0.76	0.72	0.70	0.69	0.68	0.66	0.60	0.52	0.42	0.38	0.40	0.25	0.21
16	1.60	0.84	0.84	1.55	1.60	1.60	1.60	1.44	1.31	1.37	1.26	0.97	0.75	0.71	0.68	0.67	0.67	0.65	0.59	0.51	0.40	0.37	0.38	0.23	0.20
17	1.66	0.88	0.88	1.62	1.66	1.66	1.65	1.47	1.33	1.39	1.27	0.96	0.74	0.70	0.67	0.66	0.65	0.63	0.57	0.49	0.38	0.35	0.36	0.22	0.19
18	1.72	0.93	0.93	1.69	1.72	1.71	1.70	1.50	1.34	1.41	1.28	0.95	0.73	0.68	0.66	0.64	0.64	0.61	0.56	0.47	0.37	0.34	0.35	0.20	0.17
19	1.79	0.98	0.98	1.76	1.78	1.77	1.76	1.52	1.35	1.43	1.29	0.95	0.71	0.66	0.64	0.63	0.62	0.60	0.54	0.46	0.35	0.32	0.33	0.19	0.16
20	1.86	1.04	1.05	1.84	1.84	1.84	1.82	1.55	1.37	1.44	1.29	0.93	0.69	0.64	0.62	0.61	0.60	0.58	0.52	0.44	0.33	0.30	0.31	0.17	0.15
21	1.94	1.11	1.11	1.92	1.90	1.90	1.88	1.57	1.37	1.46	1.30	0.92	0.67	0.62	0.60	0.58	0.58	0.55	0.50	0.41	0.31	0.28	0.29	0.16	0.13
22	2.02	1.19	1.19	2.01	1.97	1.97	1.94	1.60	1.38	1.47	1.30	0.90	0.65	0.60	0.57	0.56	0.56	0.53	0.47	0.39	0.29	0.26	0.27	0.14	0.12
23	2.10	1.27	1.27	2.11	2.04	2.03	2.00	1.62	1.38	1.48	1.30	0.88	0.62	0.57	0.55	0.54	0.53	0.51	0.45	0.37	0.27	0.24	0.25	0.13	0.11
24	2.19	1.37	1.37	2.21	2.11	2.10	2.06	1.63	1.38	1.48	1.29	0.86	0.59	0.55	0.52	0.51	0.50	0.48	0.42	0.34	0.25	0.22	0.23	0.12	0.09
25	2.29	1.48	1.48	2.33	2.18	2.17	2.12	1.64	1.37	1.48	1.28	0.83	0.56	0.52	0.49	0.48	0.47	0.45	0.39	0.32	0.22	0.20	0.21	0.10	0.08
26	2.38	1.60	1.61	2.44	2.25	2.24	2.18	1.65	1.36	1.48	1.26	0.80	0.53	0.49	0.46	0.45	0.44	0.42	0.36	0.29	0.20	0.18	0.19	0.09	0.07
27	2.48	1.75	1.76	2.50	2.32	2.31	2.24	1.65	1.34	1.46	1.23	0.76	0.50	0.45	0.43	0.42	0.41	0.39	0.33	0.26	0.18	0.16	0.17	0.07	0.06
28	2.50	1.93	1.93	2.50	2.38	2.37	2.29	1.64	1.31	1.44	1.20	0.73	0.46	0.42	0.39	0.38	0.38	0.35	0.30	0.24	0.16	0.14	0.15	0.06	0.05
29	2.50	2.15	2.15	2.50	2.43	2.42	2.33	1.62	1.28	1.41	1.17	0.69	0.43	0.38	0.36	0.35	0.34	0.32	0.27	0.21	0.14	0.12	0.12	0.05	0.04
30	2.50	2.42	2.42	2.50	2.47	2.46	2.36	1.59	1.24	1.38	1.13	0.64	0.39	0.35	0.32	0.31	0.31	0.29	0.24	0.18	0.11	0.10	0.10	0.04	0.03
31	2.50	2.50	2.50	2.50	2.49	2.48	2.36	1.56	1.20	1.34	1.08	0.59	0.35	0.31	0.29	0.28	0.27	0.25	0.21	0.15	0.09	0.08	0.09	0.03	0.02
32	2.50	2.50	2.50	2.50	2.49	2.47	2.35	1.52	1.15	1.29	1.03	0.54	0.31	0.27	0.25	0.24	0.23	0.22	0.18	0.13	0.08	0.06	0.07	0.02	0.02
33	2.83	2.50	2.50	2.50	2.47	2.45	2.32	1.48	1.10	1.24	0.98	0.49	0.26	0.23	0.21	0.20	0.20	0.18	0.15	0.10	0.06	0.05	0.05	0.02	0.01
34	2.50	2.50	2.50	2.50	2.45	2.43	2.31	1.44	1.04	1.19	0.91	0.43	0.22	0.19	0.17	0.16	0.16	0.15	0.12	0.08	0.04	0.04	0.04	0.01	0.01
35	2.84	2.50	2.50	2.50	2.48	2.46	2.33	1.39	0.97	1.12	0.84	0.37	0.18	0.15	0.13	0.13	0.13	0.11	0.09	0.06	0.03	0.02	0.03	0.01	0
36	2.94	2.50	2.50	2.50	2.52	2.5	2.35	1.31	0.88	1.04	0.75	0.31	0.13	0.11	0.10	0.10	0.09	0.08	0.06	0.04	0.02	0.02	0.02	0	0
37	3.05	2.50	2.50	2.50	2.55	2.52	2.35	1.22	0.78	0.93	0.65	0.24	0.1	0.08	0.07	0.07	0.06	0.06	0.04	0.03	0.01	0.01	0.01	0	0
38	3.13	2.50	2.50	2.50	2.50	2.5	2.31	1.1	0.66	0.81	0.54	0.18	0.06	0.05	0.04	0.04	0.04	0.04	0.03	0.01	0.01	0	0.01	0	0
39	3.18	2.50	2.50	2.50	2.49	2.46	2.23	0.94	0.53	0.67	0.42	0.12	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0	0	0	0	0
40	3.18	2.50	2.50	2.50	2.36	2.33	2.08	0.76	0.39	0.51	0.3	0.07	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0	0
41	3.07	2.50	2.50	3.96	2.14	2.11	1.84	0.56	0.25	0.35	0.19	0.03	0.01	0.01	0	0	0	0	0	0	0	0	0	0	0
42	2.81	2.50	2.50	3.88	1.80	1.76	1.49	0.36	0.14	0.20	0.10	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0
43	2.33	2.50	2.50	3.55	1.31	1.29	1.04	0.18	0.05	0.08	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	1.59	2.50	2.50	2.50	0.73	0.72	0.54	0.06	0.01	0.02	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0.70	2.50	2.50	1.65	0.23	0.22	0.15	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0.10	2.50	2.50	0.46	0.02	0.02	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	2.50	2.50	0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 4. Price dynamics for 1996, a relatively wet year. Well numbers match Figure 1.



Well, by distance to nearest environmental control point, closest to farthest

Week of 1998	12	24	25	17	20	3	1	7	19	23	15	13	18	22	16	9	5	6	11	8	14	10	4	2	21
1	1.50	0.71	0.71	1.39	1.61	1.62	1.64	1.66	1.64	1.66	1.63	1.48	1.32	1.28	1.25	1.24	1.24	1.21	1.18	1.08	0.96	0.93	0.94	0.72	0.67
2	1.54	0.73	0.73	1.43	1.65	1.66	1.68	1.70	1.67	1.69	1.65	1.50	1.33	1.29	1.26	1.25	1.24	1.22	1.18	1.08	0.95	0.92	0.93	0.71	0.66
3	1.59	0.76	0.76	1.47	1.69	1.70	1.72	1.73	1.70	1.72	1.68	1.52	1.33	1.29	1.26	1.25	1.24	1.22	1.18	1.08	0.95	0.91	0.92	0.70	0.65
4	1.63	0.78	0.78	1.51	1.74	1.74	1.76	1.77	1.73	1.75	1.71	1.53	1.34	1.30	1.27	1.25	1.25	1.22	1.18	1.07	0.94	0.91	0.92	0.69	0.63
5	1.68	0.81	0.81	1.56	1.78	1.79	1.80	1.80	1.76	1.78	1.73	1.55	1.35	1.30	1.27	1.26	1.25	1.22	1.18	1.07	0.94	0.90	0.91	0.68	0.62
6	1.72	0.84	0.84	1.60	1.83	1.84	1.85	1.84	1.79	1.82	1.76	1.56	1.35	1.31	1.27	1.26	1.25	1.22	1.18	1.07	0.93	0.89	0.90	0.67	0.61
7	1.78	0.87	0.87	1.65	1.88	1.88	1.90	1.88	1.82	1.85	1.79	1.58	1.36	1.31	1.28	1.26	1.25	1.22	1.18	1.06	0.92	0.88	0.89	0.66	0.60
8	1.83	0.90	0.90	1.71	1.93	1.94	1.95	1.92	1.85	1.88	1.82	1.59	1.36	1.31	1.28	1.26	1.25	1.22	1.18	1.06	0.91	0.87	0.88	0.64	0.58
9	1.88	0.94	0.94	1.76	1.98	1.99	2.00	1.96	1.89	1.92	1.85	1.61	1.36	1.31	1.28	1.26	1.25	1.22	1.17	1.05	0.90	0.86	0.87	0.63	0.57
10	1.94	0.97	0.97	1.82	2.04	2.04	2.05	2.00	1.92	1.95	1.88	1.62	1.37	1.31	1.28	1.26	1.25	1.22	1.17	1.04	0.89	0.85	0.86	0.62	0.55
11	2.01	1.01	1.01	1.88	2.10	2.10	2.11	2.04	1.95	1.99	1.90	1.63	1.37	1.31	1.27	1.25	1.25	1.21	1.16	1.03	0.88	0.84	0.85	0.60	0.54
12	2.07	1.06	1.06	1.95	2.16	2.16	2.17	2.09	1.98	2.02	1.93	1.64	1.36	1.31	1.27	1.25	1.24	1.21	1.16	1.02	0.86	0.83	0.84	0.58	0.52
13	2.14	1.10	1.10	2.02	2.22	2.22	2.23	2.13	2.01	2.06	1.96	1.65	1.36	1.30	1.26	1.24	1.24	1.20	1.15	1.01	0.85	0.81	0.82	0.57	0.51
14	2.21	1.16	1.16	2.09	2.29	2.29	2.29	2.17	2.04	2.09	1.99	1.66	1.36	1.30	1.26	1.23	1.23	1.19	1.14	1.00	0.83	0.79	0.81	0.55	0.49
15	2.29	1.21	1.21	2.17	2.36	2.36	2.35	2.22	2.07	2.13	2.01	1.67	1.35	1.29	1.25	1.23	1.22	1.18	1.13	0.98	0.82	0.78	0.79	0.53	0.47
16	2.37	1.27	1.27	2.25	2.43	2.43	2.42	2.26	2.10	2.16	2.04	1.67	1.35	1.28	1.24	1.22	1.21	1.17	1.11	0.97	0.80	0.76	0.77	0.51	0.45
17	2.45	1.34	1.34	2.34	2.50	2.50	2.49	2.30	2.13	2.19	2.06	1.68	1.34	1.27	1.23	1.20	1.19	1.16	1.10	0.95	0.78	0.74	0.75	0.49	0.43
18	2.54	1.41	1.42	2.43	2.50	2.58	2.56	2.35	2.16	2.23	2.08	1.68	1.33	1.26	1.21	1.19	1.18	1.14	1.08	0.93	0.76	0.72	0.73	0.47	0.41
19	2.50	1.50	1.50	2.53	2.50	2.50	2.63	2.39	2.18	2.25	2.10	1.68	1.31	1.25	1.20	1.17	1.16	1.12	1.07	0.91	0.74	0.70	0.71	0.45	0.39
20	2.50	1.59	1.60	2.50	2.50	2.50	2.70	2.43	2.20	2.28	2.12	1.68	1.30	1.23	1.18	1.15	1.15	1.11	1.05	0.89	0.72	0.67	0.69	0.43	0.37
21	2.82	1.70	1.70	2.50	2.50	2.50	2.77	2.46	2.22	2.30	2.13	1.67	1.28	1.21	1.16	1.14	1.13	1.08	1.03	0.86	0.69	0.65	0.66	0.41	0.35
22	2.92	1.83	1.83	2.50	2.50	2.50	2.84	2.50	2.24	2.33	2.14	1.67	1.27	1.19	1.14	1.11	1.11	1.06	1.00	0.84	0.67	0.63	0.64	0.39	0.33
23	3.02	1.98	1.98	2.50	2.50	2.50	2.91	2.53	2.25	2.34	2.15	1.66	1.25	1.17	1.12	1.09	1.08	1.04	0.98	0.81	0.64	0.60	0.61	0.36	0.31
24	3.12	2.15	2.16	2.50	2.50	2.50	2.96	2.56	2.27	2.36	2.16	1.65	1.23	1.15	1.09	1.07	1.06	1.01	0.95	0.79	0.61	0.57	0.58	0.34	0.29
25	2.50	2.38	2.38	2.50	2.50	3.07	3.01	2.58	2.28	2.37	2.17	1.64	1.20	1.12	1.07	1.04	1.03	0.99	0.92	0.76	0.58	0.54	0.55	0.31	0.26
26	3.27	2.50	2.50	2.50	2.50	3.11	3.04	2.50	2.29	2.38	2.17	1.63	1.18	1.09	1.04	1.01	1.00	0.95	0.89	0.72	0.55	0.51	0.52	0.29	0.24
27	3.32	2.50	2.50	2.50	2.50	2.50	3.06	2.50	2.30	2.40	2.18	1.62	1.15	1.06	1.00	0.98	0.97	0.92	0.86	0.69	0.52	0.48	0.49	0.26	0.22
28	3.33	2.50	2.50	2.50	2.50	3.15	3.08	2.50	2.32	2.42	2.20	1.60	1.11	1.03	0.97	0.94	0.93	0.88	0.82	0.65	0.48	0.44	0.45	0.24	0.20
29	3.33	2.50	2.50	2.50	2.50	3.17	2.50	2.50	2.34	2.45	2.21	1.58	1.07	0.99	0.93	0.90	0.89	0.84	0.78	0.61	0.45	0.41	0.42	0.21	0.17
30	3.39	2.50	2.50	2.50	2.50	3.25	3.19	2.50	2.36	2.48	2.21	1.55	1.03	0.94	0.88	0.85	0.85	0.80	0.73	0.57	0.41	0.37	0.38	0.19	0.15
31	3.53	2.50	2.50	3.45	2.50	3.36	2.50	2.50	2.37	2.50	2.21	1.51	0.98	0.90	0.83	0.81	0.80	0.75	0.69	0.52	0.37	0.34	0.34	0.16	0.13
32	3.69	2.30	2.30	3.63	2.50	3.48	3.40	2.50	2.38	2.51	2.21	1.47	0.93	0.84	0.78	0.75	0.74	0.70	0.64	0.48	0.33	0.30	0.31	0.14	0.11
33	3.87	2.46	2.47	3.83	2.50	3.60	3.51	2.50	2.37	2.50	2.19	1.42	0.87	0.78	0.72	0.70	0.69	0.64	0.58	0.43	0.29	0.26	0.27	0.12	0.09
34	4.05	2.50	2.50	2.50	2.50	3.72	3.62	2.50	2.36	2.51	2.16	1.36	0.81	0.72	0.66	0.64	0.63	0.58	0.53	0.38	0.25	0.22	0.23	0.10	0.07
35	4.25	2.50	2.50	2.50	2.50	3.85	3.73	2.50	2.33	2.50	2.13	1.29	0.74	0.65	0.60	0.57	0.56	0.52	0.47	0.33	0.21	0.19	0.19	0.08	0.06
36	4.46	2.50	2.50	2.50	2.50	3.97	3.84	2.50	2.29	2.47	2.07	1.21	0.66	0.58	0.53	0.51	0.50	0.46	0.41	0.28	0.18	0.15	0.16	0.06	0.04
37	4.69	2.50	2.50	2.50	2.50	2.50	3.94	2.50	2.23	2.42	2.00	1.12	0.58	0.51	0.46	0.44	0.43	0.39	0.35	0.23	0.14	0.12	0.13	0.04	0.03
38	4.93	2.50	2.50	2.50	2.50	2.50	4.03	2.50	2.15	2.35	1.91	1.02	0.50	0.43	0.39	0.37	0.36	0.33	0.29	0.19	0.11	0.09	0.10	0.03	0.02
39	5.17	2.50	2.50	2.50	2.50	4.32	4.10	2.50	2.05	2.25	1.80	0.91	0.42	0.36	0.32	0.30	0.29	0.26	0.23	0.14	0.08	0.07	0.07	0.02	0.01
40	5.43	2.50	2.50	2.50	2.50	2.50	4.15	2.50	1.92	2.13	1.66	0.79	0.34	0.28	0.25	0.23	0.23	0.20	0.17	0.11	0.06	0.05	0.05	0.01	0.01
41	5.68	2.50	2.50	2.50	2.50	4.46	4.18	2.77	1.76	1.97	1.50	0.66	0.26	0.21	0.19	0.17	0.17	0.15	0.12	0.07	0.04	0.03	0.03	0.01	0
42	5.93	2.50	2.50	2.50	2.50	4.47	4.15	2.50	1.57	1.78	1.31	0.53	0.19	0.15	0.13	0.12	0.12	0.10	0.08	0.05	0.02	0.02	0.02	0	0
43	2.50	2.50	2.50	2.50	2.50	4.42	4.06	2.40	1.34	1.54	1.10	0.40	0.12	0.10	0.08	0.08	0.07	0.06	0.05	0.03	0.01	0.01	0.01	0	0
44	6.32	2.50	2.50	2.50	2.50	4.29	3.88	2.14	1.09	1.27	0.86	0.28	0.07	0.06	0.05	0.04	0.04	0.03	0	0	0	0	0	0	0
45	6.39	2.50	2.50	2.50	2.50	4.03	3.58	1.80	0.82	0.97	0.63	0.17	0.04	0.03	0.02	0.02	0.02	0.02	0	0	0	0	0	0	0
46	2.50	2.50	2.50	2.50	2.50	3.60	3.13	1.41	0.55	0.67	0.40	0.09	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0	0	0
47	2.50	2.50	2.50	2.50	2.50	2.97	2.50	0.98	0.30	0.38	0.21	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	5.14	2.50	2.50	2.50	2.37	2.14	1.72	0.57	0.13	0.16	0.08	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	3.79	2.50	2.50	2.50	1.31	1.17	0.88	0.23	0.03	0.04	0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	2	2.50	2.50	2.50	0.40	0.36	0.24	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0.4	2.50	2.50	2.50	0.03	0.02	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	2.50	2.50	0.20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5. Price dynamics for 1998, a relatively dry year.

Well prices depend on the available water in the lowland streams and the proximity of a well to the lowland streams. The market model, in conjunction with the hydrology model, tracks changes in the flow of lowland streams, and assigns prices to each well depending on the impact of the well. The model therefore incorporates the environmental externality, and provides a measure of this impact via the price mechanism.

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