Graphical Models of Multivariate Volatility

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ABSTRACT

In order to understand volatility transmission between financial assets a multivariate model is essential. This paper looks at using graphical modelling to study volatility transmission. Graphical modelling is a technique that objectively test all potential influences on an index from its own past and other indices. The influences of the other indices can be contemporaneous. The results of graphical modelling are compared to the standard econometric tool for measuring multivariate volatility, the multivariate BEKK-GARCH model. The data used for this investigation is the daily closes of the Standard and Poor's 500 Composite Index (S&P 500), FTSE 100 and Nikkei 225. The period of investigation is from 3 April 2001 to 31 March 2005. The three stock indices are widely followed and over a 24 hour period and there is little overlap in trading hours. The Dickey-Fuller and Phillips-Perron tests confirm that for all three series that the log returns are first order stationary and the index prices are not stationary in mean. JEL CLASSIFICATIONS: C22, C32, C51

1 GRAPHICAL MODELLING

Graphical models in a time series context aim to find causal links between past and present observations. In addition, it also allows the study of causality among contemporaneous variables, that is, variables measured at a single point in time. Graphical modelling can be applied vector autoregressive moving average models but here the interest is restricted to vector autoregressive (VAR) models. The general form of a VAR(p) model (5) is

$$x_t = c + \Phi_1 x_{t-1} + \Phi_1 x_{t-2} + \ldots + \\ +\Phi_p x_{t-p} + \epsilon_t \qquad (1)$$

where x_t is a m x 1 vector of variables measured at time t. Also, ϵ_t is assumed to be $NID \sim (\mathbf{0}, \mathbf{\Omega})$ where $\mathbf{\Omega}$ is a general covariance matrix.

The equation 1 does not allow for contemporaneous dependence. One way of allowing contemporaneous dependence is to multiply both sides of the equation by a matrix Φ_0 giving

$$\Phi_0 x_t = d + \Phi_1^* x_{t-1} + \Phi_2^* x_{t-2} + \ldots + \\ +\Phi_p^* x_{t-p} + a_t \qquad (2)$$

There are two restrictions applied to equation 2 (7). The first is that the variance matrix of $a_t = \Phi_0 \epsilon_t$ is diagonal and the second that Φ_0 is upper triangular with a unit diagonal. Φ_0 represents the causal dependence of each variable on its contemporaneous counterparts.

In this context each causal relationship for the variance is considered a volatility channel with the squared log returns acting as proxy for variance. This means the appropriate underlying model for squared log returns is a multivariate ARCH model.

To better understand the model being fitted first consider the unconstrained multivariate ARCH model (4).

$$\begin{aligned} E(Y_t|Y_{t-1}) &= 0 \\ Var(Y_t|Y_{t-1}) &= H_t \end{aligned}$$

where the elements of H_t are affine functions of squared lagged observation.

The volatility equation is

$$H_t = c + AY_{t-1}Y_{t-1}'$$

These equations allow volatilities and covolatilities to be modelled using past squared values, $Y_{i,t-1}^2$, and cross-products, $Y_{i,t-1}Y_{j,t-1}, i \neq j$. This model requires a large number of parameters, hence the development of GARCH and MGARCH models.

A multivariate ARCH model is required with p lags and by extension this is given by

$$\begin{aligned} H_t &= c + A_1 Y_{t-1} Y_{t-1}' + A_2 Y_{t-2} Y_{t-2}' + \\ &\quad + \ldots + +A_p Y_{t-p} Y_{t-p}' \\ &= c + \sum_{i=1}^{p} A_i Y_{t-i} Y_{t-i}' \qquad (3) \end{aligned}$$

A comparison of equation 3 with 1 shows the two models are very similar. Setting $\Phi_i = A_i$ and $x_{t-i} = Y_{t-i}Y_{t-i}'$ reduces the multivariate ARCH equations to VAR equations.

With graphical modelling there are fewer parameters than a multivariate ARCH model and the model can allow for p lags. The information contained in the graphical model will be the same as that of the unconditional ARCH.

Graphical modelling creates a conditional independence graph (CIG) on which the edges represent significant partial correlations. Creating the CIG requires finding the order of the model, calculating the partial correlations on a set of admissible edges and determining which ones have significant partial correlation values. The optimal graph is found by removing edges in a stepwise procedure and testing their removal using an information criterion. Using a simple rule the causality is time dependant causal directions can be put on all the edges of the graph to give a directed acyclic graph (DAG).

The zero partial correlations indicate that the two variables are independant given all of the other variables.

In a time series context, variables are each of the series at time t and the previous p points in time, where p is the order of the VAR. Admissible edges in a graphical model are those between the present t and the past $t-1, \ldots, t-p$. Contemporaneous VAR models require the addition of edges between each of the variable at t.

2 BEKK MODEL

Multivariate GARCH models allow the study of not only volatilities but also covolatilities.

The multivariate GARCH models have the form

$$a_t = \sqrt{H_t(\theta)}\epsilon_t$$

where H_t is a positive definite matrix of conditional variances and co-variances and the random vector ϵ_t is distributed such that $E(\epsilon_t) = 0$ and $Var(\epsilon_t) = I$ the identity matrix. The models differ in how they specify H_t.

The BEKK-GARCH(1,1) model for takes the form

$$\begin{aligned} a_t &= \sqrt{H_t}\epsilon_t \\ H_t &= \Omega'\Omega + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B \end{aligned}$$

where Ω, A and B are N x N matrices. As N is the number of indices $N = 3$.

There are several restrictions on the parameters required to ensure that H_t is positive definite. For the GARCH(1,1). (1) prove that Ω is restricted to being upper triangular with positive diagonal elements and the first element of the matrices A and B must be positive. The individual volatility equations h_{ijt} are determined by multiplying the matrices.

Kevin Sheppard has developed MATLAB code to estimate the MGARCH models (3). The outputs of interest are the parameter estimates and their standard errors. These can be used to create a confidence interval. If this interval contains zero then the interpretation in this financial integration framework is that the volatility does not travel through that passage.

3 RESULTS

The following is the results of fitting a graphical model and a BEKK model to the three indices. The squared log returns act as a proxy for the second moment.

There are few underlying assumptions when fitting a graphical model. One is that the residuals are white noise. This differs significantly from GARCH models were the first moment is said to follow a random walk and restrictions were placed on the correlation structure.

The information criterion suggested that the model order should be $p = 3$. Therefore a saturated model for the squared log returns has twelve vertices, and thirty edges. Each contemporaneous variable is linked to all of the non-contemporaneous variables and the three contemporaneous variables are also linked.

The edges which have significant partial correlations are represented by the graph in Figure 1.

This data has an explicit time structure. Within any given day the Tokyo stock exchange closes first then the London stock exchange followed by New York close. This explicit structure leads to only two possible subgraphs as shown in Figure 2.

The information citerion suggested the saturated contemporaneous model fit the data best.

The initial model contains only the edges with

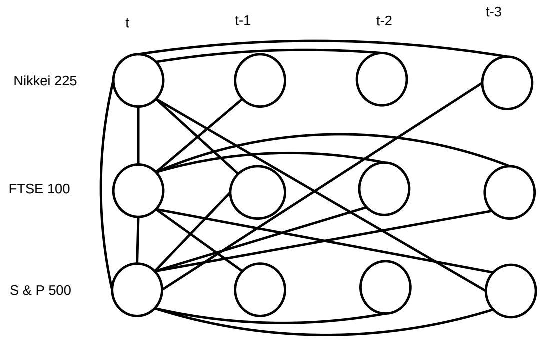

Figure 1. The CIG for the squared log returns

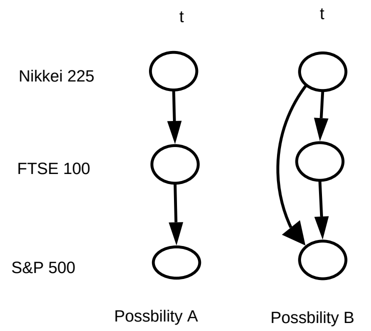

Figure 2. The two alternative DAG's for the contemporaneous variables

significant partial correlations. A stepwise selection process removes edges one at a time for the initial model if the information criterion shows that this improves the model. Improvement is measured relative to the saturated model.

The final model is given below in 3.

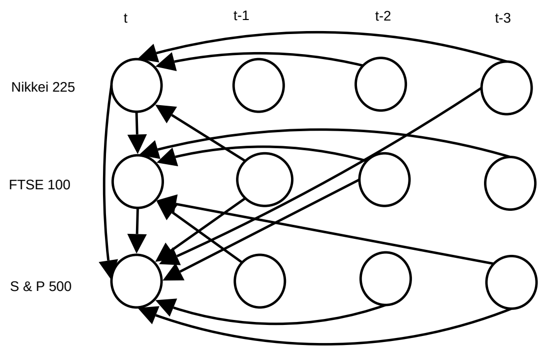

Figure 3. The DAG for the squared log returns

The interpretation of this model is complex. For each series t is effected by $t-2$ and $t-3$ but not $t-1$. This is unusual behaviour. Usually distant past has a lesser effect than recent past. The Nikkei 225 effects the FTSE 100 and the S&P 500. The FTSE 100 also effects the S&P 500.

The squared return of the FTSE 100 at $t-1$ effects the square log returns of both the Nikkei 225 at t while the S&P 500 at $t-1$ do not. This implies that the volatility channel between London and Tokyo is open in both directions. The volatility channel between New York and Tokyo only allows volatility to spillover from the US to Japan and not the other way around.

The volatility channel between the US and the UK also is open in both directions. This can be seen from the contemporaneous relationship between the two indices and the significant correlation between S&P 500 at $t-1$ and the FTSE 100 at t.

As the underlying multivariate ARCH model is similar to a VAR the graphical model can be interpreted in the same way as other graphical models. The difference is a causal link is called volatility channel. The model will have the pitfalls of the multivariate ARCH models in that a larger number of the lags will often be required. The advantage is the ability to study relationships between the models in a causal framework.

(6) studied volatility transmission of real interest rates in the G7 economies. The approach taken here extends the pairwise consideration of the interest rates to three series. A similar methodology of assessing volatility transmission is applied here to the three series. ??? re read paper

Inspection of 4 shows that volatility channels are determined by the elements of the matrix B, the β_{ij}'s. The only β values which differ significantly from zero are β_{11}, β_{22} and β_{33}.

Of the three channels the BEKK(1,1) finds to be active, all of them are yesterday's volatility of the index effecting today's volatility of that index. This happens for all the indices namely the FTSE 100, Standard and Poors 500 Composite (S&P 500) and the Nikkei 225 index.

When comparing the interpretation of the BEKK model and the graphical model for the squared returns the concept of marginalisation is required. The BEKK model used only a single time lag while the graphical model had three. The graphical model has suggested a much greater number of volatility channels are open as the BEKK model found only three volatility channels Japan-Japan, UK-UK and US-US.

The graphical model found additional channels. If considering lags 1,2 and 3 past then the volatility channels are Tokyo-US, Tokyo-UK UK-US, UK-Tokyo, US-UK, US-US, UK-UK and Tokyo-Tokyo. The Tokyo-UK relationship only shows up in the study of contemperaneous relationships.

The Jarque-Bera test was used to assess the residuals of both models to see if they resembled white noise. The results show that neither model gives rise to white noise residuals.

4 CONCLUSIONS

This paper has studied volatility transmission using Graphical modelling and GARCH models. Computationally fitting a graphical model to the squared log returns was the same as fitting any other graphical model for a vector of time series.

The resulting graphical model was complex and offered some surprising interpretations. The association between past volatility and presently observed volatility of a single series was found to be from two and three days prior but not a single day prior. This implies a lag period before the effects of the past influence the present.

The BEKK model had only the relationship between $t-1$ and t to convey the relationship between the past and present volatility. This resulted in the BEKK model forcing information into a single time lag. (2) discuss this concept.

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