Comparisons of Transient Analytical Methods for Determining Hydraulic Conductivity Using Disc Permeameters

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EXTENDED ABSTRACT

The disc permeameter is now a commonly used method for measuring the hydraulic properties of field soils. Most of the methods available for analysis of data from disc permeameters require the steady-state flow to be known. Recently methods for analysing the transient data during the earlier stages of infiltration into the soil from a disc permeameter (Vandervaere et al. 2000a,b). These methods require the differentiation of the data, which can introduce extra noise and uncertainty into the calculation procedure.

The early infiltration from a disc can be written as:

$$I = S\sqrt{t} + C_2 t \qquad (1)$$

where I is the cumulative infiltration [L], S is the sorptivity [L T$^{-1/2}$], t is time [T] and

$$C_2 = \frac{2-\beta}{3}k + \frac{\gamma S^2}{r(\theta_o - \theta_n)} \qquad (2)$$

where r is the radius of the supply surface [L], β is a constant in the interval (0,1), γ is a constant equal to 0.75, θ_o and θ_n are the final and initial volumetric water content [L^3 L^{-3}] respectively and k is the hydraulic conductivity [L T^{-1}]. Equation (1) has the same form as the two-term Philip infiltration equation. This allows use of an analysis method developed by Smiles and Knight (1976) (S&K).

Vandervaere et al. (2000a) suggest that differentiation (DI method) of eqn (1) is the best method to obtain C_1 and C_2. There are then a number of ways to obtain S and k, which are presented and compared in this paper.

One of the problems with the disc method is determining when infiltration into the sand pad ceases and infiltration into the soil commences. Different methods using visual estimation and the change in curvature seen in the DI and S&K methods are compared. It is shown that without taking this offset in time into account in eqn (1) incorrect values of S will be calculated.

Comparisons are made between S and k calculated from the C_1 and C_2 values generated with the three different methods, quadratic, DI and S&K. The differences are not significant but many more data points for k are lost when using the quadratic method. The DI method amplifies the noise in the experimental data and this reduces the regression coefficients and in certain data sets may result in an inability to determine C_1 and C_2.

Lastly we compare the S and k data generated by the three methods for the individual ring replicates and compare these with the published results of Cook and Broeren (1994) using the steady-state methods. There is no significant difference but the trend is for the S&K method to give higher values of S and the DI method higher values of k.

The transient methods should provide a rapid method for the measurement of soil hydraulic properties and the inverse methods offered here should help with analysis.

1. INTRODUCTION

The measurement of soil hydraulic properties, in particular hydraulic conductivity (k [L T^{-1}]) and sorptivity (S [L T$^{-1/2}$]), with disc permeameters has become popular since their introduction by Perroux and White (1988). The data derived from the discs of flow rate (q [L^3 T^{-1}]) with time (t [T]) can be analysed in a number of different ways to obtain values of k and S. Cook and Broeren (1994) compared six different methods In Queensland, Australia, a multi-tower bubbling tube disc permeameter has found favour. The multi-tower disc permeameter allows measurements at a number of different potentials (ψ [L]) to be made sequentially. Some difficulties have been experienced with analysis of disc permeameter data using this approach (J. Foley and D. McGarry pers. comm.) possibly due to steady-state flow not being attained before changing the potential.

Recently new methods of analysis based on transient flow conditions have become available. Vandervaere *et al*. (2000a,b) introduced a number of methods based on analytical solutions of the transient flow from a disc source. These methods will be used to reanalyse the data of Cook and Broeren (1994). These results will be compared and contrasted with the earlier results of Cook and Broeren (1994).

2. THEORY

The use of steady-state flow from a disc source into a homogenous, isotropic, uniformly unsaturated soil to derive the hydraulic properties has been reviewed by Cook and Broeren (1994). The attainment of steady-state can be a problem (Cook 1994, Foley et al. 2006) in some soils. Thus methods which use the transient flow during the initial wetting offer not only much less time to perform but also can avoid the problem of determining when steady-state is achieved.

2.1. Transient Flow Analytical Solution

Transient axisymmetric infiltration from a circular source at the soil surface has been describe by Vandervaere et al. (2000a) using a two-term equation:

$$I = C_1 \sqrt{t} + C_2 t \tag{3}$$

where I is the cumulative infiltration depth [L T^{-1}] and C_1 and C_2 are described by Haverkamp et al. (1994):

$$C_1 = S \tag{4}$$

where S is the capillary sorptivity [L T$^{-1/2}$], β is a constant in the interval (0,1), γ is a constant equal to 0.75, θ_o and θ_n are the final and initial volumetric water content [L^3 L^{-3}] respectively and C_2 is given in eqn (2) above.

Vandervaere et al. (2000a) suggest two different methods for analysing the infiltration data. The first involves fitting a parabolic function to eqn (3) with the independent variable being ($\sqrt{t}$) to obtain coefficients C_1 and C_2. However, the flow is complicated by the fact that a contact medium is used between the disc permeameter and the soil. This means that some of the initial data has to be deleted from the data set. They do not favour this method, as the choice of what data to remove from the data set is essentially arbitrary. From here on we will term this method the Quad method. Differentiation of eqn (3) results in:

$$\frac{dI}{d\sqrt{t}} = C_1 + 2C_2\sqrt{t} \tag{5}$$

The difference form of eqn (5) is:

$$\frac{dI}{d\sqrt{t}} \approx \frac{\Delta I}{\Delta\sqrt{t}} = \frac{I_{i+1} - I_i}{\sqrt{t_{i+1}} - \sqrt{t_i}} = C_1 + 2C_2\sqrt{t} \tag{6}$$

The values of C_1 and $2C_2$ are obtained from fitting a straight line function to a plot of $\Delta I / \Delta\sqrt{t}$ versus $\sqrt{t}$. A problem with this method is that differentiation will exacerbate the noise in the experimental data.

Equation (3) has the same form as the two-term Philip infiltration equation (Philip 1957). Division of eqn (3) by $\sqrt{t}$ results in:

$$I / \sqrt{t} = C_1 + C_2\sqrt{t} \tag{7}$$

Equation (6) has the same form as obtained by Smiles and Knight (1976) for the Philip infiltration equation. The values of C_1 and C_2 can be found by linear regression of $I / \sqrt{t}$ with $\sqrt{t}$. We will show that eqn (7) is just as good as eqn (6) for determining where the effect of the sand ceases while avoiding the introduction of noise into the data by differentiating. This method will be termed the S&K method. C_1 is determined from the intercept where $t = 0$, but t is not the time at which infiltration into the soil commenced. The time of commencement of infiltration into the soil, t_s, can be found as the time when the minimal value of $I/\sqrt{t}$ occurs. S for the soil is found obtaining the value of $I/\sqrt{t}$ when $t = t_s$. Vandervaere et al. (2000a) do not state this for the DI method but the time must also be corrected in this method to get the correct value of S.

Hydraulic conductivity can then be determined directly from the values of C_1 and C_2 obtained by using (Vandervaere et al. 2000b):

$$k = \frac{3}{2-\beta}\left[C_2 - \frac{\gamma C_1^2}{r(\theta_o - \theta_n)}\right] \qquad (8)$$

If β is set to 0.6 then for a log normally distributed error, the resulting error in k will be 1.4 (Vandervaere et al, 2000b). It is possible to calculate k for all members of the data set in Cook and Broeren (1994). The hydraulic conductivity can be determined using the multiple sorptivity method (MS) (Vandervaere et al. 2000b, eqn 20):

$$k\left(\frac{\psi_x + \psi_y}{2}\right) = \frac{b}{\psi_y - \psi_x}\left\{\frac{S(\psi_y)^2}{\Delta\theta_y} - \frac{S(\psi_x)^2}{\Delta\theta_x}\right\} \qquad (9)$$

where b is a constant which White and Sully (1987) found to be well approximated by 0.55, where ψ_x and ψ_y are the two potentials at which S was determined and $\Delta\theta_x$ and $\Delta\theta_y$ are the change in volumetric water content that occurs due to the infiltration of water. In the data set of Cook and Broeren (1994) the flow rate was determined using permeameters with different radii, so that the multiple radii method can also be used to estimate S and k (Vandervaere et al. 2000b):

$$S = \left\{\frac{(C_{2B} - C_{2A})(\theta_o - \theta_n)}{\gamma} . \frac{r_A r_B}{r_A - r_B}\right\}^{1/2} \qquad (10)$$

$$k = \frac{3}{2-\beta}\left[\frac{C_{2A} r_A - C_{2B} r_B}{r_A - r_B}\right] \qquad (11)$$

where r_A and r_B are the radii [L] and $r_A > r_B$, and C_{2A} and C_{2B} are the values of C_2 for the respective radii [L T^{-1}].

3. RESULTS

3.1. Illustrative examples

A typical plot of a data set from Cook and Broeren (1994) analysed with their method is shown in Figure 1. The effect of the sand can be distinguished when the data is plotted as I versus $\sqrt{t}$. This is done by visual observation and is arbitrary.

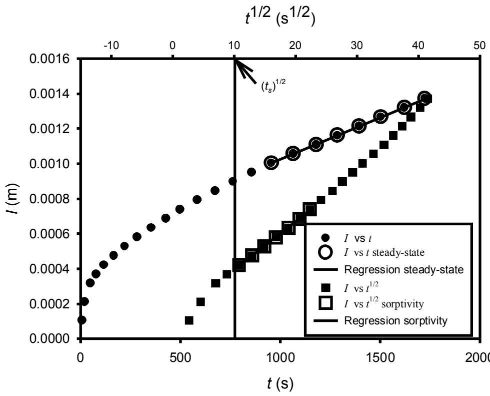

Figure 1. Typical plot of infiltration data from disc permeameter. Data from Cook and Broeren (1994) is for a potential of -0.04 m and radius of disc of 0.102 m. The vertical line indicates the estimated time that the sand pad influences infiltration.

The estimate of the S and steady-state flow rate (q) can be obtained from linear regressions of the data in Figure 1, and are 2.73 x10^{-5} m s$^{-1/2}$ (r^2 = 0.999) and 4.80 x 10^{-7} m s^{-1} (r^2 = 0.999) respectively. The hydraulic conductivity can be calculated from S and q using (Wooding 1964) and Cook and Broeren (1994)):

$$k = q - \frac{2.2 S^2}{\pi(\theta_f - \theta_i) r} \qquad (12)$$

where θ_i and θ_f are the initial and final volumetric water contents (m^3 m^{-3}) respectively. The value for k is for this measurement estimated as 4.37 x 10^{-7} m s^{-1}.

The method of fitting eqn (3) requires fitting a quadratic function to the data after the data associated with infiltration into the sand is removed (Figure 2).

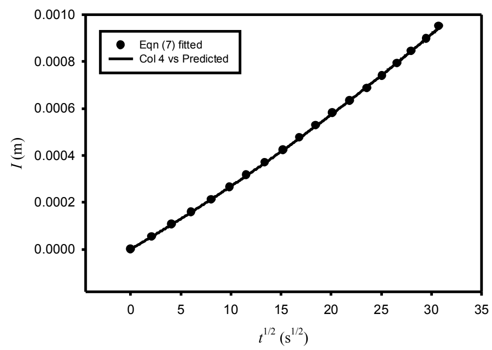

Figure 2. Data as for Figure 1 but adjusted to remove the sand infiltration. The line is a regression of eqn (3) with $C_1 = S = 2.50 \times 10^{-5}$ m $s^{-1/2}$, $C_2 = 1.85 \times 10^{-7}$ m s^{-1} ($r^2 = 1$).

The value for sorptivity found by fitting eqn (3) to the data with data points up to $t^{1/2} = 10$ $s^{1/2}$ removed is similar to that found by regression in Figure 1 for S. The estimate of k using eqn (8) with $\beta = 0.6$ is 3.2×10^{-7} m s^{-1}, which is less than predicted by the steady-state method. This presupposes that the estimate of what data to remove is correct.

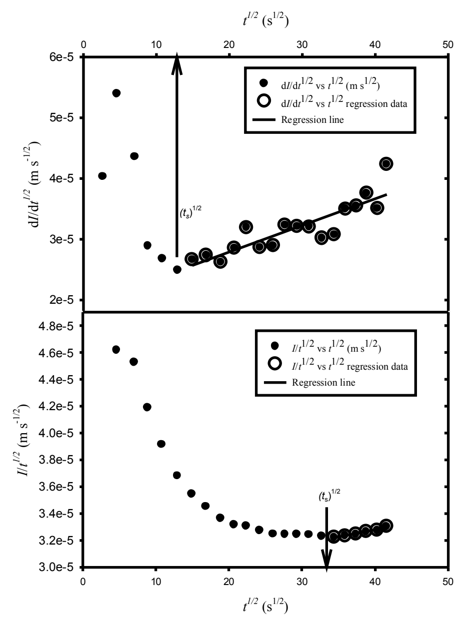

Figure 3. Data as used in Figures 1and 2 with a) differential (DI) method and b) the S&K method. Linear regressions are applied to the data range shown in the legends. Values for the regressions are given in Table 1.

The differential (DI) method of Vandervaere et al. (2000a) has as suggested above introduced a lot more noise into the data set (Figure 3a) but does fit the data well. The differential method also shows clearly an end point for when wetting of the sand appears to cease. This value of t_s is slightly greater than that estimated inspection in Figure 1. The end point is less clear in this particular example with the S&K method and it suggests that the effect of the sand occurs for much longer (Figure 3b). However in other examples the opposite is true. The data is a lot less noisy for the S&K method which is reflected in the regression coefficient being greater ($r^2 = 0.96$) than that for the DI ($r^2 = 0.76$).

From the regressions the values of S and k can be estimated (Table 1) using eqn (8) with $C_1 = S$. Here we have estimated S as the value of either $dI / dt^{1/2}$ or $I/t^{1/2}$ when $t = t_s$. Vandervaere et al. (2000a) did not realise the necessity to calculate S as is done here. In the example shown positive values of C_1 were obtained (Table 1), but often this is does not occur (Table 2).

Table 1. Values of parameters derived from the regression in Figure 3.

Method	C_1 (m $s^{-1/2}$) $\times 10^{-5}$	C_2 (m s^{-1}) $\times 10^{-7}$	S (m $s^{-1/2}$) $\times 10^{-5}$	k (m s^{-1}) $\times 10^{-7}$	r^2
DI	1.9	2.4	2.2	3.9	0.76
S&K	2.9	1.1	3.2	0.9	0.96

3.2. Comparison of methods for C_1 and C_2

The data of Cook and Broeren (1994) provided a data set with rings of two different radii (0.048 and 0.102 m) and for both rings infiltration at potentials of -0.02 and -0.04 m and for the large ring also a potential of -0.1 m. These data resulted in values of C_1 and C_2 that were calculated with all the methods described above. The sorptivity values were similar (Figure 4) for each of the methods and valid values were obtained for each replicate at each potential for all the methods.

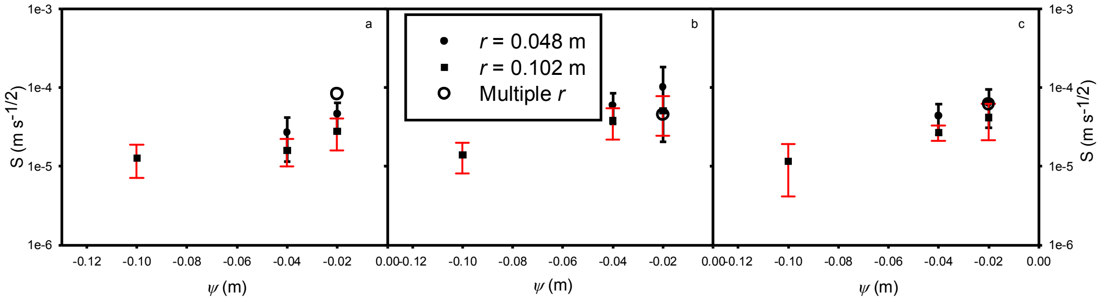

Figure 4. Comparison of S calculated with a) the DI method (eqn (6)), b) the S&K method (eqn (7)) and c) the Quad method (eqn (3)). The data for each individual ring (no. of replicates = 10) was used and the mean value with standard deviation is plotted as the point and error bars respectively. The mean values of C_2 for each ring were used to calculate S using the multiple radius method (eqn (9)).

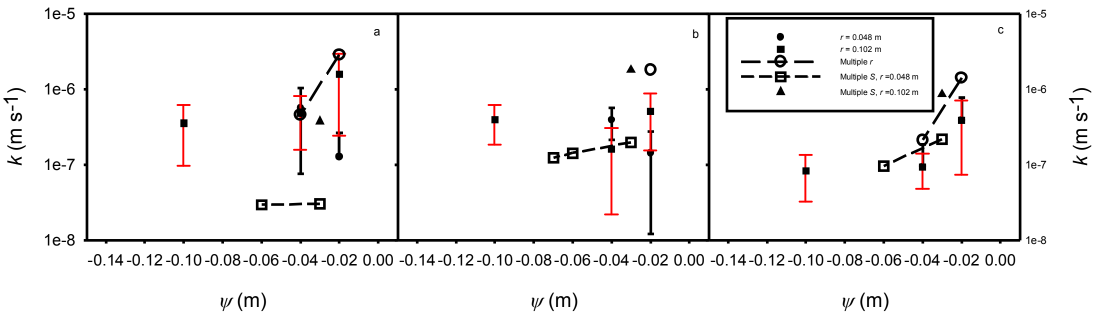

Figure 5. Comparison of k calculated with a) the DI method (eqn (4)), b) the S&K method (eqn (7)) and c) the Quad method (eqn (3)). The data for each individual ring (no. of replicates see Table 2) was used and the mean value with standard deviation is plotted as the point and error bars respectively. The mean values of C_2 for each ring were used to calculate k using the multiple radius method (eqn (11)). The mean values of S at each potential were used to calculated k by the multiple S method.

Table 2. Number of data points (n (data) used in regression to determine C_1, C_2 and k, and regression coefficient (r^2). Number of valid data points (>0) for C_1, C_2 and k.

	DI Method (eqn (6))									
	r = 0.048 m					r = 0.102 m				
ψ (m)	n (C_1)	N (C_2)	n (data)	n (k)	r^2	n (C_1)	n (C_2)	n (data)	n (k)	r^2
-0.02	10	9	8±2	3	0.38±0.22	4	10	13±5	10	0.64±0.20
-0.04	9	10	11±4	9	0.64±0.24	10	10	16±4	9	0.77±0.14
-0.10						8	10	11±2	10	0.88±0.08
	S & K Method (eqn (6))									
-0.02	10	3	6±2	1	0.94±0.08	10	8	9±1	4	0.95±0.07
-0.04	10	4	8±3	2	0.95±0.09	10	7	9±3	4	0.88±0.18
-0.10						8	10	12±2	10	0.88±0.08
	Quad Method (eqn (1))									
-0.02	10	10	13±5	0	1.0±0.0	10	10	13±5	9	1.0±0.0
-0.04	10	10	15±5	0	1.0±0.0	10	10	17±4	7	1.0±0.0
-0.10						10	10	13±2	9	1.0±0.0

The S values shown here were corrected for t_s. Without this correction for the DI method and assuming $C_1 = S$ the number of valid values for S would drop from 10 to as low as 4 (Table 2). Similarly for the S&K two values of S would be negative and invalid C_1 was taken as S (Table 2).

The regression coefficients were worst for the DI method due to the noise introduced by differentiating the data.

The number of values of k that can be calculated from a single infiltration test varies with the methods due to variations in the values of S and C_2 calculated (Table 3). The Quad method resulted in no valid values of k for the r = 0.048 m. The values of k obtained with all of the methods were similar (Figure 5), except for the values of k obtained by the multiple S method using the DI method data which are lower. This is cause by the values of S at the highest two potentials being lower for the DI method (Figure 4).

Table 3. Comparison of S and k from individual rings (replicates = 10) determined by: steady-state methods (SS) (eqn (12), Figure 1), transient differential method (DI) (eqn (6)), transient Smiles and Knight method (S&K) (eqn (7)), and quadratic method (eqn (3))

ψ (m)	S (x10^{-5} m s$^{-1/2}$)(mean ± Std. Dev.)							
	r = 0.048 m				r = 0.102 m			
	SS	DI	S&K	Quad	SS	DI	S&K	Quad
-0.02	6±4	5±2	10±8	6±3	4±1	3±1	5±3	4±2
-0.04	3±1	3±2	6±2	4±2	3±1	2±1	1±1	2±1
-0.10					1±1	1±1	1±1	1±1
	k (x10^{-5} m s$^{-1/2}$)(mean ± Std. Dev.)							
	r = 0.048 m				r = 0.102 m			
	SS	DI	S&K	Quad	SS	DI	S&K	Quad
-0.02	2±1	1±1	2*	Nd	7±3	20±10	9±8	5±5
-0.04	3±2	5±4	3±1	Nd	4±2	5±4	3±1	1±1
-0.10					2±1	3.6±0.3	8±5	1±1

*Only a single measurement was valid.

4. DISCUSSION

The disc permeameter is a convenient method for determining the hydraulic properties of soil. One of the disadvantages of this method is the requirement to determine the steady-state flow rate if Wooding's (1964) solution is to be used to calculate the soil properties. Cook (1994) found that even after 8 hours steady-state may have not been reached.

Vandervaere et al. (2000a,b) suggested a method that does not require the steady-state flow rate to be determined. Cook and Broeren (1994) also suggested a method using the values of S determined at two potentials based on White and Perroux (1989) to estimate k. Vandervaere et al. suggest differentiating the data as a means of linearising eqn (3). This unfortunately can introduce noise into the data and in some cases make it impossible to determine the signal in this noise (L. Dawes, pers comm. 2005). Here we have achieved the same effect as Vandervaere et al. by using the Smiles and Knight (1976) approach to linearising the infiltration equation. This appears to work just as well without the problem of amplifying the noise.

The other problem with this method is how to distinguish when infiltration has commenced into the soil from the infiltration data which also has infiltration into the sand pad. Both the DI and S&K methods allow the point when water starts to infiltrate the soil to be determined with more precision than the arbitrary method used by Cook and Broeren (1994). Vandervaere et al. did not seem to comprehend that the C_1 value obtained from their method is not equal to S due to this time offset created from infiltration through the sand pad. The resulting value of C_1 can and in this data set was negative for a number of replicates but when corrected for t_s all of the results for S are positive.

The values obtained by Cook and Broeren (1994) using this same set of data are similar and not significantly different from any of these transient methods on individual rings (Table 3). This suggests that use of this transient approach may be a valid method for developing rapid methods for measuring soil hydraulic properties. There is no significant difference between the methods but the consistent trend is for the S&K method to give higher S values and the DI method to give higher k value. The Quad method failed to give any valid k values when r = 0.048 and ψ = -0.04 m. This analysis would support Vandervaere et al (2000a) who suggested this method should not be used. We would recommend the use of the S&K method due to its ease of analysis and data preparation.

5. CONCLUSION

The transient analysis of disc permeameter is investigated and a number of methods for data analysis compared. Here we have compared these methods and shown that the differential (DI) and linearised method (S&K) can provide similar results. The DI method can amplify noise in data. This could be a problem in some circumstances. The ease of data preparation and analysis result in a recommendation to use the S&K method.

We have also identified a problem with the original method of Vandervaere (2000a,b) due to offset in the time causing $C_1 \neq S$ and have suggested a correction.

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