A Multi-Objective Optimization Approach to Water Management.

E. Xevi[a] and S. Khan[a]

[a] *CSIRO Land and Water Griffith PMB 3 Griffith, NSW, Australia*(emmanuel.xevi@csiro.au).

Abstract: Many optimization models exist for water management systems but there is a knowledge gap in linking bio-economic objectives with the optimum use of all water resources under conflicting demands. The efficient operation and management of a network of nodes comprising storages, canals, river reaches and irrigation districts under environmental flow constraints is very challenging. Minimization of risks associated with agricultural production requires accounting for uncertainty involved with climate, environmental policy and markets. Due to conflicts between multiple goal requirements and the competing water demands of different sectors, a multi-criteria decision-making (MCDM) framework was developed to analyze production targets under physical, biological, economic and environmental constraints. This approach is described by analyzing the conflicts between profitability, variable costs of production and pumping of groundwater for a hypothetical irrigation district.

***Keywords**: Optimization; Minimization; Multi-criteria decision-making*

1. INTRODUCTION

Many decision support systems in agricultural enterprises use the conventional linear programming approach to optimize a single objective function such as total gross margin. However, as agricultural systems become more complex, multiple objectives that are in conflict need to be addressed. Competition for scarce resources by different enterprises is a major concern in many agricultural production systems. Competition occurs at the farm level e.g. between different crops as well as at a regional level, where utilization of scarce water resources for agricultural purposes often comes into conflict with the requirement for in stream ecosystem services. For example, in bio-economic systems conflict may arise from maximizing economic returns (i.e. net revenue) as opposed to minimizing the use of resources such as water, fertilizer applications etc. On the other hand minimizing costs rather than maximizing net revenue may also be important in some water management systems. Under these conditions, multiple criteria decision-making techniques are useful tools to explore different management options. These techniques permit optimization of several objectives in many different logical formulations (Piech and Reyman, 1993). A multi-criteria approach has been used extensively to solve diverse decision problems including risk assessment in agricultural systems (Berbel, 1993). Mendoza et al., (1993) used Fuzzy Multiple Objective Linear Programming (FMOLP) techniques in forest planning where imprecise objective function coefficients are involved. Furthermore, Tecle, (1998) used Compromise Programming (CP) to develop a multi-objective decision support system for analyzing multi-resource forest management problem. The use of these techniques enables the decision maker to study the trade-offs and conflicts between, for example, profitability (measured by economic returns) and risk (measured by Partial Absolute Deviation (PAD)) This paper demonstrates the application of a MCDM technique called Goal Programming (GP) to water resource allocation problems with conflicts between irrigation water demand and in stream environmental flow requirements. First, a solution is sought for the single objective function formulation and compared to a solution of three objective functions (Net Revenue (NR), Variable Costs (VC) and Total Groundwater Pumping from the irrigation areas (TP)) using Goal Programming.

2. MODEL FORMULATION

2.1. Objective functions, decision variables and constraints

The multi-objective problem described in this paper consists of three objective functions: maximizing net returns (NR), minimizing variable cost (VC) and minimizing total supplementary groundwater pumping requirements to meet crop demand from the irrigated areas. Conceptually, NR and VC may represent the view of resource economists while minimizing total pumping may be the desired goal to avoid groundwater mining and pollution of aquifers. The management options to achieve the above objectives consist of selection of an appropriate mix of crops, optimum level of groundwater pumping and appropriate allocation of water for irrigation and environment. Constraints imposed on the system include seasonal environmental flows targets. In addition, water allocation rules and pumping targets for each month are constraints imposed on the system.

The three objective functions are formulated as follows:

$$Max \quad NR = \sum_{c} CGM(c) \times X(c) - \sum_{c}\sum_{m} \{WREQ(c,m) \times X(c) \times C_{w}\} - C_{p} \times \sum_{c}\sum_{m} P(c,m) \qquad (1)$$

$$Min \ VC = \sum_{c}\sum_{m} \left(X(c) \times WREQ(c,m) \times C_{w} \right) + \sum_{c} X(c) \times Vcost(c) \qquad (2)$$

$$Min \ TP = \sum_{c}\sum_{m} P(c,m) \qquad (3)$$

where $X(c)$ = area of crop c (Ha), CGM(c) = gross margin for crop c (\$), $WREQ(c,m)$ = water requirement for crop c in month m (ML), C_w = total cost of water per unit volume (\$/ML), C_p = cost of groundwater pumping and delivery (\$/ML), *Vcost* = variable cost (such as fertilizer and pesticides applications) per hectare other than water cost for crop c and $P(c,m)$ = volume of ground water pumped from irrigation areas for crop c in month m (ML).

The model consists of a network of nodes that connect supply nodes to irrigation or urban areas (demand nodes). The links connecting the nodes include river reaches that may carry environmental flows as well as irrigation canals. The continuity equation for each node (i) assuming no storage at the node is given by the following:

$$\sum_{j} Q(i,j) = \sum_{k} Q(k,i) \qquad (4)$$

where $Q(i,j)$ = flow of water from node i to node j, $Q(k,i)$ = flow of water from node k to node i.

The physical and environmental constraints imposed on the model are given by the following:

Total water use in the irrigation areas should not exceed total allocation in a given month:

$$\sum_{c} \left(X(c) \times WREQ(c,m) \right) \le Allocation(m)$$

$$m = 1,..,12 \qquad (5)$$

The sum of all crop areas is equal to the total farm area:

$$\sum_{c} X(c) = TArea \qquad (6)$$

Environmental flows in each month should equal or exceed target flows:

$$Env\_f(m) \ge Environmental\ flow(m)$$

$$m = 1,..,12 \qquad (7)$$

Total pumping (TP) from the irrigation area in any month should be less than or equal to allowable pumping.

$$\sum_{c} P(c,m) \le Pump(m) \quad m = 1,..,12 \qquad (8)$$

where *Allocation(m)* = monthly water allocation for irrigation areas (ML), *TArea* = Total irrigable farm area (Ha), *Env_f(m)* = environmental flow (ML) in month m, *Environmental flow(m)* = target environmental flow in month m and *Pump(m)* = allowable pumping in the irrigated areas for month m.

Two auxiliary equations were used to restrict the minimum cropped area to a given value when the crop area becomes a basic variable in the solution vector:

$$-X(c) + mArea \le TArea \times Y(c)$$

$$X(c) \le TArea \times (1 - Y(c)) \qquad (9)$$

where *mArea* = minimum crop area (Ha) and *Y(c)* = binary variable for crop c. For the illustration problem given in the next section the minimum crop area was assigned a value of 1000 Ha for all calculations.

3. THE EXAMPLE PROBLEM

In order to demonstrate model application a hypothetical 3-node network is chosen to illustrate the concepts outlined above. The network consists of a supply node (e.g. reservoir), a demand node (e.g. irrigation area), a distribution node and an environmental flow link. The network is schematically illustrated below (see Figure 1). The supply schedule and environmental flow targets are usually stipulated by water sharing plans and flow rules of a river system. These flows may be dependent on climate, aesthetics, social, economic and environmental factors.

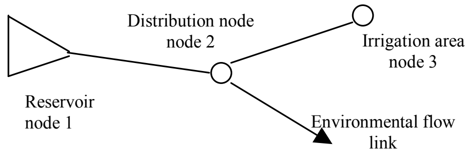

Figure 1. Schematic of nodal network.

The demand node (node 3) is made up of a total irrigable land of 30,000 Ha with a potential for growing six crops (maize, wheat, canola. rice, oats and barley). Groundwater pumping from the irrigable area is permitted to satisfy crop water demand if surface water supplies are not sufficient. Each of these crops is characterized by crop water requirements and growing period within the year. The continuity equation (4) was recast for the example problem as:

$$IRR\_node3(m) + pump(m) = \sum_{c} X(c) \times WREQ(c, m)$$

$$m = 1,..,12 \qquad (10)$$

$$w\_node2(m) = S\_node1(m)$$
$$- Chn\_losses(link(1,2)) \times Chn\_l(link(1,2)) \qquad (11)$$

$$w\_node2(m) - Env\_f(m) = IRR\_node3(m)$$
$$+ Chn\_losses(link(2,3)) \times Chn\_l(link(2,3)) \qquad (12)$$

where *IRR_node3(m)* = surface water available at node 3 for the irrigation area in month *m*, *w_node2(m)* = surface water available at the distribution node 2 in month *m*, *S_node1* = reservoir supply in month m, *Env_f(m)* = environmental flow in month *m*, *Chn_losses* = channel seepage per unit length, *Chn_l* = channel length(m) and *link(i,j)* = link joining node *i* to *j*.

Reference evapo-transpiration (ET) and rainfall data for the example problem are shown in Tables 1 and 2. The pay-off matrix and the corresponding crop mix was determined using the three objective functions given by (1) to (3) and constraint equations (5) to (12) for the dry, average and wet seasons and shown in Tables 3 to 5. The elements of the pay-off matrix were obtained by optimizing each of the objectives (1), (2) and (3) individually and then calculating the values of the remaining objectives using the solution vector of the decision variables. For example, the first row of Table 3 shows results from maximizing NR. When net revenue is maximized, its maximum value is $34,348,685 and the cost associated with it is $20,180,526 and total groundwater pumping from the irrigated area was 16,632 ML. The crop mix obtained by maximizing net revenue is Rice (4789 Ha), Canola (19518 Ha), Oats (1000 Ha) and Maize (4693 Ha). The diagonal elements of the pay-off matrix in Table 3, 4 and 5 are the optimum values for each individual goal. The results clearly indicate the degree of conflict between the three objectives. However, the pay-off matrix in Table 3 indicates that for the dry season there is not much difference between minimizing total cost and minimizing total pumping. Obviously, the decision maker is very likely to be interested in a combination of maximum NR, minimum cost and minimum total pumping. However, because the objectives are in conflict, some sort of compromise solution must be found. Several MCDM methods are used to obtain solutions including Multi-Objective Programming (MOP), Compromise Programming (CP) and Goal Programming (GP). MOP methods generate a set of efficient solutions sometimes called Pareto optimal solutions and can be very difficult to

Table 1. Rainfall (ML/Ha or x100 mm) for dry, average and wet seasons.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Dry	0.15	0.03	0.14	0.11	0.3	0.21	0.14	0.39	0.17	0.26	0.11	0.13
Average	0.22	0.12	0.28	0.27	0.27	0.29	0.4	0.3	0.35	0.37	0.26	0.28
Wet	0.49	0.18	0.33	0.32	0.73	0.49	0.42	0.42	0.45	0.48	0.32	0.36

Table 2. Reference Evapo-transpiration (ET, ML/Ha) for dry, average and wet seasons.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Dry	2.92	2.41	1.94	1.22	0.69	0.47	0.54	0.83	1.27	1.91	2.49	2.94
Average	2.72	2.24	1.87	1.12	0.67	0.46	0.52	0.74	1.11	1.72	2.24	2.63
Wet	2.65	2.16	1.84	1.08	0.59	0.41	0.43	0.7	1.02	1.67	2.16	2.58

Table 3. Pay-off matrix and crop-mix for dry season

Pay-off Matrix				Crop-Mix (Ha)					
Optimization Goal	Net Revenue ($)	Total Cost ($)	Total Pumping (ML)	Rice	Wheat	Barley	Canola	Oats	Maize
Net Revenue	34348685	20180526	16632	4789			19518	1000	4693
Total Cost	26107450	14873443	0		13803		1000	10605	4592
Total Pumping	27032049	15569178	0	1000	11350	1000	1000	10605	5044

Table 4. Pay-off matrix and crop-mix for average rainfall season

Pay-off matrix				Crop-mix (Ha)					
Optimization Goal	Net Revenue ($)	Total Cost ($)	Total Pumping (ML)	Rice	Wheat	Barley	Canola	Oats	Maize
Net Revenue	37811126	1961400	7345	9845			19155		1000
Total Cost	20417517	12639862	3998			13828	6372	8800	1000
Total Pumping	28486042	15977407	0	1000	16250	1000	6277	1000	4472

Table 5. Pay-off matrix and crop-mix for wet rainfall season

Pay-off matrix				Crop-mix (Ha)					
Optimization Goal	Net Revenue ($)	Total Cost ($)	Total Pumping (ML)	Rice	Wheat	Barley	Canola	Oats	Maize
Net Revenue	39407159	19865143	0	11488			17512		1000
Total Cost	17582653	12437679	0			23123	3276	2600	1000
Total Pumping	28896551	15490436	0	1000	1000	1000	23400	2600	1000

implement when the number of objectives is large. On the other hand, CP looks for a solution as close as possible to the "Ideal Point". This point is normally taken as the individual optimal solutions.

3.1. The Goal Programming Model

GP solves the multiple objective problem by introducing the objectives into the problem as constraints and setting targets to be achieved. The objectives are included in the problem by adding positive (p_i) and negative (n_i) deviation variables that describe over-achievement and under-achievement of each goal.

The weighted version of goal programming model (WGP) was used in this example. The model is defined to minimize only the undesirable deviations from defined targets:

$$Min\,Z = \beta_1 \times n_1 + \beta_2 \times p_2 + \beta_3 \times p_3 \quad (13)$$

subject to:

$$\sum_c X(c) \times CGM(c) - \sum_c \sum_m WREQ(c,m) \times X(c) \times C_w - C_p \times \sum_c \sum_m P(c,m) + n_1 - p_1 = T\_rev \quad (14)$$

$$\sum_c \sum_m WREQ(c,m) \times X(c) \times C_w + \sum_c \{Vcost(c) \times X(c)\} + n_2 - p_2 = T\_cost \quad (15)$$

$$\sum_c \sum_m P(c,m) + n_3 - p_3 = T\_pump \quad (16)$$

and constraints (5) to (12). The weights β_i are defined as:

$$\beta_i = \frac{\alpha_i}{\sum_1^3 \alpha_i} \qquad i = 1,..,3 \quad (17)$$

where T_rev = target revenue, T_cost = target cost, T_pump = target pumping and α_i = relative weights assigned to the individual goals.

Assuming that all goals are of equal importance i.e. $\alpha_1 = \alpha_2 = \alpha_3$, and setting the target values of the goals to values on the diagonal of the pay-off matrix of Table 5 (i.e. net revenue = \$39,407,159, total cost = 12,437,679 and total pumping = 0) the following solution was obtained: crop areas for the three different seasons are shown in Table 6 and the corresponding deviational variables are shown in Table 7. Figure 2 shows the actual and targeted environmental flow for dry, average and wet seasons. Figure 3 shows the actual water allocated to the irrigation areas for dry average and wet seasons as computed by the model.

Table 6. Crop areas (Ha) for dry, average and wet seasons.

	dry	average	wet
Rice			3319
Wheat	1000		23081
Barley			
Canola	19286	20200	
Oats	10605	8800	2600
Maize	2322	1000	1000

Table 7. Positive (p) and negative (n) deviational variables for dry, average and wet seasons (\$ for indices 1 and 2 and ML for index 3).

	dry		average		wet	
index	*n*	*p*	*n*	*p*	*n*	*p*
1	12375000	0	12301000	0	11099000	0
2	0	2635600	0	1884000	0	1672700
3	0	0	0	0	0	0

The net revenue was under-achieved by more than one million dollars in all seasons while cost exceeded target by over one and half million dollars. The pumping target was achieved.

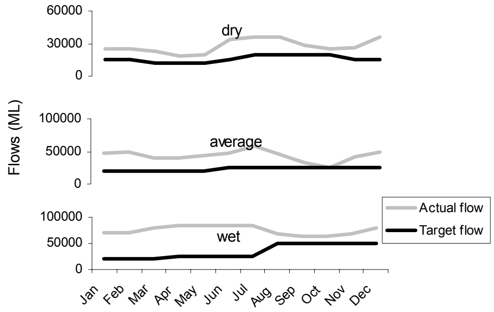

Figure 2. Actual and targeted environmental flows for dry, average and wet seasons.

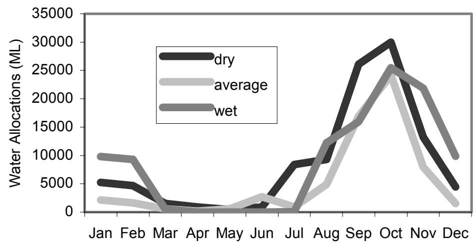

Figure 3. Water allocation to irrigated areas (ML) for dry, average and wet seasons.

The sensitivity analysis of applying different weights is demonstrated below. If we apply twice as much weight to the NR goal (i.e. $\alpha_1 = 2$, $\alpha_2 = \alpha_3 = 1$) the following results were obtained.

Table 8. Crop areas (Ha) for dry, average and wet seasons.

	dry	average	wet
Rice	2215	6994	11488
Wheat		1000	
Barley	1000		
Canola	12503	9063	14912
Oats	10605	8320	2600
Maize	3678	4622	1000

Table 9. Positive (p) and negative (n) deviational variables for dry, average and wet seasons ($ for indices 1 and 2 and ML for index 3).

	dry		average		wet	
index	*n*	*p*	*n*	*p*	*n*	*p*
1	10233000	0	4751900	0	640380	0
2	0	4054700	0	5983600	0	6909700
3	0	0	0	0	0	0

.

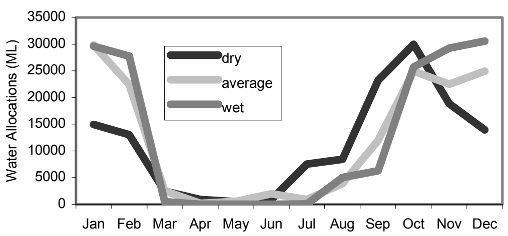

Figure 4. Water allocation to irrigated areas (ML) for dry, average and wet seasons.

The crop areas in Table 8 are remarkably different from those in Table 6. Comparing the values in Table 7 to Table 9, it can be seen that weighting of the net revenue objective higher than the other two objectives reduced the under-achievement for net revenues by almost 94% ($11,099,000 to $640380) while increasing over-achieved cost from $1,672,700 to $6,909,700 in the wet season. Figure 4 shows the corresponding water use in the irrigated areas.

4. CONCLUSIONS

Most water management systems are concerned with satisfying conflicting demands of various groups and MCDM techniques provide the mechanism for resolving these conflicts. They provide better results than simple linear programming (LP) solutions because they integrate the effect of all the objectives simultaneously. There are an increasing number of highly sophisticated LP solvers that could easily be adapted to solve MCDM problems using Goal Programming (GP) or Weighted Goal Programming (WGP) as illustrated with the example problem. The application of MCDM techniques to the simple nodal-network example problem demonstrates its ability to provide solutions that integrate different goals and trade-offs. The pay-off matrix for the three goals illustrates the degree of conflict between the different goals and trade-offs. The effect of different ET and rainfall (dry, average and wet) on NR, crop areas, environmental flows and water allocated to the irrigation areas was clearly demonstrated. Furthermore, the sensitivity of the weights assigned to the different goals was shown to have marked impact on optimal crop areas and the degree of under- and over-achievement of the selected targets for all the three goals. Maximization of NR was almost equivalent to minimization of total pumping under dry climatic conditions. By attaching different weights to goals, sets of decision variables (crop area, water allocations) could be formulated for different seasons that could aid in policy formulations and decision-making. Although goal programming is a useful tool to analyze MCDM problems, there is a difficulty of selecting the target values and weights for the different goals.

5. REFERENCES

Berbel, J., Risk programming in Agricultural Systems: A multiple criteria analysis. *Agricultural Systems,* 41, 275-288, 1993.

Mendoza. G. A., B.B. Bare, and Z. Zhou, A Fuzzy Multiple Objective Linear Programming Approach to Forest Planning Under Uncertainty. *Agricultural Systems,* 41, 257-274, 1993.

Piech, B. and T. Rehman, Application of Multiple Criteria Decision Making Methods to Farm Planning: A case Study. *Agricultural Systems,* 41, 305-319, 1993

Tecle, A. A Multiobjective Decision Support System for Multiresource Forest Management. *Group Decision and Negotiation* 7, 23-40,1998.