Models for Mine Site Rehabilitation

L. Caccetta and P.J. Kelsey

Department of Mathematics and Statistics, Curtin University of Technology, GPO Box U1987, Perth Western Australia 6845 (caccetta@maths.curtin.edu.au)

Abstract: Over the past decade or so mine site rehabilitation has become increasingly important. Indeed, mining leases today include conditions relating to the preservation of the surrounding environment and the rehabilitation of the mine site once the mining has been completed. There are two basic requirements of mine site rehabilitation: (i) that the site is safe, stable and non-eroding at the end of the life of the mine; and (ii) pollutants such as acid-producing waste be buried and capped with highly impermeable material. The first of these requirements gives rise to the land surface reshaping problem which is defined as finding the surface which satisfies the requirements (usually expressed in terms of wall slopes) and which minimises the material movement. In this paper we consider this problem. An integer linear programming model is presented. A solution method using the Lagrangian multiplier technique is proposed. Results from a small producing mine are presented.

Keywords: Mine optimization; Mine-site rehabilitation; Integer linear programming

1. INTRODUCTION

In Australia mine site rehabilitation and the adherence to specified environmental practices during the operation of a mine and at the end of the life of a mine are determined by myriad state government legislation as well as the conditions contained within the individual mining leases. A rehabilitation program for a mine-site should encompass the following:

- Ensure the safety of the area.
- Minimise erosion and create stable aesthetic landforms within the lease area.
- Ensure that the lease area is non-polluting in terms of air (dust) and water emissions.
- Rehabilitate the site to restore it to its pre-mining condition or to enable development of the site in a manner agreeable to the local communities.
- Minimise the maintenance requirements of the rehabilitated site.

In many cases it is difficult to achieve even the first three of the above aims and the mine-site remains a source of pollution for many years beyond its closure. However, ever-increasing environmental awareness has prompted many rehabilitation programs and there are many examples of success [Mine and Quarry Rehabilitation in South Australia, 1988; Fawcett and Sinclair, 1997].

Mine-site rehabilitation requirements vary greatly from site to site in Australia. The current practice is that mining voids are not back filled [Hollands, 1997; Smith and Hillis, 1996] but waste dumps and tailing ponds are rehabilitated to prohibit environmental pollution beyond the lease boundary [Orr and Veivers, 1997]. The requirement to completely rehabilitate the mini-site to its pre-lease condition is rarely enforced, though in the United States this is becoming increasingly common particularly in regions of prime agricultural land [Williams, 1997].

The rehabilitation process should be planned and executed in conjunction with the mining of the ore body as rehabilitating a mine-site once the mining has ceased is generally prohibitively expensive. Waste material from the mine has to be assessed and acid-producing waste has to be buried and capped with highly-impermeable material to prohibit acid mine drainage and the associated pollution of adjacent waterways and ground-water reservoirs [Orr and Veivers, 1997]. Clearly mine-site rehabilitation entails extensive material movement (millions of cubic metres) and careful scheduling is required to optimise the process and minimise this material movement, particularly the double handling of materials.

In this paper, the simple problem of achieving a stable and safe landform with minimal material movement is considered. Given a particular landform (in this case a mine-site), the land surface reshaping problem seeks to reshape this landform to one which conforms to desired slope criteria, with least material movement. The land surface reshaping problem, as defined here, considers the material in the landform to be uniform. The different handling that would eventuate if the material had different characteristics (e.g. acid-producing, non acid-producing, top soil) is not considered.

The land surface reshaping problem is modelled mathematically and an optimum solution method is presented. The paper is organised as follows. An integer linear programming formulation is presented in Section 2. Solution methods are discussed in Section 3. In Section 4 the subgradient optimisation method is applied on the Lagrangian relaxation model using data from a real mine. Conclusions are presented in Section 5.

2. FORMULATION

The economic viability of the modern day mine is highly dependent upon careful planning and management. The operation and management of a large open pit mine having a life of several years is an enormous and complex task. Optimisation techniques can be used to resolve a number of important problems that arise [see Caccetta and Giannini, 1986, Caccetta and Hill, 2000]. The "block model" is extensively used in optimisation.

2.1 Block Model

An important task in mine management is the establishment of an accurate model for the deposit. Though a number of models are available, the regular 3D fixed-block model is the most commonly used and is the best suited to the application of computerised optimisation techniques [Gignac, 1975]. This model is based on the ore body being divided into fixed-size blocks. The block dimensions are dependent on the physical characteristics of the mine, such as pit slopes, dip of deposit and grade variability as well as the equipment used. The centre of each block is assigned, based on drill hole data and a numerical technique, a grade representation of the whole block. Using the financial and metallurgical data the net profit of each block is determined.

To satisfy the mine site safety and stability requirements, wall slope restrictions are imposed on each block. These are given as a set (usually 4 to 8) of azimuth-dip pairs. From these we can identify for each block x the set S_x of blocks which must be removed before block x can be mined. This collection of blocks $x \cup S_x$, is usually referred to as a "cone". It can easily be generated using the minimum search pattern (MSP) algorithm of Caccetta and Giannini [1988].

2.2 The Land Surface Reshaping Problem

The landform to be shaped is represented by a block model. The block model is defined so as to include the highest and lowest points on the surface and be of great enough extent so as to avoid any anomalies at the boundaries. Many blocks are of course air blocks. The desired final slopes are specified and may vary with azimuth as well as height throughout the block model.

The land surface reshaping problem may be defined as specifying the surface which conforms to the desired slope criteria which may be achieved with the least material movement. The amount of material removed (cut) must be equal to the amount of material deposited (fill) and this material movement must be a minimum. This is displayed pictorially in Figure 1.

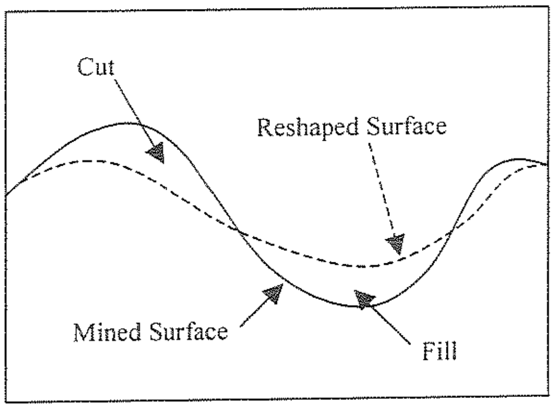

Figure 1. Schematic View of the Land Surface Reshaping Problem.

We now formulate the land surface reshaping problem as an integer linear programming problem. For a model of N blocks, let x_n be a variable corresponding to block n, $n = 1,2,\ldots,N$, such that

$$x_n = \begin{cases} 1, & \text{if a block n is above the reshaped surface,} \\ 0, & \text{otherwise.} \end{cases}$$

Let c_n be a weight corresponding to the blocks of the model defined as:

$$c_n = \begin{cases} -1, & \text{if a block n is above the mined surface (air block),} \\ 1, & \text{if a block n is below the mined surface.} \end{cases}$$

The objective function of minimising the material movement can be written as:

Minimise $\sum_{n=1}^{N} c_n x_n$.

Linear constraints on block removal can be introduced to ensure that the reshaped surface conforms to the safe wall slope criteria. A block may only be removed if all its overlying blocks have been removed. That is, the block corresponding to a variable x_i can only be removed if all overlying blocks have been removed. More specifically, if block j is an overlying block, then we have the restriction $x_i - x_j \leq 0$. If each block in the model has at most R overlying blocks and E is defined to be an (N.R) by N dimensional matrix of (0,1,-1) coefficients for the block removal constraints, then these constraints are given by

$$EX \leq 0,$$

where $X = (x_1, x_2, \ldots, x_N)^T$ is an N element column vector.

Suppose there are p blocks of positive weight which are above the reshaped surface (cut) and m blocks of negative weight which are below the reshaped surface (fill). If M is the total number of blocks of negative weight then

$$\sum_{n=1}^{N} c_n x_n = (-1)(M - m) + p = -M + m + p.$$

So minimising $\sum_{n=1}^{N} c_n x_n$ is equivalent to minimising the amount of material removed (p) plus the volume filled (m), which is one of the requirements of the problem definition.

The other requirement is the necessity for the amount of material removed to be equal to the amount of material filled, that is p = m. Since the number of blocks above the reshaped surface $\left(\sum_{n=1}^{N} x_n\right)$ is M − m + p, this can be expressed as requiring

$$\sum_{n=1}^{N} x_n = M .$$

Thus the integer programming formulation of the land surface reshaping problem is:

Minimise $\quad CX \quad (1)$

subject to

$$EX \leq 0 \quad (2)$$

and

$$\sum_{n=1}^{N} x_n = M \quad (3)$$

where X is a column vector with 0-1 components, $C = (c_1, c_2, \ldots, c_N)$ is a row vector of costs and E is the matrix defined earlier.

The above MILP formulation is in fact analogous to the mining to a tonnage (scheduling) optimisation problem considered in Caccetta and Giannini [1990]. Thus solution methods for mine scheduling can be used to solve the land surface reshaping problem. We discuss these further in the next section.

3. SOLUTION METHODS

The direct application of powerful MILP software packages such as CPLEX will solve only small sized problems and are unlikely to yield good solutions to practical sized problems. Branching algorithms utilizing the cutting plane method may yield good solutions for practical sized problems. In this section we detail how the Lagrangian relaxation method can be used to obtain a solution.

We replace the equality (equation 3) by the inequalities:

$$\sum_{n=1}^{N} x_n \leq M \quad (4)$$

and

$$-\sum_{n=1}^{N} x_n \leq -M . \quad (5)$$

Relaxing these constraints by introducing non-negative multipliers λ_1 and λ_2, we can write down the objective function for the resulting Lagrangian relaxation problem as:

Maximise

$$-\sum_{i=1}^{N} (c_i x_i) - \lambda_1\left(\sum_{i=1}^{N} x_i - M\right) + \lambda_2\left(\sum_{i=1}^{N} x_i - M\right)$$

$$= \sum_{i=1}^{N} (\lambda_2 - \lambda_1 - c_i)x_i + (\lambda_1 - \lambda_2)M$$

$$= -(C + \lambda A)X + \lambda M,$$

where A is an N dimensional row vector of ones and $\lambda = \lambda_1 - \lambda_2$ is the Lagrangian multiplier that may take on positive or negative values. So the Lagrangian relaxation problem may be stated as:

$$\text{Maximise} \quad -(C + \lambda A)X + \lambda M \qquad (6)$$

subject to

$$EX \leq 0 \qquad (7)$$

with E and X as previously defined.

Let $l(\lambda)$ be the optimal solution of the above problem. Then the desired solution for the original problem (1) – (3) is found by solving:

$$\lambda^* = \min \{l(\lambda) : \lambda \text{ unrestricted in size}\} \qquad (8)$$

The solution to this dual problem is difficult because of its discrete nature. In practice duality gaps may arise. However, for a fixed λ the problem (6) – (7) has, as briefly explained below, an identical structure to the open pit mine production scheduling problem [see Caccetta et al., 1998] as well as the ultimate open pit limit problem [Caccetta et al., 1991]. Note that (7) specifies the wall slope requirements.

The ultimate open pit limit problem is that of determining the contour which satisfies the safe wall slope and which yields the maximum profit. Mathematically, this problem can be formulated as follows. Consider an ore body with N blocks. Let p_i be the net value (profit) of block i and S_i the set of overlying blocks associated with block i. The ultimate pit limit problem can be written as:

$$\text{Maximise} \quad \sum_{i=1}^{N} p_i x_i \qquad (9)$$

subject to

$$x_i - x_j \leq 0 \text{ for all i and } j \in S_i \qquad (10)$$

$$\text{and} \quad x_i = 0, 1 \text{ for all i} \qquad (11)$$

Note that (10) can be written as:

$$EX \leq 0 \qquad (10^1)$$

where E is an appropriate matrix of (0,1,-1) coefficients and X is a column vector of binary variables.

From the solution of (9) – (11) we have the final pit contour defined as:

$$\Gamma = \{i : x_i = 1\}.$$

The optimal contour Γ corresponds to the maximal closure in a graph. In the graph model of an ore body, each block is represented by a vertex, the mining restrictions are represented by arcs (the arc (x,y) signifies that block x requires the mining of block y) and the value of a block is represented by a weight on the vertex representing it. A closure of graph is a subset of vertices with no arc directed to a vertex not in the set. The value of the closure is the sum of weights of the vertices in it. Thus the ultimate pit limit problem can be expressed as a graph optimisation problem.

The optimal contour can be effectively determined using a network flow method or the Lerchs-Grossmann algorithm [Caccetta et al., 1994]. For a given λ, the problem (6) – (7) is just the ultimate pit limit problem with objective function coefficients given by $-(C + \lambda A)$.

The production scheduling problem incorporates additional constraints that relate to factors such as: mill throughput; volume of material extracted; blending requirements; stockpile capacity; and various logistic constraints. Dualising these constraints results in a problem with a similar form to that of (6) – (7).

Optimisation methods for solving the mine production scheduling problem include: a number of heuristics; branch and cut; and Lagrangian relaxation [Caccetta and Hill, 2000; Caccetta et al., 1998]. The most successful computational results are those of Caccetta and Hill [2000].

The Lagrangian relaxation problem (6) – (7) can be tackled by the application of the subgradient optimisation algorithm [Held et al., 1974; Sandi, 1979]. This is a numerical technique for solving non-differentiable optimisation problems. It allows a sequence of λ's (or λ_1 and λ_2) to be derived which eventually converges to produce an optimal solution. Theoretically, it is guaranteed to determine the optimal λ, but in practice convergence may be slow.

The computational problems that arise relate to "duality gaps". Caccetta et al., [1998] implemented the subgradient optimisation algorithm for the mine scheduling problem. This same algorithm can be used for solving the problem (6) – (7). The only difference is that here λ may be negative.

For a given λ the relaxed problem is solved using the network flow method. The Lerchs-Grossmann algorithm can also be used and tends to be more efficient for mines which have wall slope restrictions that give rise to a large number of elements (arcs) in the search pattern.

4. COMPUTATIONAL RESULTS

We have implemented the subgradient optimisation algorithm originally developed for the mine scheduling problem [Caccetta et al., 1998]. Here we briefly discuss an application to a small real open-pit mine. The actual block values were scaled to protect the anonymity of the mine.
The block model consists of 20,979 five metre cubic blocks on 9 levels, 37 columns and 63 sections. The safe wall slope criteria for the pit are 55° to the north and south, 60° to the east and 50° to the west. The ultimate pit limit contains 611 ore blocks and 1231 waste blocks. The mine had a life of 5 periods and a production schedule was produced for the 5 periods using the subgradient optimisation technique along with a heuristic [Caccetta et al., 1998]. The final pit contour was reshaped as discussed below.

The subgradient optimisation algorithm is used to adjust a single Lagrange multiplier λ to obtain the maximum solution of the objective function (6) which satisfies the slope constraints as well as the $\sum_{n=1}^{N} x_n = M$ constraint. For this particular problem the optimum solution is obtained with a negative λ. This transpires because of the relative lack of worth of the positive blocks in the model. For a positive value of λ the maximum pit contour does not contain enough blocks to enable the $\sum_{n=1}^{N} x_n = M$ (i.e. p = M) constraint to be satisfied.

The final pit is reshaped using two different maximum slope criteria. The first case is for a maximum allowable slope throughout the pit of 30°. The results from the reshaping process are in Table 1. A duality gap with respect to the $\sum_{n=1}^{N} x_n = M$ constraint is apparent. M is equal to 1443, the duality gap is between 1439 and 1461. The better result has the number of blocks cut equal to 304 and the number to be filled equal to 308. The original pit had a depth of 9 blocks. The reshaped pit has a depth of 6 blocks and is of much greater extent laterally than the original pit.

This pit is also reshaped using a maximum slope criteria of 15° throughout the pit. The results from the reshaping process are in Table 1. The duality gap with respect to the $\sum_{n=1}^{N} x_n = M$ constraint is larger in this case. This is primarily because the MSP associated with the 15° maximum slopes contains more elements.

Table 1. Results of Reshaping.

Desired M	1443	
Possible M's	1439	1461
Number of Blocks Cut	304	318
Number of Blocks Filled	308	300
Total (Cut + Filled)	612	618

(a) Slope 30°

Desired M	1443	
Possible M's	1440	1484
Number of Blocks Cut	537	568
Number of Blocks Filled	540	527
Total (Cut + Filled)	1077	1095

(b) Slope 15°

5. CONCLUSIONS

This paper developed a mathematical formulation of the important land surface reshaping problem. A mixed integer linear programming model is presented and solved using the subgradient optimisation algorithm. The relaxed problems are solved using the methods of the ultimate pit limit problem. A real mine is used to illustrate the method. The solution methodology presented provides a fast effective tool which enables mine planners to examine various reshaping scenarios. It is a considerable improvement on the currently used procedures which are essentially manual.

6. REFERENCES

Caccetta, L. and L.M. Giannini, Optimisation techniques for the open pit limit problem, *Proceedings of the Australasian Institute of Mining and Metallurgy*, 291(8) 57-63, 1986.

Caccetta, L. and L.M. Giannini, The generation of minimum search patterns in the optimum design of open pit mines, *Proceedings of the Australasian Institute of Mining and Metallurgy*, 293(5) 57-61, 1988.

Caccetta, L. and L.M. Giannini, Application of operations research techniques in open pit mining, in Byong-Hun Ahn (ed.), *Asia-Pacific Operations Research: APORS '88*, Elsevier Science Publishers B.V. North-Holland 707-724, 1990.

Caccetta, L., L.M. Giannini and P. Kelsey, Optimum open pit design: A case study, *Asia Pacific Journal of Operations Research*, 8(2) 166-178, 1991.

Caccetta, L., L.M. Giannini and P. Kelsey, On the implementation of exact optimisation techniques for open pit design, *Asia-Pacific Journal of Operations Research*, 11, 155-170, 1994.

Caccetta, L. and S. Hill, An application of branch and cut to open pit mine scheduling, submitted for publication, 2000.

Caccetta, L., P. Kelsey and L.M. Giannini, Open pit mine production scheduling, *Third Regional APCOM Computer Applications in the Minerals Industries International Symposium, The Australasian Institute of Mining and Metallurgy Publication Series 5*, 65-72, 1998.

Fawcett, M.N.R. and G. Sinclair, Rehabilitation of final voids at the Pine Creek goldfields, *Proceedings of the Post-Mining Landform Stability and Design Workshop*, L.C. Bell and R.W. McLean (ed.), 107-109, 1997.

Gignac, L., Computerized ore evaluation and open pit design, *Proceedings 36th Annual Mining Symposium of the Society of Mining Engineers*, AIME, New York, 46-53, 1975.

Held, M., P. Wolfe and H.P. Crowder, Validation of subgradient optimisation, *Mathematical Programming 6*, North Holland Publishing, 62-88, 1974.

Hollands, K.M., Legislative requirements for final voids in New South Wales, *Proceedings of the Post-Mining Landform Stability and Design Workshop*, L.C. Bell and R.W. McLean (ed.),107-109, 1997.

Mine and Quarry Rehabilitation in South Australia, *Special Publication Vol 4*, Department of Mines and Energy, Adelaide, South Australia, 1988.

Orr, M.S. and P. Veivers, Waste dump rehabilitation at Mt Leyshon gold mine, *Proceedings of the Post-Mining Landform Stability and Design Workshop*, L.C. Bell and R.W. McLean (ed.), 67-74, 1997.

Sandi, C., Subgradient optimisation, in Christofides, N., Mingozzi, A., Toth, P. and Sandi, C. (ed.), *Combinatorial Optimisation*, 73-91, John Wiley and Sons, 1979.

Smith, P. and R. Hillis, BHP Iron Ore protocol for mine closure, *Proceedings of the Australasian Institute of Mining and Metallurgy 1996 Annual Conference: Diversity: The Key to Prosperity*, Vol 1/96, 67-71, AUSIMM, Australia, 1996.

Williams, D.J., Overseas trends in post-mining waste rock dump design, *Proceedings of the Post-Mining Landform Stability and Design Workshop*, L.C. Bell and R.W. McLean (ed.), 83-92, 1997.