Models for the Location of Emergency Facilities

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Abstract: The location of emergency service facilities such as ambulance and fire stations are a major concern and challenge for modern city planners. In addition to the challenge of effectively meeting the service demands, managers of service facilities need to deal with budget cuts as well as the ever increasing volume of data and information made available by modern technology. The location problems, which arise in emergency services, can be broadly classified into three categories namely strategic, tactical and operational. Strategic problems involve the location of fixed facilities for example hospitals, emergency centers, etc. Tactical problems relate to the location and relocation of emergency vehicles at any point in time. Operational problems are concerned with procedures to be followed by staff (paramedical) when calls are received. Our concern here is with the strategic and tactical categories. The optimization problems that arise are computationally difficult and complex. This paper discusses a number of models for this difficult class of problems. Dominance concepts are introduced for problem size reduction.

Keywords: Emergency; Location; Facilities; Coverage; Models

1. INTRODUCTION

The provision and utilization of effective and efficient emergency services is an important optimization problem encountered in all parts of the world. Typically, the optimization problem is to find the 'best' location of facilities that meets the desired requirements. The objective to be optimized is usually a function that measures the quality of service or the cost/output of the system. The requirements usually relate to factors such as demand, response time and capacity. In addition to the location one also needs to determine the allocation of "customers" to facilities.

The fundamental objectives of emergency facility location can be summarized into two categories the first refers to those designed to cover demand within a specified response time (covering models) and the second refers to those designed to minimize the average weighted distance or response time (*p*-median models).

The optimization models developed to solve such problems often yield good outcomes for the location of emergency facilities of which some examples are given below.

- In Montreal Canada, a heuristic method (tabu search) was used to solve the ambulance location problem. A graph model of the network using double coverage criteria was considered (7-minute and 14-minute response time). Computational results showed that this method was very efficient in improving ambulance coverage in modern cities [Gendreau et al., 1998].
- An optimization model was also developed and applied to the allocation of emergency vehicles in Louisville, Kentucky. As a result the ambulance response time decreased by 36% [Repede and Bernardo, 1994].
- An efficient model was developed to reassess the ambulance deployment in Tucson, Arizona. The model was to evaluate potential changes in paramedic services provided in the city [Goldberg et al., 1990].
- In Bangkok, Thailand the ambulance location was reorganized by applying a deterministic and simulation model to the location problem. The results led to a reduction in the number of ambulance in the city but maintaining the level of service. This resulted in a cost reduction in providing the service [Fujiwara et al., 1987].

The objective of this paper is to discuss some models for emergency facilities with a special reference to the p-median problem. The paper is organized as follows. In Section 2 we discuss the covering models and the p-median models. Some solution methods of the p-median problem are discussed in Section 3. In Section 4 we introduce the concept of dominance for reducing the problem size. Conclusions are presented in Section 5.

2. MODELS

Decision models for locating emergency facilities have been around for sometime now. Most of the early models assumed that emergency facilities would always be available once they are positioned Badri et al (1998).

We begin with the simplest covering models and the p-median problem. We use the following notation:-

$I = \{1,....,m\}$, the set of demand locations,

$J = \{1,...,n\}$, candidates sites for facilities,

d_{ij} = the shortest distance between location i and location j,

x_{ij} = 1 if the customer at location i is allocated to facility at location j, 0 otherwise,

y_j =1 if a facility is established at location j, 0 otherwise,

p = the number of facilities to be established,

x_j =1 if a server is stationed at j, 0 otherwise,

y_i = 1 if node i is covered, 0 otherwise,

S = time or distance standard for coverage,

N_i =the set of nodes j located within the standard distance or time from demand node i,

a_i = the population at the demand node i.

2.1 Location Set Covering Problem (LSCP)

Mathematically the LSCP developed by Toregas et al. [1971] can be formulated as

$$\text{Minimize } Z = \sum_{j \in J} x_j \tag{1}$$

subject to

$$\sum_{j \in N_i} x_j \geq 1 \ \forall i \in I; \ x_j \in \{0,1\}, \ \forall j \in J \tag{2}$$

The objective stated in (1) minimizes the number of facilities. Constraint (2) ensures that the demand at each node i is covered by at least one server located within the time or distance standard S.

The need to provide each demand point with a facility within a specified standard time or distance often results in solutions with large number of facilities, many of which cover fringe locations having a small demand. One approach to solve this problem has led to the development of the Maximal Covering Location Problem (MCLP) by Church and ReVelle [1974].

2.2 Maximal Covering Location Problem (MCLP)

The MCLP seeks to maximize the number of demands covered within the time standard by a fixed number of facilities. The mathematical formulation of a MCLP can be written as:

$$\text{Maximize } Z = \sum_{i \in I} a_i y_i \tag{3}$$

subject to

$$y_i \leq \sum_{j \in N_i} x_j \ , \forall i \in I \tag{4}$$

$$\sum_{j \in J} x_j = p \tag{5}$$

$$x_j, y_i \in \{0,1\} \ , \ \forall j \in J \quad i \in I$$

The objective (3) maximizes the sum of covered demands. Constraint (4) indicates that demand i can only be covered if at least one server is located within the time or distance standard S. Constraint (5) ensures that p facilities are located.

The above models assume the demand node receives complete benefit if is within the coverage distance. However, the benefit associated with a demand node/facility pair changes gradually with the distance between the demand and nearest facility. The p-median problem developed by Hakimi [1964] addresses this issue by examining the relationship between the distance between the facility and the demand point.

2.3 The *P*-Median Problem

The p-median problem consists of determining the location for p service centers so as to minimize the overall costs to supply the customers given that each customer is supplied by the service center nearest to it. The p-median problem arises naturally in locating plants or warehouses to serve other plants or warehouses. It can also be used to locate emergency centers [Serra and Marinov, 1998, Mirchandani, 1980].

The mathematical formulation of a p-median problem with a uniform weight (i.e. each node has the same demand) can be specified as follows,

$$\text{Min} \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \quad (6)$$

subject to

$$\sum_{j \in J} x_{ij} = 1, \quad \forall i \in I \quad (7)$$

$$\sum_{j \in J} y_j = p \quad (8)$$

$$x_{ij} \leq y_j \quad \forall i \in I, \forall j \in J \quad (9)$$

$$y_j \in \{0,1\}, \; x_{ij} \in \{0,1\} \quad (10)$$

The objective (6) is to minimize the total distance from customers or clients to their nearest facility. Constraint (7) shows that the demand of each customer or client must be met. From constraint (8) the number of facilities to be located is p. Constraint (9) shows that customers must be supplied from an open facility, and constraint (10) specifies the binary nature of the variables. If (6) is replaced by

$$\text{Min} \sum_i \sum_j a_i d_{ij} X_{ij},$$

then we have the p-median problem with varying demands (a_i is the demand at node i).

Several extensions have been proposed for the covering and the p-median based models to improve their efficiency [Daskin et al., 1988]. See also ReVelle [1989] for a detail review of extension to covering models. Extensions to the p-median problem that account for its stochastic nature has been given by Fitzsimmons [1973], Weaver and Church [1985] and Swoveland et al. [1973].

3. SOLUTION METHODS FOR THE *P*-MEDIAN PROBLEM

The p-median problem is a computationally difficult problem to solve [the problem is *NP*-hard on general networks]. Most solution methods are heuristic because of the large number of variables and constraints that arise. The heuristics are based on: genetic algorithms, simulated annealing, tabu search, node partitioning, node insertion, node substitution and various hybrids [see Hosage and Goodchild, [1986]; Golden and Skiscism, [1986]; Glover [1990]]. Some of these heuristics together with Lagrangian relaxation, which is one of the most successful exact methods, are briefly discussed below.

3.1 Lagrangian Relaxation

Lagrangian relaxation is based on the principle that removing constraint from a problem makes the problem easier to solve. Generally, Lagrangian relaxation removes a constraint and solves the revised problem but introduces a penalty for violating the removed constraint. The solution procedure for solving the problem is stated below.

The Lagrangian relaxation for the p-median is given as

$$L(\lambda) = \min \sum_i \sum_j d_{ij} x_{ij} + \sum_i \lambda_i \left(1 - \sum_j x_{ij} \right) \quad (11)$$

subject to constraints (8)-(10).

The expression $r_j = \sum_i \min\{0, d_{ij} - \lambda_i\}$ (12)

is used to minimize the objective function (11) for the fixed values of the Lagrange multipliers. We set

$$x_{ij} = \begin{cases} 1, \text{ if } y_j = 1 \text{ and } d_{ij} - \lambda_i < 0 \\ 0, \text{ otherwise} \end{cases} \quad (13)$$

The lower and upper bounds of the objective function are determined by using the variables of modified and unmodified problems respectively. The next step involves the use of subgradient optimization to update the value of the Lagrange multipliers [see Daskin, 1995].

$$\lambda_i^{m+1} = \max\left\{0, \lambda_i^m - t^m \left(\sum_j x_{ij}^m - 1 \right) \right\} \quad (14)$$

$$t^m = \frac{A^m (UB - L^m)}{\sum_i \left\{ \sum_j x_{ij}^m - 1 \right\}^2} \quad (15)$$

where

A^m = a constant on the mth iteration,

t^m = the stepsize at the m[th] iteration of the Lagrangian procedure,

UB = the best (smallest) upper bound on the P-median objective function,

L^m = the value of the objective function using the solution obtained from the relaxed problem,

x_{ij}^m = the optimal value of the allocation variable at the m[th] iteration.

An optimal solution is found if the lower bound is equal or very close to the upper bound. Narula et al. [1977] and Galvao [1980] and Beasley [1993] have successfully applied the subgradient optimization to solve a number of problems with up to 900 nodes. However for the larger problems tested the computational time is excessively large.

3.2 Heuristics

We begin our discussion by observing that it is a simple task to allocate a set of m customers to p facilities J' with fixed locations. We just determine

$$d_{ij_i^*} = min\left\{d_{ij}\right\},\ 1 \le i \le m,\ j \in J' \qquad (16)$$

and allocate customer i to facility j_i^*. This provides us with a simple tool for generating feasible solution and for exploring alternate solutions through exchange of facility locations. Simple constructive heuristics can be easily designed. We now describe three simple heuristics which are competitive with other methods and which are widely used today.

3.3 Teitz and Bart [1968] Heuristic

This is one of the first heuristics developed for the p-median problem. The heuristic starts by generating a feasible solution and then attempts to improve the current solution through a swapping operation, which relocates a facility to an unused site. The process continues until no further improvements in the objective function value are possible. The solution thus obtained is a local optimum.

3.4 Densham and Rushton Heuristics [1992]

Motivated by the observation that the Teitz-Bart method spends considerable computational time with non-improving swaps, Densham and Rushton [1992] proposed the GRIA (Global-Regional Interchange Approach) procedure. This method uses two exchange procedures, a local exchange and a global exchange. The global exchange selects the best site to drop (amongst the p) and the best site to add (amongst the n-p). The local exchange identifies for each site j in the current solution a set of neighboring sites N_j (which may consist of the customers that are served by the site) and selects the best site in N_j to replace site j.

3.5 Myopic Algorithm for the *P*-Median Problem

The myopic heuristic is a greedy type, which, works in the following way. Firstly a facility is located in such a way as to minimize the total cost for all customers. Facilities are then added one by one until p is reached. For this heuristic the location that gives the minimum cost is selected. Ardalan [1988] heuristic also works using the same procedure. The main problem with this approach is that once a facility is selected it stays in all subsequent solution. Consequently, the final solution attained may be far from optimal.

The above three heuristics are the simplest to understand and implement. The available computational evidence, though limited, supports the use of Teitz-Bart and the Densham-Rushton heuristics. Certainly they outperform the Genetic and Simulated Annealing algorithms. In applications, The Teitz-Bart and the Densham-Rushton heuristic are restarted a number of times in order to obtain a good solution. Usually, both heuristics start with a random initial solution. The stopping rule that is used is to restart the procedure until the best-generated solution has been identified a specified number r of times. In some numerical experiments with r = 10 the number of restarts has ranged from 16 to 2515 [see Church and Sorensen; 1994].

Recently some researchers [Salhi; 2002, Rolland et al. 1996; Voss; 1996] have focused on the application of the Tabu Search method to the p-median problem. Preliminary results are encouraging, but more work needs to be done to establish the superiority of this approach. This method utilizes a swap move and a number of criteria (tabu tenure, aspiration level, intensification strategy etc.) to guide the search.

4. DOMINANCE CRITERIA FOR THE *P*-MEDIAN PROBLEM

The previous section briefly explained some of the algorithms that have been proposed for the p-median problem. There has not been any serious comparative analysis of the various algorithms. Indeed, computational work has been somewhat limited. Practical problems with several thousand nodes are not uncommon. Exact methods for such applications are computationally expensive or infeasible. Heuristic algorithms (eg. Teitz-Bart and Densham-Rushton) may require many restarts to generate a good solution or, as in the case of the Myopic algorithm give a solution far from optimal. One possible way of achieving some improvement is by reducing the size of the problem. Our aim here is to introduce some ideas in this direction. We illustrate our ideas with a simple example and do not indulge in computational work as this is outside the scope of this paper.

Problem size reduction is a widely used strategy in addressing large and computationally difficult optimization problems. We now introduce some reduction methods for the p-median problem using the concept of dominance.

Consider a weighted p-median problem given as $D=(d_{ij})$. Note that each row (column) of D identifies a demand (facility) location. We say **column k dominates column l** if $d_{ij} \le d_{il}$ for all $i \ne j$. We use the term **strongly dominates** in the case of strict inequality. Observe that locating a facility at a dominated location l would provide no advantage to locating a facility at k except possibly in serving the demands of customers in location l. Further, strongly dominated columns would only be use for "self-serve". Consequently dominated columns can be dropped to generate a feasible solution and later be considered for "self-service".

We extend slightly the concept of dominance as follows. We say **columns** ***k*** **and** ***l*** **dominate column** ***j*** if $d_{ij} \ge min\{d_{ik}, d_{il}\}$ for all $i \ne j$. In this case there is no advantage in using location j (except for serving customers in location j) when locations k and l are used. So again we can drop the dominated column j if sites k and l are used. The term strongly is used as before. We illustrate this concept with the following example.

Example 1: Consider the p-median problem with the weighted distance matrix D gives as:

$$\begin{bmatrix} 0 & 10 & 66 & 29 & 91 \\ 10 & 0 & 68 & 58 & 45 \\ 66 & 68 & 0 & 100 & 92 \\ 29 & 58 & 100 & 0 & 84 \\ 91 & 45 & 92 & 84 & 0 \end{bmatrix}$$

Observe that column 1 strongly dominates columns 3 and 5 and column 2 strongly dominates column 4. So we can consider the reduced matrix D' consisting of columns 1 and 2.

We would never locate a facility at locations 3, 4 or 5 except for "self-serve". If $p = 2$ the optimal solution for D' is to locate facilities at location 1 and 2. Facility 1 would serve customers in locations 1, 3 and 4 whilst facility 2 serves customers in locations 2 and 5. Using this as an initial solution we consider the original problem with matrix D. Now the current solution has objective function value of 140. If we utilize any of facility locations 3, 4 or 5 then we need the optimal solution of the 1-median problem for matrix D'. This solution, which, places a facility at location 1, has objective function value of 196.

The values of the candidate solutions, which use location 1 and a "self-serve" facility, are:

Table 1: Candidate Solutions

Self-Serve Facility	3	4	5
Objective Function Value	130	167	105

Thus the optimal 2-median solution is a facility at location 1 to serve locations 1, 2, 3 and 4 and self-serve facility at location 5.

For p>2 we can, since columns 3 and 4 are strongly dominated and columns 1 and 5 strongly dominate column 2, allocate the (p-2) most expensive customers in the above (p = 2) optimal solution to (p-2) self-serve facilities at locations 2, 3 or 4. Again this provides the optimal solution in this example.

The above example can also be used to illustrate the problem with the Myopic heuristic. With p = 1 the heuristic would locate the single facility at site 2. With one facility fixed at site 2, it would then choose site 3 for the location of the second facility. Continuing in this way we obtain the solution for all values of p. These along with the solution generated using dominance are given in the following table.

Table 2: Illustration of Dominance

P	Solution			
	Dominance		Myopic	
	Facilities	Objective	Facilities	Objective
2	{1,5}	105	{2,3}	113
3	{1,3,5}	39	{1,2,3}	74
4	{1,3,4,5}	10	{1,2,3,5}	29

The Myopic can yield a poor solution because earlier chosen sites cannot be changed. Of course, both the Teitz-Bart and the Densham-Rushton heuristics overcome this problem through swaps.

The dominance concept we have introduced above can be used with any computational procedure and can assist by reducing the problem and identifying certain sites as potential self-serve sites. Further, given a set of sites, dominance can be used to identify candidates for inclusion in considering swaps.

5. CONCLUSIONS

This paper discusses the computational methods that can be used for determining the optimal location of facilities in a given network. Available methods are of limited use in practical sized problems involving a large number of demand/site locations. This paper introduces a reduction method based on dominance for the p-median problem. The method can be used with any algorithm. It can identify potential sites for "self serve" and identify candidates for swapping moves.

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