A Simulation of the Heckscher-Ohlin Theorem

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Abstract: The Heckscher-Ohlin (H-O) theorem is one of the classical results in international trade theory. It asserts that capital-rich countries export capital intensive commodities, while labor-rich countries export labor intensive commodities. In the real world, however, this tendency has not been observed. Two restrictive assumptions are required for this theorem to hold. One is the identity of utility functions between the two trading countries, and the other relates to production functions. In this paper, simulations are conducted to identify which assumption is more important in order for the H-O theorem to hold. Suppose that there are two countries in a two-commodity, two-factor model, where workers possess labor, while capitalists possess capital. Production functions (constant returns to scale) and utility functions are assumed to be of Cobb-Douglas type. In the first simulation, 10000 pairs of parameters on production and utility functions are selected randomly, where production functions on both countries are identical and utility functions can be different. The H-O property is observed for approximately 70% of the solutions. In the second, the same simulation is conducted where utility functions in both countries are identical and production functions can be different. Then, the H-O property is observed for approximately 50% of the solutions. In other words, when the production structure differs between the two trading countries, there is no tendency for the H-O theorem to hold, and the identity assumption of production functions is shown to be more important than the one of utility functions.

Keywords: International trade, Heckscher-Ohlin, Leontief paradox, Simulation.

1. INTRODUCTION

Since Ohlin [1933] explicitly asserted that capital-rich countries export capital intensive commodities, while labor-rich countries export labor intensive commodities, the Heckscher-Ohlin (H-O) theorem has been one of the fundamental theorems in international economics. In this theorem, two countries, say country A and country B, engage themselves in trade with each other. Country A is endowed with labor, Le_A, and capital, Ke_A, while country B, Le_B and Ke_B, respectively. If $Le_A/Ke_A>Le_B/Ke_B$, then country A is called the labor-rich country while country B, the capital-rich one. Suppose that they produce the same two commodities, with labor input for the i-th commodity, L_i, and capital input, K_i. (i=1,2) Furthermore, assume that for each country, say the first commodity (sector) is labor intensive and the second commodity (sector) is capital intensive; $L_1/K_1>L_2/K_2$. According to the H-O theorem, at the general equilibrium, country A exports the first commodity while country B exports the second commodity: the H-O property. With respect to this theorem, however, the Leontief paradox is also well-known. From an econometric analysis on the United States of America, Leontief [1953] asserted that there is no tendency for the H-O property to hold. This phenomenon is called the Leontief paradox. Even now, the H-O property is examined empirically by many economists [Davis & Weinstein, 1999].

Sufficient conditions for the Heckscher-Ohlin theorem are as follows:

- Production technologies are the same between the two countries.
- Constant returns to scale in production.
- Perfect competition prevails.
- Imperfect specializaition prevails.
- Homothetic utility functions are the same between the two countries.

If the first condition is not satisfied, the H-O theorem does not necessarily holds, as is shown in Layard & Walters [1978]. The purpose of this paper is to extend their analysis. Assuming the production and utility functions are Cobb-Douglas type, which may be different between the two countries, we conduct simulations. Thus, specifying parameters on those functions and endowment allocations randomly, we compute the probability of trade equilibrium which shows the H-O property, and examine which identity assumption is more important in order for the H-O theorem to hold.[1]

2. A CASE IN WHICH THE HECKSCHER-OHLIN THEOREM DOES NOT HOLD: DIFFERENCE IN THE PREFERENCES BETWEEN THE TWO COUNTRIES

Before conducting the simulation with random selection of parameters, it is ascertained that there exists a case in which the Heckscher-Ohlin theorem does not hold when the preference relations differ between the two countries. In this case, the production functions for the two countries are assumed to be of the same Cobb-Douglas type, where the one for the first sector is $f_{1A}= f_{1B}=K_1^{2/3} L_1^{1/3}$ and the one for the second sector is $f_{2A}= f_{2B}=K_2^{1/3} L_2^{2/3}$. It is assumed that country A is endowed with 49 units of Labor, Le_A, and 50 units of capital, Ke_A.

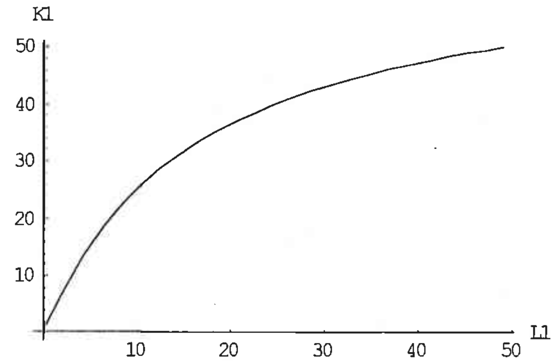

Figure 1. Efficiency Locus of the First Sector

In the first sector, inputs are selected from the efficiency locus:

$$K_1=200\ L_1/(49 + 3\ L_1)$$

This locus is depicted in Figure 1, so that $K_1/L_1> Ke_A/ Le_A$ always holds. On the other hand, in the second sector, inputs are selected from the efficiency locus:

$$K_2 =50\ L_2/(196\text{-}3\ L_2)$$

This locus is depicted in Figure 2, so that $K_2/L_2< Ke_A/ Le_A$ always holds. Thus, the first sector is capital-intensive, and the output of this sector is named good y. The second industry is labor-intensive, and its output is named good x.

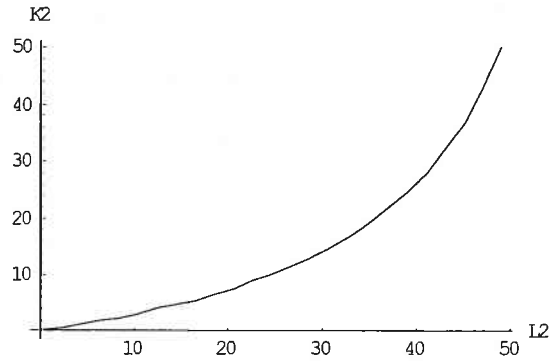

Figure 2. Efficiency Locus of the Second Sector

We proceed with the assumptions on the consumption side. The utility functions are different between the two countries. For the purpose of ease of derivation of the demand function, a Cobb-Douglas utility function is assumed with x-, and y-goods, variables. There are two types of classes; workers and capitalists. Utility functions for workers, u_L, and capitalists, u_K, are specified as follows. Within a country, the two classes are assumed to have the same utility function. In country A, $u_{LA}=u_{KA}=xy^2$. The workers class is assumed to possess solely Le_A, while the capitalist class is assumed to possess solely Ke_A. This Classical assumption follows that of Fukiharu [2000]. As for country B it is assumed to be endowed with 50 units of Labor, Le_B, and 49 units of capital, Ke_B, where $u_{LB}=u_{KB}=xy$.

In this country, the workers class is assumed to possess solely Le_B, while the capitalist class is assumed to possess solely Ke_B.

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[1] In this paper, *Mathematica* is heavily utilized for the computation and drawing figures. For the *Mathematica* programs, see my home page:

http://home.hiroshima-u.ac.jp/fukito/index.htm

Thus, country A is capital-rich, while country B is labor-rich. Following the traditional general equilibrium model, workers and capitalists maximize utility subject to an income constraint, while firms maximize profit.

There are eight markets in this trade model; two commodity markets in each country and two input markets in each country. Exports and imports are taken into account in each commodity market. There are eight prices to be determined in equilibrium; commodity prices in each country, p_{xA}, p_{yA}, p_{xB}, p_{yB}, and input prices in each country, p_{LA}, p_{KA}, p_{LB}, p_{KB}. Due to the assumption of constant returns to scale in each sector, only input prices need to be determined in equilibrium. Furthermore, the assumption of identity of production functions for the two countries results in factor-price-equalization; i.e. $p_{LA}= p_{LB}$ and $p_{KA}= p_{KB}$. Thus, only the factor prices in one country need to be computed in equilibrium. In other words, equilibrium in this model is defined by the existence of wage rate and rental fee in one country which equate supply and demand in labor and capital markets, where those inputs are used for the production to satisfy not only domestic demand but also foreign demand.

Applying the *Mathematica* command "Solve", we can compute the unique general equilibrium with trade under this specification; the capital-rich country A imports capital-intensive good y, on the amount of $401\ 5^{1/3}/2726^{2/3}$, exports labor intensive good x on the amount of $401\ 5^{2/3}/\ 29\ 1363^{1/3}$ and the equilibrium rental fee for capital is 940/841 when the wage rate is normalized to one. In other words, when different utility functions are allowed between the two countries, the Heckscher-Ohlin theorem does not necessarily hold.

3. SIMULATION 1: DIFFERENCE IN THE UTILITY FUNCTIONS BETWEEN THE TWO COUNTRIES

In the previous section, such parameters of the production and utility functions were selected such that the Heckscher-Ohlin theorem did not hold. In this section, parameters on production and utility functions and initial endowments are selected randomly and simulation of the previous model is conducted, to check whether a general equilibrium with trade has a tendency to support the H-O property. The function, check3[2], constructed in this section, computes the general equilibrium with trade, given parameters for the production and utility functions, and endowments. The general equilibrium is computed using the Newton Method ("FindRoot"; command name in *Mathematica*) and provided in the third element. The function check3 provides $Ke_A/Le_A - Ke_B/Le_B$ in the first element and $K_1/L_1 - Ke_A/Le_A$ in the second element. Thus, if the first element is positive, country A is relatively capital-rich country, while if the second element is positive the first sector is capital-intensive. This function is essentially a collection of the series of programs in the previous section.

By randomly selecting 10000 pairs of parameters on identical production functions, (different) utility functions, and (different) endowments, we examine how many cases show the Heckscher-Ohlin property. The result is that 6703 cases show the H-O property. Note, however, that a remark is in order when the Newton Method is applied. In this method, the initial value must be set before the application, and depending on that initial value, the convergence to equilibrium is not always guaranteed as exhibited in the simulation[3]. Thus, it may be safe to conclude that when the production functions are different between the two countries, the Heckscher-Ohlin theorem holds with the probability of approximately 70%.

4. A CASE IN WHICH THE HECKSCHER-OHLIN THEOREM DOES NOT HOLD: DIFFERENCE IN THE TECHNOLOGIES BETWEEN THE TWO COUNTRIES

In this section, an example is constructed in which the Heckscher-Ohlin theorem does not hold when the last condition (identical production functions) is not satisfied. It is assumed that the Cobb-Douglas production functions may differ between the two countries, while the same commodity is capital-intensive for the two countries. Country A's Cobb-Douglas production functions are assumed to be $f_{1A}=K_1^{2/3}\ L_1^{1/3}$, $f_{2A}=K_2^{1/4}\ L_2^{3/4}$, while Country B's are $f_{1B}=K_1^{3/4}\ L_1^{1/4}$, $f_{2B}=K_2^{1/3}\ L_2^{2/3}$. As for the initial endowments it is assumed that Le_A=499 and Ke_A=500, while

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[2] See my home page.

[3] See my home page.

Le_B=500 and Ke_B=499 for country B. Thus, country A is a capital-rich country. It must be noted that in the case of different technologies the factor-price- equalization property does not hold. Under this specification of parameters, the *Mathematica* command "Solve" cannot compute the equilibrium. The equilibrium rental fee of capital, p_{KA}, must be computed by equating the following excess demand for labor in country A, edLA, to zero;

$$edLA = -9481/3 - 704000\ 2^{14/15}/(243\ 3^{11/20}\ p_{KA}^{1/12}) + 31437\ 3^{17/20}\ p_{KA}^{11/12}/(16\ 2^{13/15}) + 8500\ p_{KA}/3$$

If p_{KA} is chosen between 0 and 2, Figure 3 guarantees the existence of general equilibrium near p_{KA}=1.

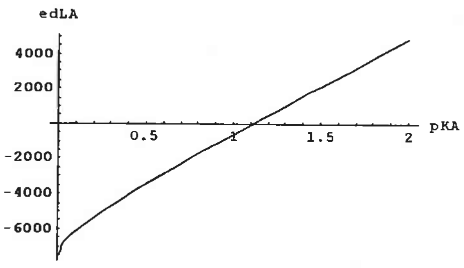

Figure 3. Existence of Equilibrium

If p_{KA} is chosen between 0 and 1000000, Figure 4 guarantees the uniqueness of general equilibrium

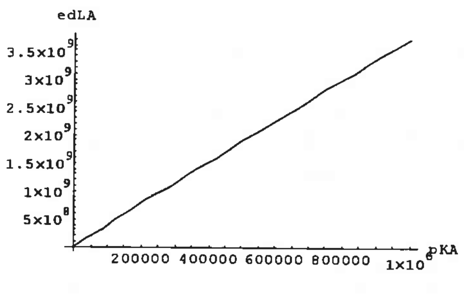

Figure 4. Uniqueness of Equilibrium

After graphically checking the existence of the unique solution, we obtain the result using the "Newton Method"; the equilibrium rental fee=1.10928 and country A's import of capital intensive good y=1.71637. Thus, when the different production functions are allowed between the two countries, the Heckscher-Ohlin Theorem does not necessarily hold.

5. SIMULATION 2: DIFFERENCE IN THE TECHNOLOGIES BETWEEN THE TWO COUNTRIES

In this section, parameters on production and utility functions and initial endowments are selected randomly and simulation of the previous model is conducted, to check whether a general equilibrium with trade has a tendency to exhibit the H-O property. It must be noted that when the production functions are selected randomly, there may arise a case in which good x is capital-intensive for country A and it is labor-intensive for country B. Thus, those parameters on production functions must be selected randomly, which may be different between the two countries, and the same commodity is capital-intensive for the two countries.

From 10000 cases, 4985 cases show the H-O property. Of course, from 10000 cases, some failed to convergence using the Newton process. However, there is no reason to believe that these cases have a tendency to either deny or affirm the Heckscher-Ohlin theorem. Thus, we may safely conclude that the Heckscher-Ohlin theorem holds with 50% probability when technologies differ between the two countries. In other words, when technologies differ between the two countries, there is no tendency for the capital-rich country to export capital-intensive commodity. In this sense, the Leontief paradox is a normal phenomenon.

6. CONCLUSIONS

Two equality assumptions are required for the Heckscher-Ohlin theorem to hold; the identity of utility functions and of production functions between the two countries. The aim of this paper is to construct a two-country, two-good, two-input trade model and examine which identity assumption is more important for the Heckscher-Ohlin theorem to hold by conducting simulation experiments.

From the above simulations we may conclude firstly that even if different utility functions are allowed between the two countries, there is a tendency for the Heckscher-Ohlin theorem to hold. Second, if different production functions are allowed between the two countries, there is no tendency for the Heckscher-Ohlin theorem to hold. Thus, similarity of production technologies between the two countries is more important for the Heckscher-Ohlin theorem than the one of utility functions.

7. REFERENCES

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