Mathematical Modeling of Forest Fire Spread Initiation

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Abstract: In this paper the theoretical investigation of the problem of forest fire spreading through the forest in windy situations was carried out. The research was made by means of the mathematical modeling methods of physical processes. It was based on numerical solution of two dimensional Reynolds equations for the description of turbulent flow taking into account for diffusion equations chemical components and equations of energy conservation for gaseous and condensed phases. To obtain discrete analogies a method of controlled volume of Patankar - Spalding was used. In this context, a study - mathematical modeling of the conditions of forest fire spreading that would make it possible to obtain a detailed picture of the change in the velocity, temperature and component concentration fields with time, and determine as well as the influence of different conditions on the forest fire spread, is of interest.

Keywords: Forest fire; Mathematical modeling; Control volume

1. INTRODUCTION

The general mathematical model of forest fire was obtained by Grishin [1992]. Let us examine a problem of radiation-convection heat and mass exchange of forest fuels in all forest strata with gaseous combustion products and radiation from the forest fire source. The fire source is modeled as a volume layer of burning forest fuels with known temperature and increasing area of burning. It is assumed that the forest during a forest fire can be modeled as a two-temperature multiphase non-deformable porous reactive medium. Let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products). To describe the transfer of energy by radiation we use a diffusion approximation, while to describe convective transfer controlled by the wind and gravity, we use Reynolds equations.

2. FOREST FIRE EQUATIONS

Let the coordinate reference point x_1, x_2, x_3=0 be situated at the center of the forest fire source, axis $0x_3$ directed upward, axis $0x_1$ and $0x_2$ directed parallel to the Earth's surface. Because of the horizontal sizes of forest massif more than height of forest – h, system of equations of general mathematical model of forest fire was integrated between the limits from height of the roughness level - 0 to h. Suppose that

$$\int_0^h \phi \, dx_3 = \bar{\phi} h ,$$

$\bar{\phi}$ - average value of ϕ. The problem formulated above is reduced to a solution of the following system of equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = Q - (\dot{m}^- - \dot{m}^+)/h ; \quad (1)$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}(-\rho \overline{v_i' v_j'}) - \rho s c_d v_i |\bar{v}|$$
$$- \rho g_i - Q v_i \quad + (\tau_i^- - \tau_i^+)/h , \bar{g} = (0, g) ; \quad (2)$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j}(-\rho c_p \overline{v_j' T'}) + q_5 R_5 -$$
$$- \alpha_v (T - T_s) + (q_T^- - q_T^+)/h ; \quad (3)$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j}(-\rho \overline{v_j' c_\alpha'}) + R_{5\alpha} - Q c_\alpha +$$
$$+ (J_\alpha^- - J_\alpha^+)/h, \ \alpha = 1,3; \quad (4)$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k}\frac{\partial U_R}{\partial x_j}\right)-k(cU_R-4\sigma T_S^4)+$$
$$+(q_R^- - q_R^+)/h=0; \qquad (5)$$

$$\sum_{i=1}^{4}\rho_i c_{pi}\varphi_i\frac{\partial T_S}{\partial t}=q_3R_3-q_2R_2+ \qquad (6)$$
$$+k(cU_R-4\sigma T_S^4)+\alpha_V(T-T_S);$$

$$\rho_1\frac{\partial\varphi_1}{\partial t}=-R_1,\ \rho_2\frac{\partial\varphi_2}{\partial t}=-R_2,$$
$$\rho_3\frac{\partial\varphi_3}{\partial t}=\alpha_C R_1-\frac{M_C}{M_1}R_3,\ \rho_4\frac{\partial\varphi_4}{\partial t}=0; \qquad (7)$$

$$\sum_{\alpha=1}^{3}c_\alpha=1, p_e=\rho RT\sum_{\alpha=1}^{4}\frac{c_\alpha}{M_\alpha},\vec{v}=(v_1,v_2); \qquad (8)$$

$$Q=(1-\alpha_c)R_1+R_2+\frac{M_c}{M_1}R_3, R_{51}=-R_3-$$
$$-\frac{M_1}{2M_2}R_5,\ R_{52}=\nu(1-\alpha_c)R_1-R_5, R_{53}=0. \qquad (9)$$

The system of equations (1)–(9) must be solved taking into account the following initial and boundary conditions:

$$t=0: v_1=0, v_2=0,\ T=T_e, c_\alpha=c_{\alpha e}; \qquad (10)$$
$$x_1=x_{10}: v_1=V_e, v_2=0, T=T_e, c_\alpha=c_{\alpha e}; \qquad (11)$$
$$x_1=x_{1e}: \frac{\partial v_1}{\partial x_1}=0, \frac{\partial v_2}{\partial x_1}=0, \frac{\partial c_\alpha}{\partial x_1}=0, \frac{\partial T}{\partial x_1}=0; \qquad (12)$$
$$x_2=x_{20}: \frac{\partial v_1}{\partial x_2}=0, \frac{\partial v_2}{\partial x_2}=0, \frac{\partial c_\alpha}{\partial x_2}=0, \frac{\partial T}{\partial x_2}=0; \qquad (13)$$
$$x_2=x_{2e}: \frac{\partial v_1}{\partial x_2}=0, \frac{\partial v_2}{\partial x_2}=0, \frac{\partial c_\alpha}{\partial x_2}=0, \frac{\partial T}{\partial x_2}=0. \qquad (14)$$

Here and above $\frac{d}{dt}$ is the symbol of the total (substantial) derivative; α_V is the coefficient of heat and mass exchange; t is time; x_i, v_i, $(i,j = 1, 2)$ are the Cartesian coordinates and the velocity components; index α=1,2,3, where 1 corresponds to the density of oxygen, 2 - to carbon monoxide CO (combustible products of pyrolysis), 3 – concentration of inert components of air; V_e – equilibrium wind velocity (average value of velocity), ρ - density of gas phase, p - pressure; c_p – constant pressure specific heat of the gas phase, c_{pi}, ρ_i, φ_i – specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel). Values of $\tau_i^+,\tau_i^-,J_\alpha^+,J_\alpha^-,q_T^+,q_T^-$ characterize the exchange by impulse, mass of α - components and energy with boundary layer of atmosphere and lower layer of forest [Grishin, 1992]. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a pine forest [Grishin, 1992]. To define source terms, which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae (9) were used for the rate of forming of the gas-dispersed mixture Q, outflow of oxygen R_{51} and changing carbon monoxide R_{52}. The source of ignition is defined as a function of time and turned off after the forest fire initiation.

Thus, the solution of the system of equations (1) - (9) with initial and boundary conditions (10) - (14) may result in defining the fields of velocity, temperature, component concentrations and radiation density. To close the system, the components of the tensor of turbulent stresses, and the turbulent heat and mass fluxes are determined using the local-equilibrium model of turbulence [Grishin, 1992].

The boundary-value problem (1) – (14) was solved numerically using the method of splitting according to physical processes. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting was then integrated. A discrete analog for equations (1) – (9) was obtained by means of the control volume method using the SIMPLE -like algorithm [e.g. Patankar, 1980; Perminov, 1995].

The accuracy of the program was checked by the method of inserted analytical solutions. Analytical expressions for the unknown functions were substituted in (1)–(9) and the closure of the equations were calculated. This was then treated as the source in each equation. Next, with the aid of the algorithm described above, the values of the functions used were inferred with an accuracy of not less than 1%. The effect of the dimensions of the control volumes on the solution was studied by diminishing them. The time interval was selected automatically.

3. TYPICAL CALCULATED RESULTS AND DISCUSSION

Fields of temperature, velocity, component mass fractions, volume fractions of phases and concentration of component gas phase were obtained numerically. At $V_e \neq 0$, the wind field in the forest canopy interacts with the gas-jet obstacle that forms from the forest fire source and from the ignited forest canopy base. On the windward side the movement of the air flowing past the ignition region accelerates. Figures 1 – 10 present the distribution of temperatures for gas and solid phase

($\overline{T} = T / T_e$, $\overline{T}_S = T_S / T_e$, $T_e = 300K$), concentration of oxygen, volatile combustible products of pyrolysis concentrations ($\overline{c}_\alpha = c_\alpha / c_{1e}$, $c_{1e} = 0.23$) and vectorial field of velocity for different instants of time when a wind velocity V_e= 5 m/s and moisture of forest combustible materials – 0.6: t=5 s (Figures1 - 5) and t=10 s (Figures 6 - 10). We can note that the isotherms is moved in the forest canopy and deformed by the action of wind. Similarly, the fields of component concentrations are deformed. It is concluded that the forest fire begins to spread.

High temperature zone is removed in x_1 – direction. The velocity of forest fire spread is about 7 m/s (Figure 1 and Figure 6). The zone of burning is extended in x_2 – direction as well but more slowly.

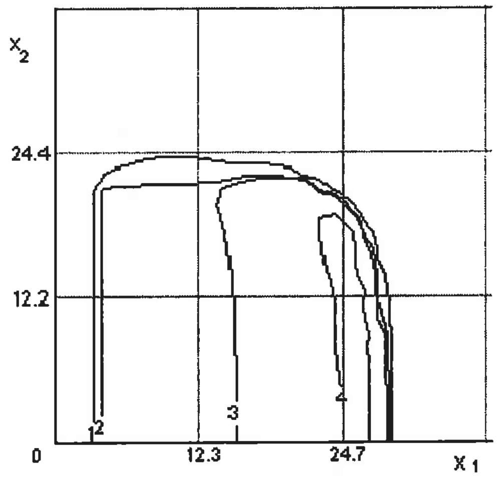

Figure1. Isotherms of the forest fire for t=5 s; $1 - \overline{T} = 1.5, 2 - \overline{T} = 2.6., 3 - \overline{T} = 3.5, 4 - \overline{T} = 5.$

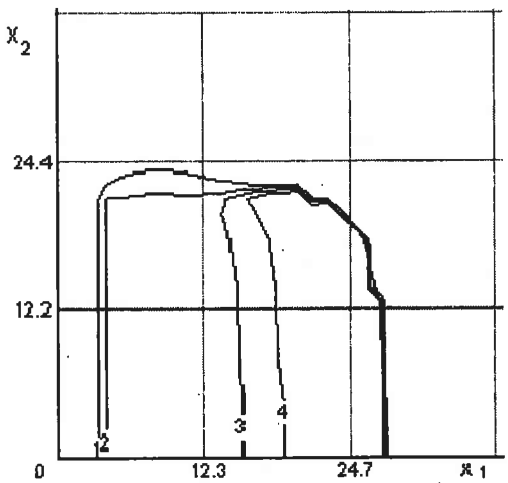

Figure 2. Isotherms of solid phase at t=5 s: $1 - \overline{T}_S = 1.5$, $2 - \overline{T}_S = 2.6$, $3 - \overline{T}_S = 3.5$, $4 - \overline{T}_S = 5$.

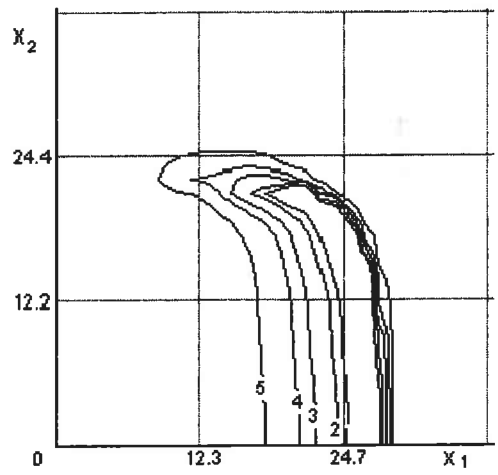

Figure 3. The distribution of oxygen $\overline{c}_1$
$1 - \overline{c}_1 = 0.5$, $2 - \overline{c}_1 = 0.6$, $3 - \overline{c}_1 = 0.75$, $4 - \overline{c}_1 = 0.85$, $5 - \overline{c}_1 = 0.9$.

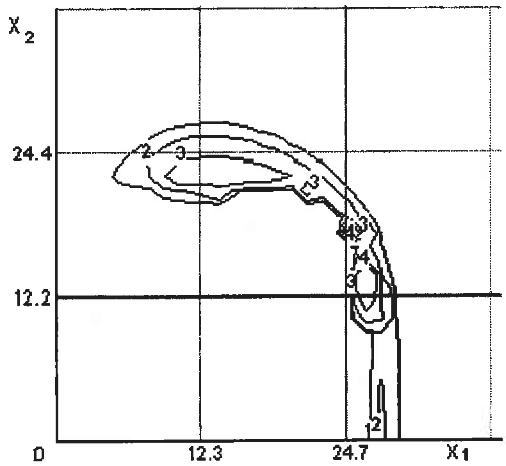

Figure 4. The distribution of $\overline{c}_2$ at t=10 s:
$1 - \overline{c}_2 = 0.01$, $2 - \overline{c}_2 = 0.03$, $3 - \overline{c}_2 = 0.12$, $4 - \overline{c}_2 = 0.4$.

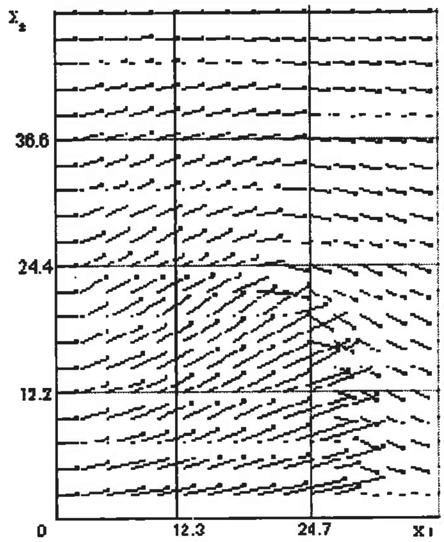

Figure 5. Vectorial field of velocity at t=5 s.

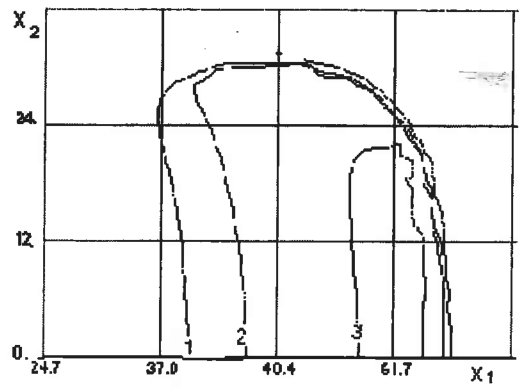

Figure 6. Isotherms of the forest fire for t=10 s:
$1-\overline{T} = 2.6, 2-\overline{T} = 3.5., 3-\overline{T} = 5.$

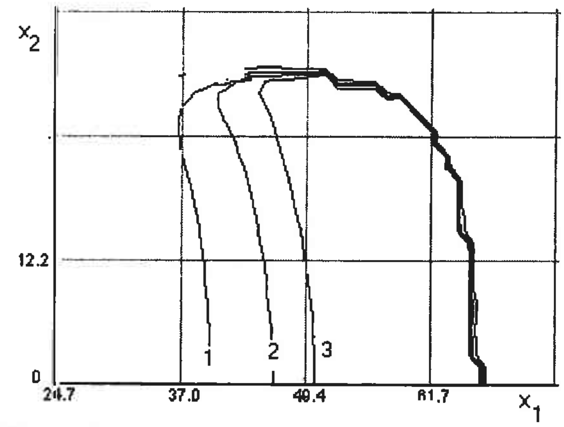

Figure 7. Isotherms of the solid phase for t=10 s:
$1-\overline{T}_s = 2.6, 2-\overline{T}_s = 3.5, 3-\overline{T}_s = 5.$

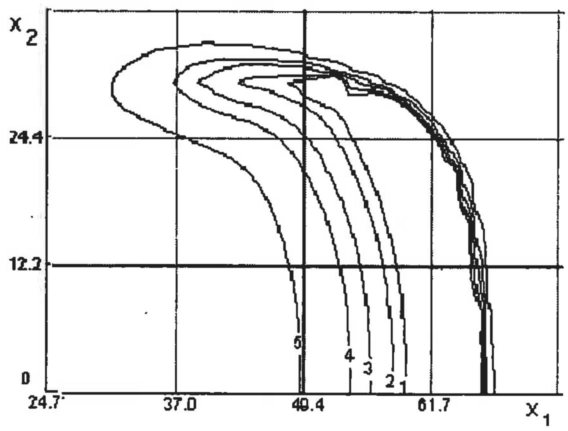

Figure 8. The distribution of oxygen $\overline{c}_1$ at t=10 s:
$1-\overline{c}_1=0.5$, $2-\overline{c}_1=0.6$, $3-\overline{c}_1=0.75$, $4-\overline{c}_1=0.85$, $5-\overline{c}_1=0.9$.

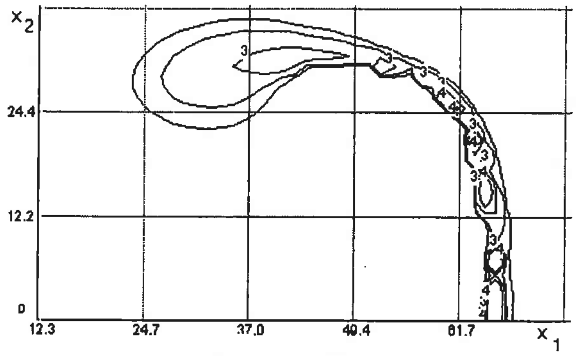

Figure 9. The distribution of $\overline{c}_2$ at t=10 s:
$1-\overline{c}_2=0.01$, $2-\overline{c}_2=0.03$, $3-\overline{c}_2=0.12$, $4-\overline{c}_2=0.4$.

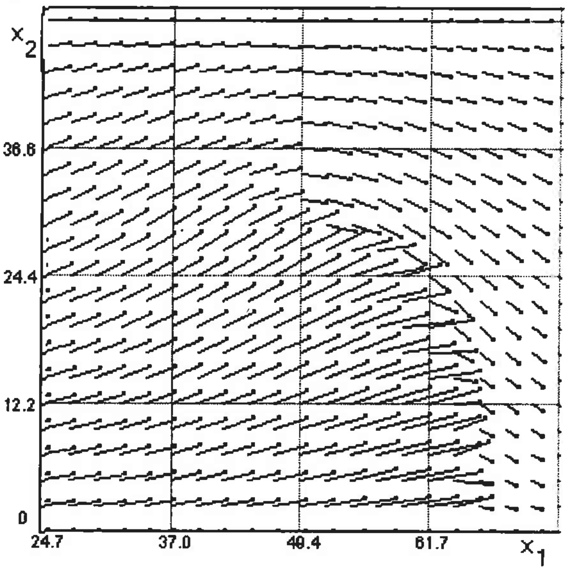

Figure 10. Vectorial field of velocity at t=10 s.

Figures 4 and 9 show the maximum concentration of combustible products of pyrolysis occur in front of the burning zone. It is seen in Figures 3 and 8 that the oxygen concentration is gradually decrease due to process of combustion. The maximum reduction of $\overline{c}_1$ occurs at points of maximum temperature.

Of interest is the investigation of moisture of forest fuel impact on the process of forest fire spread. Calculations demonstrated that the size of burning zone (at $w=0.7.$) is less than at $w=0.6$. It has been shown that the increase of forest combustible materials moisture decreases the rate of forest fire spread.

The mathematical model above was realized as a program for IBM PC. The availability of convenient software for user graphic interface make it possible to change initial data (under dialogue conditions), to display the results on the monitor screen during the problem solution, to obtain hard copies (graphs and tables).

4. REFERENCES

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