

# Effects of Nonlinearity in Storage-discharge Relationship on Design Flood Estimates

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**Abstract:** Rainfall-based flood estimation techniques are commonly adopted in hydrological design. Runoff routing models are widely used in Australia, which employ either a linear or nonlinear storage-discharge relationship. Australian Rainfall and Runoff recommends the use of a nonlinear storage-discharge relationship; though, many have proposed other types of storage-discharge relationships. This paper adopts a Monte Carlo simulation technique to examine the effects of nonlinearity in storage-discharge relationship on design flood estimates. This simulates thousands of rainfall events, which are then routed through a runoff routing model to generate streamflow hydrographs. The nonlinearity parameter in the storage discharge relationship is varied to quantify its effects on design flood estimates. It is found that nonlinearity parameter has significant effect on design floods, particularly for larger floods. Results from three study catchments indicate that a nonlinear storage-discharge relationship is preferable to linear relationship.

**Keywords:** Runoff routing; Storage-discharge relationship; Flood estimation; Design floods

## 1. INTRODUCTION

In rainfall-based flood estimation techniques, runoff routing models are commonly adopted e.g. RORB [Laurenson and Mein, 1997], WBNM [Boyd et al., 2001 and Rigby et al., 1999], RAFTS [Willing and Partners, 1988], URBS [Carroll, 1994]. A central component of these models is a conceptual storage and storage routing procedure. Australian Rainfall and Runoff [I. E. Aust., 2001] recommends to use a nonlinear storage-discharge relationship for estimation of large floods. Some investigators have proposed other forms of storage-discharge relationships. For example, Bates and Pilgrim [1983] proposed an asymptotically linear relationship. Sriwongsitanon et al. [1998] proposed a nonlinear relationship with an intercept and Zhang and Cordery [1999] proposed a linear relationship. Wong [1989] and Bates et al. [1993] had limited success in establishing variation patterns between optimal parameter values of  $k_c$  and  $m$  of RORB model. Bates and Pilgrim [1986] describe a nonlinear runoff routing model with a storage-discharge relation different from the power law function, which approximates linear behaviour at high flows. RORB model has also provisions to adopt

storage-discharge relationships different from the power function. However, little use has yet been made of these alternative forms of storage-discharge relations [I. E. Aust., 2001]. The question as to whether the nonlinear storage-discharge relationship is appropriate for estimation of large floods has never been answered [Zhang, 2000].

This paper presents an empirical study to assess the effects of nonlinearity in storage-discharge relationship on design flood estimates. This uses a large number of simulated rainfall events with a simple conceptual storage model to examine the effects of nonlinearity on design floods covering a large range of frequency.

## 2. APPROACH

A simple storage-discharge relationship is adopted in this study, expressed by:

$$S = kQ^m \quad (1)$$

where  $S$  is catchment storage in  $m^3$ ,  $k$  is storage delay parameter in hours,  $Q$  is the rate of outflow

in  $m^3/s$  and  $m$  is a nonlinearity parameter. This model becomes linear when  $m = 1$ . The value of  $m$  is generally taken as 0.80 for many applications. This study examines three values of  $m$ , which are 0.8, 0.9 and 1.0.

To examine the effects of assumed storage-discharge relationship on floods having a large range of frequency; rainfall events are simulated using a Monte Carlo Simulation technique. This generates 3 to 6 partial series rainfall events per year in a catchment, which have the potential to produce significant surface runoff. From the observed pluviograph data, independent rainfall events are identified as a period of rainfall preceded and followed by at least 6 consecutive hours of no rainfall, following the approach of Hoang et al. [1999]. For each complete storm, a rainfall burst (called storm-core) is identified, which has the highest rainfall intensity ratio compared to the 2-year average recurrence interval (ARI) design rainfall [Hoang et al., 1999; Rahman et al., 2001a & b].

The distribution of storm-core duration ( $D_c$ ) is obtained from the historical pluviograph data recorded on the catchment, the distribution of rainfall intensity ( $I_c$ ) is expressed in the form of intensity-frequency-duration (IFD) curves and observed temporal patterns ( $TP_c$ ) of storm-core events are expressed in dimensionless form and stored in a data bank for random selection during event simulation.

An initial loss continuing loss model is used for runoff production. The values of initial losses are computed from concurrent pluviograph and streamflow data recorded in the study catchment. A theoretical distribution is fitted to  $IL_c$  data. Continuing loss ( $CL_c$ ) is assumed to be constant from rainfall event to event for a catchment.

In the simulation of a rainfall event, a value of  $D_c$  is generated from its probability distribution, followed by generation of an  $I_c$  data from IFD table and random selection of  $TP_c$  data from the data bank of rainfall temporal patterns. These form a gross rainfall hyetograph, as shown in Figure 1. A value of  $IL_c$  is then generated from its probability distribution, which together with fixed  $CL_c$ , forms a net rainfall hyetograph. A large number of rainfall hyetographs (in the order of thousands) are simulated. The adopted Monte Carlo simulation technique is described in more detail in Rahman et al. [2001a & b]. These simulated rainfall hyetographs are then routed through the runoff routing model with different sets of  $(m, k)$  values to obtain streamflow hydrographs. The peaks of these hydrographs are

noted and subjected to a non-parametric frequency analysis to construct a derived flood frequency curve. The derived flood frequency curves for different sets of  $(m, k)$  values are plotted to examine the effects of nonlinearity ( $m$ ) on design floods.

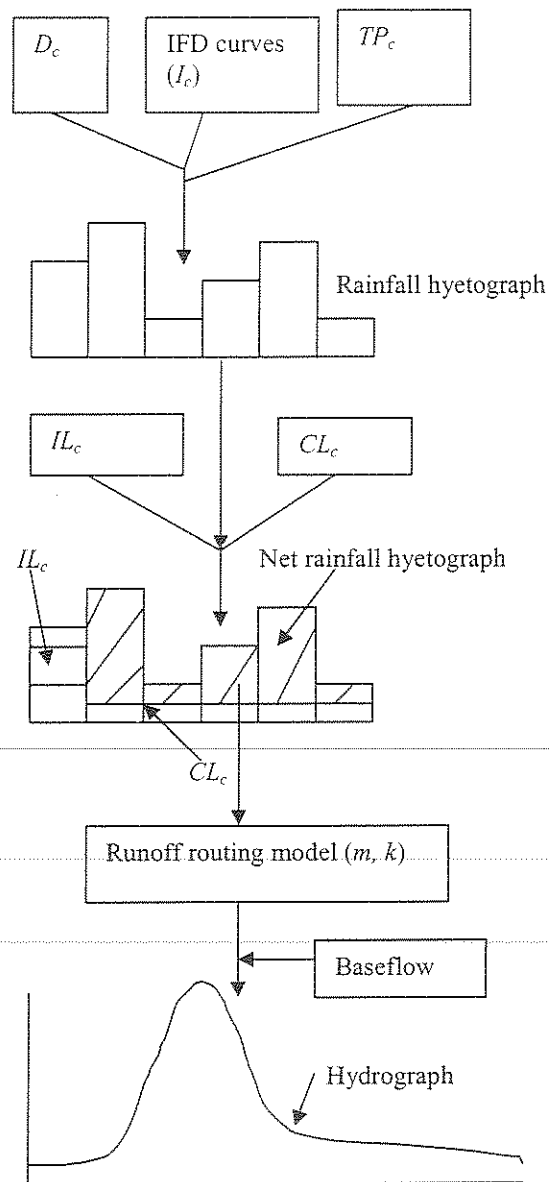


Figure 1. Rainfall and streamflow events simulation.

### 3. STUDY CATCHMENTS

Three catchments from Victoria are selected: Avoca River at Amphitheatre (catchment area,  $A = 78 \text{ km}^2$ ), Boggy Creek at Angleside ( $A = 108 \text{ km}^2$ ) and Tarwin River East Branch at Dumbalk ( $A =$

127 km<sup>2</sup>). These are unregulated and rural catchments.

#### 4. RESULTS

Three values of  $m$  are considered,  $m = 1.0$  (linear storage-discharge relationship),  $m = 0.8$  (nonlinear storage-discharge relation, frequently adopted in practice) and an intermediate value,  $m = 0.9$ . Since,  $k$  value changes with  $m$ , a different  $k$  is obtained for each  $m$  value from calibration of the runoff routing model. For each catchment, a number of observed pluviograph and streamflow events are selected for calibration. Three sets of  $(m, k)$  values are obtained for each of the three catchments, as shown in Table 1. The  $k$  values are found to be consistent with the recommended adjusting factor,  $(Q_p/2)^{m-m_1}$ , where  $Q_p$  is the peak discharge and  $m$  the old and  $m_1$  the new value of  $m$  [Laurenson and Mein, 1997].

**Table 1.** Values of  $(m, k)$  from calibration.

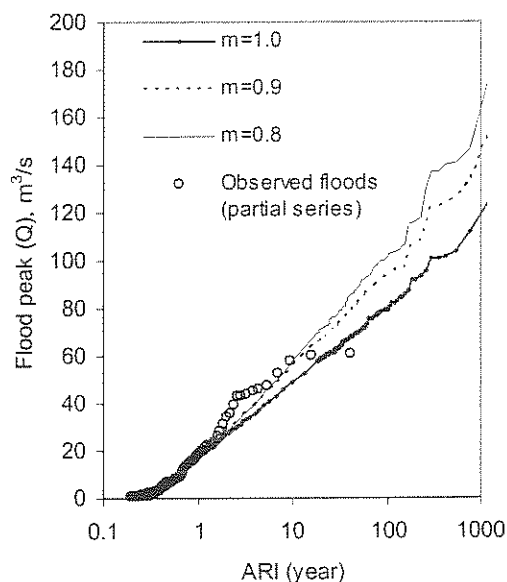
Catchment	Set 1	Set 2	Set 3
Avoca River	(1.0, 7)	(0.9, 9)	(0.8, 13)
Boggy Creek	(1.0, 17)	(0.9, 24)	(0.8, 34)
Tarwin River	(1.0, 14)	(0.9, 20)	(0.8, 27)

A derived flood frequency curve is constructed for a catchment from 10,000 simulated rainfall and streamflow events for each set of  $(m, k)$  values. An input rainfall hyetograph and other variables/parameters (e.g.  $CL$ , baseflow) remain same for a catchment in three runs with three sets of  $(m, k)$  values, which allows assessment of the effects of changing  $m$  value on design floods.

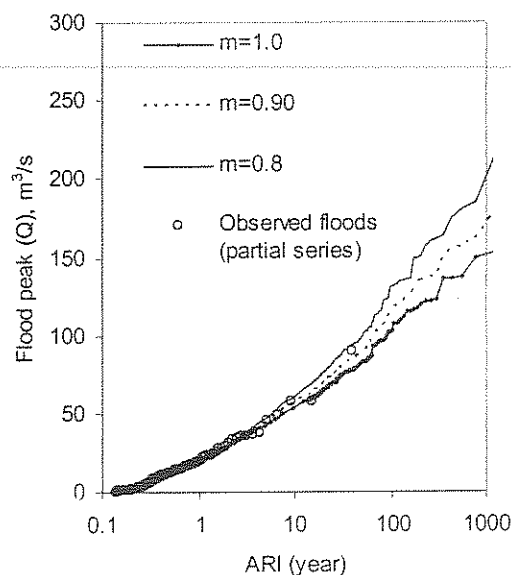
The derived flood frequency curves for Avoca River, Boggy Creek and Tarwin River catchments are shown in Figures 2, 3 and 4, respectively. These show that flood peaks with different  $m$  values do not vary noticeably at smaller ARIs (up to about 5 years), but there are remarkable differences for greater ARIs. The percentage differences in flood magnitudes from linear storage-discharge relationship ( $m = 1$ ) and two other degrees of nonlinearities ( $m = 0.9$  and  $m = 0.8$ ) are found to be remarkably high, particularly at higher ARIs, as shown in Figure 5. The above results indicate that flood frequency curves are remarkably sensitive to the adopted  $m$  value, particularly at higher ARIs.

The observed partial series flood data at the selected catchments are superimposed on Figures 2-4. For Avoca River catchment (Figure 2), it

appears that for up to 10 year ARI floods,  $m = 0.8$  better approximates the observed flood series. For ARIs greater than 10 years, a greater  $m$  value tends to better approximate the observed flood series. It may be noted that the observed largest flood data for Avoca River catchment does not match well with the remaining historical flood data, and hence might have been subjected to data error. Ignoring this data point,  $m = 0.9$  appears to best approximate the observed flood series.



**Figure 2.** Effects of changing  $m$  on flood peaks for Avoca River catchment.



**Figure 3.** Effects of changing  $m$  on flood peaks for Boggy River catchment.

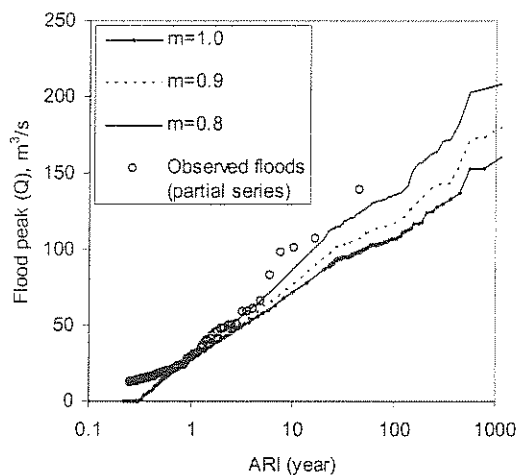


Figure 4. Effects of changing  $m$  on flood peaks for Tarwin River catchment.

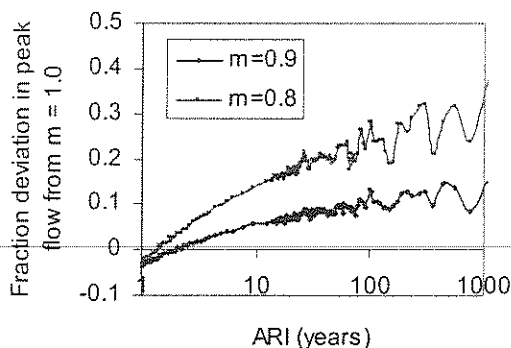


Figure 5. Fraction deviation in peak floods from linearity ( $m = 1.0$ ) to nonlinearities,  $m = 0.9$  and  $m = 0.8$  (Boggy Creek catchment).

For Boggy Creek catchment (Figure 3), it is found that  $m = 0.9$  best approximates the observed flood series. For Tarwin River catchment, it appears that  $m = 0.8$  best approximates the observed flood series. It should be noted here that there are only few data points in the observed flood series beyond 20 years ARI, which makes it difficult to assess the suitability of an  $m$  value for larger floods.

## 5. CONCLUSION

The paper presents an empirical study to identify an appropriate value of the exponent  $m$  in the storage-discharge relationship of the runoff routing

models used in flood estimation. Following conclusions may be drawn from the study:

- Flood peaks generated by the runoff routing models are quite sensitive to  $m$  value, particularly for large floods.
- For the three study catchments, a nonlinear storage-discharge relationship appears to better approximate the observed flood series than the linear relationship.
- There is no unique  $m$  value that is equally valid for all the three study catchments. This indicates the importance of determining an appropriate  $m$  value for a particular catchment.
- Monte Carlo simulation technique offers a powerful means to determine an appropriate  $m$  value for a particular catchment covering a wide range of flood frequencies.

## 6. ACKNOWLEDGMENTS

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