

Bilevel optimisation as a solution method for an agri-environmental principal-agent problem

B.L. Barnhart^{a,b}, **M.B. Bostian**^c, **S. Hasan**^d , **J.C.R. Smart**^{d,e}  and **C. Fleming**^{d,f} 

^a National Council for Air and Stream Improvement, Inc., Corvallis, Oregon, USA,

^b Department of Applied Economics, Oregon State University, Corvallis, Oregon, USA,

^c Department of Economics, Lewis & Clark College, Portland, Oregon, USA

^d Australian Rivers Institute, Griffith University, Nathan, Queensland, Australia,

^e School of Environment and Science, Griffith University, Nathan, Queensland, Australia,

^f Griffith Business School, Griffith University, South Bank, Queensland, Australia.

Email: bbarnhart@ncasi.org

Abstract: Recent computational advances in environmental modelling have enabled modellers to predict the impacts of spatially distributed management practices on environmental quality throughout agricultural, forested, urban, and mixed-use watersheds. In addition, real and hypothetical incentive policies – as well as the interactions between policymakers and policy followers – have been simulated using agent-based modelling techniques as well as optimisation and multi-criteria decision-making methods.

In this paper, we use bilevel optimisation as a solution method for solving an agri-environmental principal-agent problem—that is, to create spatially targeted environmental incentive policies to improve water quality. In constructing the problem and solution framework, we draw parallels between agent-based and bilevel approaches as means to simultaneously consider both the objectives of the policymakers and policy followers. Our case study investigates the Tully catchment, which is dominated by sugar cane farming and a major contributor of nutrient runoff from northeastern Australia to the Great Barrier Reef Lagoon. We compare uniform and spatially targeted policies that offer payments for agricultural producers to implement discrete reductions in fertilizer application rates, and the resulting policy solutions highlight the optimal trade-offs between policy cost and nutrient reductions. In addition, we show that targeting policy incentives based on soil type achieves greater efficiencies (i.e., less policy cost, and less nutrient runoff) than simply offering different incentives for each fertilizer reduction. By leveraging knowledge of the spatial distribution of soil type throughout the catchment, our results suggest that policymakers can construct more efficient policies that will ensure adoption and achieve considerable nutrient load reductions at feasible costs. This framework for optimizing incentive policies could be extended to include more complicated and realistic policy options, and it could also be applied in other watersheds dominated by agricultural, forested, urban, and mixed land uses.

Keywords: *Environmental modelling, agent-based models, bilevel optimisation*

1. INTRODUCTION

The presence of excessive nutrients in aquatic systems can be detrimental to water quality and ecosystem health (Guignard *et al.* 2017). Agricultural production, which depends on the use of nitrogen and phosphorus agrochemical fertilizers, is a leading source of nutrients to many aquatic systems, primarily through leaching and surface runoff (Guignard *et al.* 2017; Tilman *et al.* 2002). In the Great Barrier Reef Lagoon off the coast of northeastern Australia, nitrogen runoff from sugar cane farmland serves as a key water quality threat (van Grieken *et al.* 2019; Webster *et al.* 2012). Therefore, a long-held objective remains to optimise agricultural management in such a way that simultaneously preserves both yields and environmental quality (van Grieken *et al.* 2019; Raffensperger, Prabodanie, and Kostel 2017; Smart *et al.* 2016).

Numerous studies have leveraged computational advances in environmental modelling to simulate impacts of spatially distributed management practices on environmental quality throughout a watershed (Wellen, Kamran-Disfani, and Arhonditsis 2015). By using multi-objective optimisation algorithms along with cost estimates for implementing various management practices, studies have demonstrated that optimally efficient policies – that is, those that minimize cost and maximize water quality – are achievable by spatially targeting particular management practices (Xu *et al.* 2018; Yang and Best 2015). The reasons for these results vary, but they can often be attributed to spatially heterogeneous environmental and landscape properties that vary throughout a watershed, including soil quality, temperature and rainfall, and elevation and/or slope changes, which ultimately impact local management costs and environmental benefits.

In addition to the advances in environmental modelling in the context of agri-environmental systems, advances in agent-based modelling have provided the ability to simulate the interactions of individuals and groups within socio-environmental systems (Filatova *et al.* 2013). Principal-agent models are particularly relevant for formulating and evaluating agri-environmental incentive policies, where there is an asymmetric information structure between the principal (*i.e.*, policymaker) and the agents (*i.e.*, landowners, stakeholders). Numerous studies have utilized principal-agent frameworks along with integrated models to simulate the effects of alternative agri-environment policies. For example, Cho and Blandford (2018) highlight a case study in Norway where they examine a peatland retirement program to reduce agricultural greenhouse gas emissions using a principal-agent framework. In addition, Gómez-Limón, Gutiérrez-Martín, and Villanueva (2019) use a principal-agent model to design optimal schemes for improving farmland biodiversity through incentives.

While optimisation solution methods and multi-criteria decision making have been widely applied to agri-environmental systems for designing optimal policies (Kaim, Cord, and Volk 2018), few of these methods directly account for the interactions between policymakers and policy followers as with agent-based models. One approach that does explicitly model group interactions is the relatively new bilevel optimisation, which describes two nested optimisation problems, where each is dependent on the solution of the other (Sinha, Malo, and Deb 2017b, 2017a). A few studies have applied bilevel optimisation to targeting agri-environmental policy and explicitly model optimized decision-making of policymakers and policy followers. In particular, Whittaker *et al.* (2017) implement a bilevel optimisation framework using a hybrid genetic algorithm in order to produce optimally targeted incentive policies to improve water quality. Barnhart *et al.* (2017) implement a similar bilevel optimisation framework and compare implementations using three different evolutionary optimisation algorithms.

In this paper, we leverage the recent advances in bilevel optimisation to simulate interactions between multiple landowners and a single policymaker and draw parallels to terminology used in agri-environmental principal-agent models. We first introduce the framework and then test the approach using data from the Tully catchment in northeastern Australia, which is dominated by sugar cane production. By optimizing the spatial distribution of incentive policies to reduce fertilizer application rates, the bilevel optimisation solution method is able to achieve optimal solutions that meet both the objectives of the policy maker and the landowners. In addition, we show that targeting policy incentives based on soil type achieves greater efficiencies (*i.e.*, less policy cost, and less nutrient runoff) than simply offering different incentives for each management decision. By leveraging knowledge of the spatial distribution of soil quality throughout a watershed, policymakers can construct optimal policies that will ensure adoption and limit environmental impacts at feasible costs. Specific implementation details using a genetic algorithm are discussed, as well as suggestions for further research, which include extending the framework to more complicated and realistic policy options and applying the methodology in other watersheds.

2. METHODOLOGY

2.1. Bilevel optimisation solution method

The bilevel optimisation framework, which describes two nested optimisation problems that characterize the decision-making process of two individuals or groups and are interdependent, lends well to principal-agent problems that are characterized by multiple, misaligned objectives. It has been compared to the widely known Stackelberg game (Sinha, Malo, and Deb 2017b, 2017a), in which two players seek to optimize their strategy, yet their outcomes are dependent on the previous player's decision (Sinha, Malo, and Deb 2017b). This section will briefly describe a bilevel optimisation framework in an agri-environmental context, and principal-agent concepts will be incorporated to highlight the closeness of the methods. For more technical detail on the application of bilevel optimization to principle-agent problems, refer to Cecchini *et al.* (2013).

In an agri-environmental context, the principal (e.g., a policymaker) represents one level of optimization. This principal seeks to determine a set of decisions that will achieve some set of objectives. As an example, a policy maker distributes incentive payments with the intent to minimize the total policy cost while maximizing environmental improvement. Meanwhile, the agents (e.g., landowners) are presented with a set of policy options and must decide how to respond – whether to accept or reject the policy – given their own individual optimization objectives. For example, the individuals could be offered individual payments to implement a particular management practice. These individuals will consider their own objectives (e.g., resource efficiency, local environmental benefit, profit maximization) and decide whether or not to accept the payment. Bilevel optimization then links these two optimization processes, and therefore solutions to the bilevel optimization problem offer optimal policy alternatives that match the objectives of both the principal and the agents.

For this study, we will focus on the specific example of allocating payments to reduce fertilizer usage in sugar cane production in the Tully catchment, which will be described in detail in section 2.2. The bilevel multi-objective problem can be formulated as follows:

$$\begin{aligned} & \min_{c_i(x_j)} \{C(c_i(x_j)), N(n_i(x_j))\} \\ & \text{s.t. } x_j \in \operatorname{argmax}_{c_i(x_j)} \{GM_i(x_j) + c_i(x_j)\} \end{aligned} \quad (1)$$

Here, the first line, which is referred to as the upper level, represents the principal (i.e., policymaker) who offers a series of payments $c_i(x_j)$ to each agent i (i.e., agricultural producers) for adopting particular management practices x_j (in this case, the reduction of fertilizer applications by some fixed amount). From the perspective of the policymaker, an optimal policy is one that both minimizes the total policy cost $C(c_i(x_j)) = \sum c_i(x_j)$ and the total nutrient runoff from the catchment $N(n_i(x_j)) = \sum n_i(x_j)$, where the lowercase $c_i(x_j)$ and $n_i(x_j)$ represent each agent's individual contributions to the total policy cost and nutrient runoff for a given management practice x_j , respectively.

Meanwhile, the second line, which is referred to as the lower level, represents the response of the agents to a proposed policy. After being offered a payment $c_i(x_j)$ to adopt a particular management practice x_j (in this case, to reduce fertilizer usage by some fixed amount), each agent i determines which practice to adopt in order to maximize their own profits, defined as the sum of the gross margin achieved from a given practice and the payment received from the policy: $GM_i(x_j) + c_i(x_j)$. The resulting choices of x_j for each agent contributes to the calculation of the upper level objectives $C(x)$ and $N(x)$.

Detailed implementation of the bilevel optimisation framework using a genetic algorithm is discussed in section 2.3. Note that in this problem, the upper level has two objectives, and the lower level has one objective (for each agent). Therefore, at the lower level, there are no trade-offs between multiple objectives. This greatly simplifies implementation, since the lower level sends only a single solution from each agent back up to the upper level. If each agent had more than one objective, and thus produced a frontier of optimal solutions due to conflicting objectives, then the upper level would need to construct optimistic and pessimistic frontiers by anticipating the “best” and “worst” agent choices to essentially bound feasible choices of the agents. Further information on how bilevel optimisation can handle asymmetric information exchange is covered in Sinha, Malo, and Deb (2017b).

2.2. Description of the Tully catchment

The optimisation framework above will be implemented for the Tully catchment (approximately 18 °S 146 °N) in northeastern Australia. The sugar cane production region of the catchment was broken into 4,020 cells, each 250 m x 250 m, as shown in Figure 1. This particular catchment is in the Wet Tropics bioregion and is characterised by high annual average rainfall (4100 mm) (van Grieken *et al.* 2019). There is a general consensus that the nitrogen fertilizer application rate is the most important determinant of nitrogen pollution in this region (Webster *et al.* 2012) and that nitrogen surface runoff from sugarcane farms reach the coast and the Great Barrier Reef Lagoon quickly, with little opportunity of nitrogen removal through in-soil or in-stream processes (van Grieken *et al.* 2019).

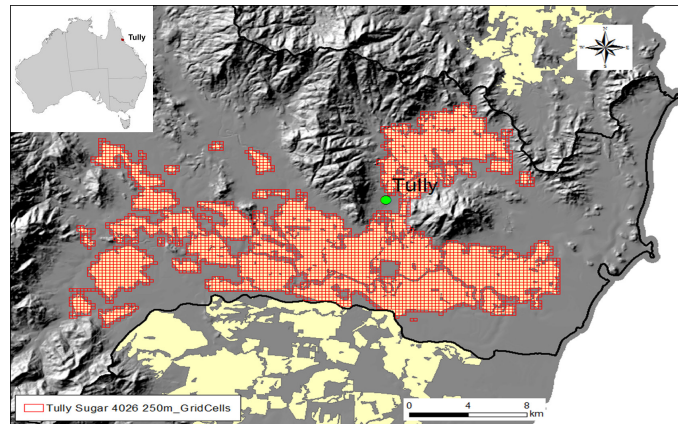


Figure 1. Overview of the Tully catchment on the coast of northeastern Australia.

In the present study, the relationships between nitrogen application rate (kg N/ha) and average annual dissolved inorganic nitrogen losses (kg DIN/ha), and between nitrogen application rate and estimated annuity gross margins (AU \$/ha) were obtained from Roebeling *et al.* (2007) (Figures 4 and 6) for four fertilizer application rates: 210, 180, 120, and 60 kg N/ha over four soil classes in the Tully catchment. Using these relationships, cell-specific data on DIN losses and gross margins were calculated for each fertilizer application rate, knowing sugar cane area and soil class composition for each cell (Smart *et al.* 2016). These data were stored as a comma-separated value look-up table to be used during the optimisation implementation, which will be described in subsequent sections.

2.3. Optimisation implementation for two policy scenarios

As described in the previous section, the Tully catchment was divided into 4,020 cells, each 250 m x 250 m. Each cell is treated as an agent (i.e., landowner), and the principal (i.e., policymaker) will construct alternative policy scenarios that are offered to each agent. This is clearly a simplification of reality that can be further refined with future work—in particular, by more appropriately defining agents as controlling portions of land based on ownership boundaries.

For this application, both uniform and soil-type targeted policies were optimized. A graphical depiction of the algorithm implementing the uniform policy is depicted in Figure 2.

In this policy, the policymaker offers a fixed payment to each landowner in exchange for implementing a fertilizer application rate (i.e., c_{210} , c_{180} , c_{120} , c_{60}). The naming convention used for the payments refers to the kg N/ha of fertilizer applied to each cell. The policymaker offers different payment amounts for implementing each fertilizer application rate, but the same payments are offered to all cells in the catchment. Note that c_{210} is set to zero because the policy maker does not offer a payment for adopting the highest fertilizer application rate.

The green box in Figure 2 describes how each agent (i.e., cell) receives the payment offers from the principal (i.e., policymaker) and decides to implement the practice that maximizes the sum of the gross margin and payment. Note that whether each cell accepts one of the payments depends entirely on the local conditions in that particular cell. The decisions made within the green box in Figure 2 then determines the gross margin,

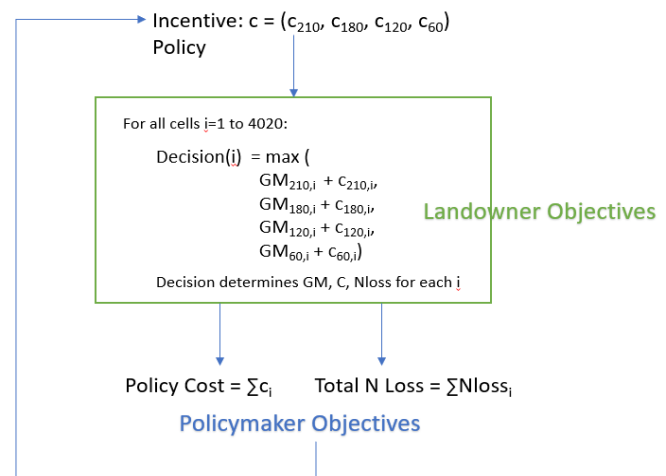


Figure 2. Optimization flowchart depicting the upper level (policymaker) objectives and the lower level (landowner) objectives. This particular example shows a uniform policy with four alternative management practices.

incremental cost of the policy, and the resulting nitrogen loss from that cell. The information from all of the cells is then passed to the upper level (i.e., principal, policy maker) to calculate the policy outcomes.

The policymaker objectives are to minimize the total policy cost, which is calculated by adding up all the payments made to all of the cells, and to minimize the total nitrogen runoff, which is calculated by adding up the nitrogen runoff from all of the cells.

This first policy optimisation explored is a uniform policy in which the policymaker offers the same payments to all cells in the catchment. In addition, a second policy was generated that offered different payments depending on the soil type of each cell. There were four different soil types throughout the catchment, so the soil-type-targeted policy offered 16 possible payments to each of the cells (i.e., $[(c_{210, 1}, c_{210, 2}, c_{210, 3}, c_{210, 4}), (c_{180, 1}, c_{180, 2}, c_{180, 3}, c_{180, 4}), (c_{120, 1}, c_{120, 2}, c_{120, 3}, c_{120, 4}), (c_{60, 1}, c_{60, 2}, c_{60, 3}, c_{60, 4})]$, where the second subscript denotes the soil type. Note that the four payments for the highest fertilizer application rate ($c_{210, 1}, c_{210, 2}, c_{210, 3}, c_{210, 4}$) were set to 0, since the policymaker does not offer a payment to incentivize adoption of the highest fertilizer rate.

To implement the bilevel optimisation framework described in equation (1) and Figure 2, a genetic algorithm (GA) was used. GAs are widely used iterative algorithms that find solutions to optimisation problems with one or more objectives. In particular, GAs represent sets of input parameters as individuals within a population. By evaluating the performance of each individual, the algorithm determines each individual's fitness with respect to meeting the objectives. Selection and mating are then performed, and mutations are introduced into the individuals with some probability of occurrence, then the algorithm again evaluates the new solutions. After many iterations, convergence occurs, and the resulting solutions typically represent a set of optimal solutions that exhibit trade-offs between the different optimisation objectives.

The bilevel optimisation problem described above was implemented using a genetic algorithm from the *inspyred* package (Garrett 2012) in Python 3.4. *Insyred* implements a variety of different optimisation solution methods, but the widely used nondominated sorting genetic algorithm [NSGA-II; (Deb et al. 2002)] was chosen for this study. NSGA-II is typically used for multi-objective optimisation problems that have 1-3 objectives, so the method is considered to be sufficient for this study. It utilizes Pareto ranking (or nondomination) as well as crowding distance to sort and select solutions that span the entire objective space (Deb et al. 2002). A population of 96 individuals evolved for 1,000 generations. We used the simple binary crossover (SBX) with a SBX distribution of 10 and a crossover rate of 0.75. The probability of mutation (mutp) was set to $1/N$ where N is the number of decision variables. No specific convergence metric was set for this implementation, yet convergence was confirmed through visual inspection of the solutions, since they did not vary after approximately 200 generations.

The two policies described above have 4 and 16 decision variables, respectively, since there are 4 possible management practices and 4 soil types. However, the highest fertilizer application rate was not incentivized, so the actual numbers of decision variables are 3 and 12. The algorithm was run on a computer with Windows 10 an Intel Core i7-8700 central processing unit (CPU) @ 3.2 Ghz with 64 Gb of random-access memory (RAM).

3. RESULTS

The solutions to the upper level objectives for the two policies are shown in Figure 3. The left panel shows the total policy cost vs. the total loss of nitrogen for the catchment. The no-policy total loss of nitrogen is approximately 750 tonnes; the right panel shows the nitrogen loss as a percentage reduction from that baseline.

Note that the maximum nitrogen runoff reductions achievable by both policies were approximately 70% from the baseline, and the total cost for achieving these reductions was approximately \$9 million AUS. This is due to the fact that only four fertilizer application rates were included in our setup, and these solutions require that all cells adopt the lowest level of fertilizer application rate possible. In practice, sugar cane farmers will have a spectrum of fertilizer application rates to choose from; however, our data limited us to only consider four alternatives. In addition, both sets of solutions show discontinuities within their frontiers. This is also likely caused by the discrete policy choices.

Another key result is that the soil type-targeted policy was more efficient than the uniform policy. As shown in Figure 3, the soil-specific targeted policy points (blue) provided lower total policy costs for equivalent nitrogen losses, and likewise, for a given total policy cost, the targeted policy achieved lower N loss, or, equivalently, greater reductions in N loss.

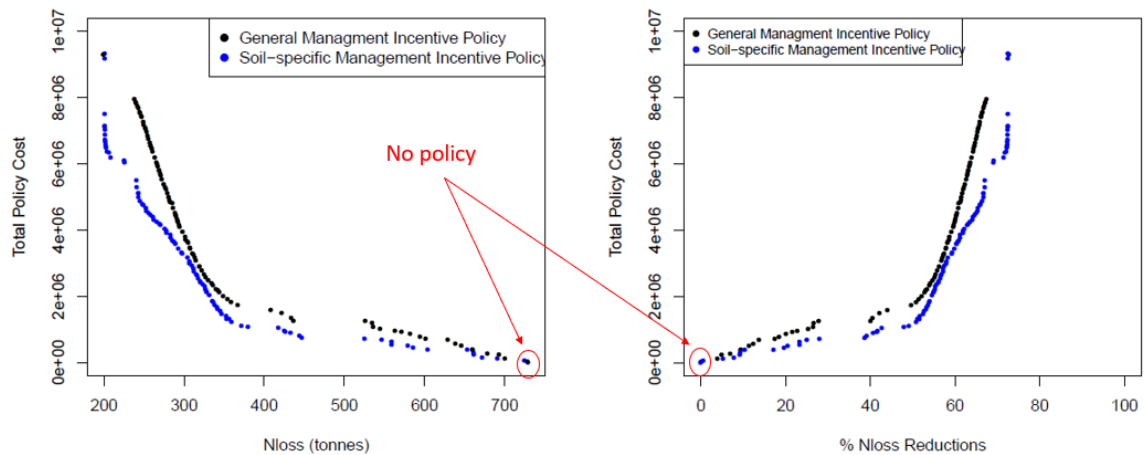


Figure 3. Final resulting trade-offs between upper level objectives (i.e., total policy cost and total N loss) plotted for two specific policies: a uniform payment to adopt different management practices (black) and a policy that offers payments that vary depending on the soil type of a given cell (blue). The red circles indicate the outcomes achieved under the no-payment scenario.

4. DISCUSSION AND CONCLUSION

This paper introduced bilevel optimisation as a solution method for principle-agent problems and specifically described how the method can be used to determine optimal targeting of agri-environmental incentive policies while taking into account the objectives of the policymaker and the landowners.

We provided an example implementation of this methodology using the Tully catchment in northeastern Australia, which is dominated by sugar cane production. By using data on the relationships between nitrogen fertilizer application, gross margin from cane yields, and nitrogen loss, we constructed two incentive policies that offered incentive payments to persuade landowners to reduce their fertilizer application and subsequently reduce the total nitrogen loss from the catchment. Both a uniform policy, in which a fixed payment was provided for switching to alternative fertilizer applications, and a soil-specific policy, in which fixed payments were offered but varied depending on the soil type of the location, were optimized in a bilevel optimisation framework using a genetic algorithm.

The results showed that targeting policy payments based on soil type provided more efficient results (i.e., lower total policy cost and lower nitrogen loss) than the uniform policy. These results match findings from previous studies. For example, Bostian *et al.* (2015) found variation amongst the trade-offs between maximum profit and nitrogen loading from individual farms subject to a fertilizer tax policy in Oregon's Willamette Valley (USA). Also, Whittaker *et al.* (2017) targeted multiple agri-environmental policies at the catchment, zip code (i.e., sub-catchment), and individual farm levels and found similar policy efficiency improvements. All of these approaches suggest that utilizing targeted payments offers more flexibility and does not suffer from overpayment issues that occur with uniform policy.

Further research should be conducted to explore the spatial distribution of the policy solutions shown in Figure 3, as well as their environmental impacts. In addition, the present approach could be expanded to include more policy alternatives to enhance the realism of the policies. For example, the upper level environmental objective was to minimize the sum of all nitrogen losses from of all cells. However, this edge-of-field approach to measuring nitrogen loss may be insufficient to fully characterize the impacts of excessive nutrients that deposit in the Great Barrier Reef Lagoon. Therefore, simple transfer coefficients or full water quality and watershed models could be used to better characterize the transport of excess nitrogen through water bodies and ultimately to the Reef. In addition, only four fertilizer application rates were used for the case study. Data on additional fertilizer application rates are available and should be incorporated to produce more realistic policy alternatives. Policy realism could also be improved by offering a sub-scale at which the policy is offered. That is, we only investigated a uniform policy and a soil-type-specific policy. Further work could divide the catchment into sub-catchments based on geography, or perhaps land ownership, to provide more easily implementable policy alternatives. Finally, only a few studies have applied bilevel optimisation to solve agri-environmental policy targeting problems in general. Therefore, further work should apply this framework to other watersheds in pursuit of other environmental outcomes – including various metrics of water quality, air quality, and carbon sequestration, to name a few – in order to broaden its applicability.

REFERENCES

- Barnhart, B., Lu, Z., Bostian, M., Sinha, A., Deb, K., Kurkalova, L., Jha, M., and Whittaker, G. (2017). *Handling practicalities in agricultural policy optimization for water quality improvements*. In *Proceedings of the Genetic and Evolutionary Computation Conference*, 1065-72. ACM.
- Bostian, M., Whittaker, G., Barnhart, B., Färe, R., and Grosskopf, S. (2015). Valuing water quality tradeoffs at different spatial scales: An integrated approach using bilevel optimization. *Water Resources and Economics*, 11, 1-12.
- Cecchini, M., Ecker, J., Kupferschmid, M., and Leitch, R. (2013). Solving nonlinear principal-agent problems using bilevel programming. *European Journal of Operational Research*, 230, 364-73.
- Cho, W., and Blandford, D. (2018). Bilateral information asymmetry in the design of an agri-environmental policy: An application to peatland retirement in Norway. *Journal of Agricultural Economics*.
- Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: Nsga-ii. *IEEE Transactions on Evolutionary Computation*, 6, 182-97.
- Filatova, T., Verburg, P.H., Parker, D.C., and Stannard, C.A. (2013). Spatial agent-based models for socio-ecological systems: Challenges and prospects. *Environmental Modelling & Software*, 45, 1-7.
- Garrett, A. (2012). *Inspyred (version 1.0.1)*.
- Gómez-Limón, J.A., Gutiérrez-Martín, C., and Villanueva, A.J. (2019). Optimal design of agri-environmental schemes under asymmetric information for improving farmland biodiversity. *Journal of Agricultural Economics*, 70, 153-77.
- Guignard, M.S., Leitch, A.R., Acquisti, C., Eizaguirre, C., Elser, J.J., Hessen, D.O., Jeyasingh, P.D., Neiman, M., Richardson, A.E., and Soltis, P.S. (2017). Impacts of nitrogen and phosphorus: From genomes to natural ecosystems and agriculture. *Frontiers in Ecology and Evolution*, 5, 70.
- Kaim, A., Cord, A.F., and Volk, M. (2018). A review of multi-criteria optimization techniques for agricultural land use allocation. *Environmental Modelling & Software*, 105, 79-93.
- Raffensperger, J.F., Prabodanie, R.R., and Kostel, J.A. (2017). A smart market for nutrient credit trading to incentivize wetland construction. *Journal of hydrology*, 546, 248-61.
- Roebeling, P., Webster, A., Biggs, J., and Thorburn, P. (2007). Financial-economic analysis of current best management practices for sugarcane, horticulture, grazing and forestry industries in the tully-murray catchment. *Report to the Marine and Tropical Sciences Research Facility (MTSRF), Reef and Rainforest Research Centre (RRRC), Cairns, Australia*.
- Sinha, A., Malo, P., and Deb, K. (2017a). Evolutionary algorithm for bilevel optimization using approximations of the lower level optimal solution mapping. *European Journal of Operational Research*, 257, 395-411.
- . (2017b). A review on bilevel optimization: From classical to evolutionary approaches and applications. *IEEE Transactions on Evolutionary Computation*, 22, 276-95.
- Smart, J., Syezlin, H., Volders, A., Curwen, G., Fleming, C., and Burford, M. (2016). A tradable permit scheme for cost-effective reduction of nitrogen runoff in the sugarcane catchments of the great barrier reef. *Report to the National Environmental Science Programme, Cairns, Qld, Australia*.
- Tilman, D., Cassman, K.G., Matson, P.A., Naylor, R., and Polasky, S. (2002). Agricultural sustainability and intensive production practices. *Nature*, 418, 671.
- van Grieken, M., Roebeling, P., Bohnet, I., Whitten, S., Webster, A., Poggio, M., and Pannell, D. (2019). Adoption of agricultural management for great barrier reef water quality improvement in heterogeneous farming communities. *Agricultural systems*, 170, 1-8.
- Webster, A., Bartley, R., Armour, J., Brodie, J., and Thorburn, P. (2012). Reducing dissolved inorganic nitrogen in surface runoff water from sugarcane production systems. *Marine Pollution Bulletin*, 65, 128-35.
- Wellen, C., Kamran-Disfani, A.-R., and Arhonditsis, G.B. (2015). Evaluation of the current state of distributed watershed nutrient water quality modeling. *Environmental science & technology*, 49, 3278-90.
- Whittaker, G., Färe, R., Grosskopf, S., Barnhart, B., Bostian, M., Mueller-Warrant, G., and Griffith, S. (2017). Spatial targeting of agri-environmental policy using bilevel evolutionary optimization. *Omega*, 66, 15-27.
- Xu, H., Brown, D.G., Moore, M.R., and Currie, W.S. (2018). Optimizing spatial land management to balance water quality and economic returns in a lake erie watershed. *Ecological Economics*, 145, 104-14.
- Yang, G., and Best, E.P. (2015). Spatial optimization of watershed management practices for nitrogen load reduction using a modeling-optimization framework. *Journal of environmental management*, 161, 252-60.