

Exploiting ecological non-trophic models in representations of warfare

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Abstract: Traditional combat models such as the Lanchester model are typically limited to two competing populations and exhibit either exponential growth or decay solutions. Although these early models were well suited to the type of warfare in the early twentieth century, they are no longer as directly applicable to the complex modern military operating environment. Despite these shortcomings, the Lanchester model was used in the Operations Research of Force Design even in World War II, as evidenced in the classic work *Methods of Operations Research* of Morse and Kimball [1950]. Our work seeks to enrich such models to account for modern and future complexities, particularly around the role of inter-agency engagement in operations.

To this end, we account for the presence of civilian or non-combatant populations, which have long been an unfortunate part of the combat setting as they are directly impacted by the warfare surrounding them. Typically this non-combatant group consists of the host population in the space where combat occurs. The other type of non-combatant groups, a development since the 19th century and with ongoing evolution today, are agencies, be they governmental or non-governmental, that undertake work in conflict environments to support local populations. Depending on whether they are governmental or otherwise, these agencies have a range of formal and informal relationships with both sides of the conflict. As the agency non-combatant populations play no direct role in combat, their interactions with the two combatant forces are well suited to be modelled through the recent developments in non-trophic ecological models.

The networked non-trophic ecological model is one of the most recent developments in ecological modelling that incorporates a great number of positive and negative interactions, both trophic (consumptive) and non-trophic (non-consumptive), between multiple species in a “multiplex” network. In a similar manner in which the Lanchester combat model can be viewed as an adaptation of the Lotka-Volterra model for two species in a predator-prey relationship, the networked non-trophic ecological model can be exploited as a viable representation of modern combat in which non-combatant groups exist.

The combat model presented in this paper provides a global representation of asymmetrical combat between two forces in the modern setting where non-combatant populations are present. In our model, the non-combatant population is present as a neutral agency supporting the native population to the extent that they are non-combatants, but where there can be leakage from this group to the insurgent fighting force. Correspondingly, the opposing intervention force is under obligations to enable an environment where the neutral agency may undertake its work. A key result of our model is that, in contrast to the typical exponential growth or decay solutions of the Lanchester system, with the inclusion of a third group limit cycles and bifurcations can now occur which we interpret in light of the warfighting application of the model.

Keywords: *Combat models, non-trophic interactions, Lanchester’s laws, future force, operating environment*

1 INTRODUCTION

The Lanchester equations for combat [Lanchester, 1916; Taylor, 1983] were proposed in 1916 as a compact mathematical representation for the engagement between two opposing forces. The tragic irony is that, as a model of unmoving undifferentiated forces, they were uniquely suited to the stalemate, mud and trench bound slaughter of the years around which they were published, as evidenced by the Battle of the Somme. With some novel adaptation, the equations have remained a tool of interest in Operations Research [Lin and MacKay, 2014; MacKay, 2015], even in the more dynamic circumstances of World War II [Lucas and Turkes, 2004; Hung *et al.*, 2005]. Modern warfare, in contrast, has increased in complexity – with concepts such as manoeuvre, networked communications systems, and the presence of non-combatant groups in the operational environment. This increase in complexity has driven recent developments in the literature to advance the Lanchester equations e.g. multiple combatants [Kress *et al.*, 2018], geographically distributed combat interactions [McLemore *et al.*, 2016], and network centric warfare [Kim *et al.*, 2017] to name but a few. This paper will present a new approach to modelling the presence of non-combatant groups, drawing upon recent developments in ecological systems modelling [Fontaine *et al.*, 2011; Kéfi *et al.*, 2012, 2016].

The Lanchester equations are an adaptation of the Lotka-Volterra model for two species in a predator-prey, or trophic relationship. This is the simplest model of an ecological system that is straightforwardly generalised to multiple species in, for example, a food web. However, as increasingly recognised, some species in an ecological network play no trophic role at all – are neither preyed upon, nor prey others. Rather, they reside in niches amongst other foraging species, facilitating the growth of one or another element [Kéfi *et al.*, 2012; Fontaine *et al.*, 2011; Berlow *et al.*, 2004; Melián *et al.*, 2009; Mougi and Kondoh, 2012; Gross, 2008]. In the modern military operating environment there are also such entities. Above all, there is the host population in the operational environments where, in recent conflicts, external armed forces have confronted each other; Afghanistan is a case in point. The other type of non-combatant group are agencies, be they governmental (aid agencies of Departments of State or Foreign Affairs) or non-governmental (Red Cross/Crescent or *Médecins Sans Frontières*), that undertake work in conflict environments to support local populations. Depending on whether they are governmental or otherwise, these agencies have a range of formal and informal relationships with both sides of the conflict: with formal military organisations who are obligated to guarantee them protection [Office for the Coordination of Humanitarian Affairs, 2007] or provide logistics support [Australian Agency for International Development, 2011]; and with armed insurgents on, the other side, who seek to exploit the ambiguities around ‘unprivileged combatants’ [Sassoli *et al.*, 2011; Lewis *et al.*, 2015] to blend in and out of the very population to which these agencies provide assistance.

In this paper we draw upon these recent representations of non-trophic interactions in ecology to model such non-combatant support agencies in modern operational environments. The broader goal is to enrich available force design models to account for such present and future environments with high inter-agency complexity. We present a simple characteristic scenario and explore the types of dynamic behaviours that arise when such a third group, labelled ‘Green’, is inserted into the classic Blue-on-Red models in the so-called Blue-Green-Red (BGR) model. Using two scenarios of equal and unequal forces, in contrast to the typical exponential growth or decay solutions of the Lanchester system, with a third group now limit cycles and bifurcations can occur which we interpret in light of the warfighting application of the model.

2 NON-TROPHIC INTERACTIONS IN THE BGR COMBAT MODEL

Recent developments in ecological modelling have placed a focus upon the incorporation of non-trophic interactions in the study of complex ecological networks [Fontaine *et al.*, 2011; Kéfi *et al.*, 2012, 2016]. These non-trophic interactions, commonly categorised as either positive or negative, represent a range of diverse interactions between species that extend well past the commonly modelled interactions considered in typical Lotka-Volterra models [Brauer *et al.*, 2001]. Some examples of positive non-trophic interactions are improved recruitment, refuge provisioning, and increased survival. In the case of negative non-trophic interactions some examples are competition for space, predator interference, and increased mortality [Kéfi *et al.*, 2016]. In this work we seek to exploit this non-trophic framework as a representation of warfare through the formulation of a combat model that generalises the typical Lanchester combat model [Lanchester, 1916; Taylor, 1983]. Prior work has explored ideas from ecological modelling in the Lanchester combat model through logistic growth [Syms and Solymar, 2015] and multiple populations [Feichtinger *et al.*, 1996]. Our work thus builds on this in enriching the variety of interactions available in ecological modelling for warfare modelling.

Our combat model is based upon the work done by Kéfi *et al.* [2016] and the bioenergetic consumer-resource

models in Brose [2008]; Yodzis and Innes [1992]. In this model we explore a specific example of a global BGR model that highlights the asymmetries present in a potential combat scenario between an external intervention force Blue (B) and an insurgent/native force Red (R). Here Green (G) is present as a neutral agency supporting, or *facilitating*, the local population to the extent that they are non-combatants, but where there can be leakage from this group to the Red fighting force. Correspondingly, Blue is under external obligations to enable an environment where Green may undertake its work.

The dynamics of this asymmetric combat between Blue and Red is given by the following system

$$\begin{aligned}\dot{B} &= r_B^* B \left(1 - \frac{B}{K_B}\right) - F_{RB} R - x_B B, \\ \dot{R} &= r_R^* R \left(1 - \frac{R}{K_R}\right) - F_{BR} B - x_R^* R, \\ \dot{G} &= r_G^* G \left(1 - \frac{G}{K_G}\right) - x_G^* G,\end{aligned}\tag{1}$$

where B , R , and G are the sizes of the Blue, Red, and Green forces/populations respectively. Recruitment occurs through a logistic growth term with a carrying capacity $K_{(\cdot)}$ for each population. Here the non-trophic interactions (typically denoted through the superscript (*) here) in the improved recruitment rates are given by

$$r_B^* = r_B f(R), \quad r_R^* = \frac{r_R + r_{\max_R} G}{1 + G}, \quad r_G^* = \frac{r_G + r_{\max_G} B}{1 + B},\tag{2}$$

where $r_{(\cdot)}$ is the intrinsic recruitment rate for each population and $r_{\max_{(\cdot)}}$ is the maximum recruitment rate in the presence of facilitators. In this formulation we have included the function $f(R)$ in the improved recruitment rate of the Blue force. The function $f(R)$ accounts for the phenomenon that Blue's recruitment strategy would be dependent on the size of the Red force. As an example we might choose $f(R)$ such that $f(0) = 0$ and $f'(R) > 0$, thus in the absence of a red force, Blue has no need to recruit and thus its force size will gradually diminish in time due to the attrition solely driving its dynamics. In the presence of Red force, the level of recruitment will be driven by the size of the Red force. A couple of examples for the choice of $f(R)$ are $f(R) = \tanh(\alpha R)$ and $f(R) = \frac{\alpha R}{1 + \alpha R}$, where α can be viewed as a surge rate parameter. Combat between Blue and Red is described through the so-called functional response of combat, similar to that of the functional response of predation in ecological models [Holling, 1959], and is given by

$$F_{RB} = \frac{w_R b_{RB} B^{1+q}}{1 + w_R t_R b_{RB} B^{1+q}}, \quad F_{BR} = \frac{w_B b_{BR}^* R^{1+q}}{1 + w_B t_B b_{BR}^* R^{1+q}},\tag{3}$$

where b_{ij} is the intrinsic attack rate of i upon j , $w_{(\cdot)}$ can be interpreted as an engagement rate to account for the fact that a force has to divide its attention amongst adversaries and is given by $1/(\text{no. adversarial engagements})$, and $t_{(\cdot)}$ can be viewed as the combat engagement time that accounts for a force searching for and then engaging with its adversary, and the Hill coefficient $q \in \{0, 1\}$ yields type II and type III responses respectively [Holling, 1959]; essentially the larger the value of q the sharper the saturation of F to its maximum value. Observe that for small b_{BR}^* , F_{BR} scales as b_{BR}^* , however for large values, $F_{BR} \approx 1/t_B$. In other words, for increasingly large fire-rate the functional response is only as good as the time it takes to find targets. This will be important in the following. The non-trophic interaction through the Green in the functional response occurs through refuge provisioning for Red and modifies the attack rate of Blue upon the Red as follows

$$b_{BR}^* = \frac{b_{BR} + b_{\min_{BR}} G}{1 + G},\tag{4}$$

where $b_{\min_{BR}}$ is the minimum attack rate of Blue upon Red in the presence of Green.

The last term in the system equations can be viewed as an intrinsic decay term that might cover combat fatigue, cost of engagement and non-combat casualties etc. The intrinsic decay rates with non-trophic interactions are:

$$x_R^* = x_R - \frac{(x_R - x_{\min_R}) G}{1 + G}, \quad x_G^* = x_G - \frac{(x_G - x_{\min_G}) B}{1 + B} + \frac{(x_{\max_G} - x_G) R}{1 + R},\tag{5}$$

where $x_{(\cdot)}$ is the intrinsic decay rate and the parameters $x_{\min(\cdot)}$ and $x_{\max(\cdot)}$ are the minimum and maximum decay rates in the presence of facilitators and competitors respectively.

3 LIMIT CYCLES AND BIFURCATIONS IN THE BGR COMBAT MODEL

The initial numerical study of our combat model in Eq.(1) consists of two different scenarios. These are entirely fictitious but sufficiently general to capture the diversity of behaviours of the model. The first scenario, denoted as Scenario 1, represents roughly parametrically equal Blue and Red forces fighting in the asymmetric system and the second scenario, denoted as Scenario 2, represents parametrically unequal forces where a technologically superior Blue force engages with an inferior Red force. Here the superiority of Blue is represented through an improved carrying capacity, improved engagement and minimum fire rates, a shorter engagement time, and decreased intrinsic decay rate. The parameter values for these two scenarios are given on Table 1. In both of these toy scenarios, we choose the initial conditions $B(0) = R(0) = G(0) = 2$. We select $f(R) = \tanh(\alpha R)$ for the Blue recruitment rate.

Time series plots of the three populations over time for a range of discrete choices of values of b_{BR} , the Blue to Red fire rate, are provided in Figure 1 for both Scenarios. In the case of Scenario 1 (Figure 1) we observe that for small values of b_{BR} ($b_{BR} < 1$), Red emerges victorious as Blue lacks the firepower to achieve outright success in combat. When $b_{BR} = 1$ the two forces have equal attack rates and a stalemate is reached in which neither side is able to achieve victory as stable fixed and non-trivial solutions for all three populations are reached. For $b_{BR} = 4$ Blue wins after diminishing cycles but must remain constant - effectively as an occupation force. For large values of b_{BR} ($b_{BR} > 1$) persistent limit cycles emerge. In the case of $b_{BR} = 7$, in the context of combat we can interpret the result as Blue winning each combat engagement and then withdrawing from the combat theatre due to a lack of sufficient Red forces to warrant continued engagement. However since Red is not truly defeated, once sufficient Blue has withdrawn Red experiences a surge in growth which prompts Blue to re-enter combat.

In the case of Scenario 2 (Figure 1) these limit cycles appear for smaller values of b_{BR} . For $b_{BR} = 1$ Blue achieves limited success through repeated defeat of Red, withdrawal, resurgence of Red and surge from Blue, but over time with lower Blue resources needed to contain Red. A small Blue force remains - a low level occupation. However here for large values of b_{BR} we observe that Red is the eventual victor regardless of the fact that Blue, as the superior force, decidedly wins in each prior engagement. The interpretation of Red's victory is that once Blue has withdrawn almost the entirety of its force, Red is eventually able to surge faster than Blue is able to return to combat and hence emerges victorious.

Table 1. List of parameters, their descriptions and their default values for the two toy scenarios. The difference in parameters between the two scenarios is highlighted in boldface.

Parameter	Description	Scenario 1	Scenario 2
r_B	Intrinsic recruitment rate of Blue	3	3
r_R	Intrinsic recruitment rate of Red	1	1
r_G	Intrinsic recruitment rate of Green	1	1
$r_{\max R}$	Maximum recruitment rate of Red	2	2
$r_{\max G}$	Maximum recruitment rate of Green	2	2
K_B	Carrying capacity of Blue	10	20
K_R	Carrying capacity of Red	10	10
K_G	Carrying capacity of Green	10	10
w_B	Engagement rate of Blue	1	2
w_R	Engagement rate of Red	1	1
t_B	Engagement duration of Blue	1	0.5
t_R	Engagement duration of Red	1	1
b_{RB}	Red to Blue fire rate	1	1
$b_{\min BR}$	Minimum Blue to Red fire rate	0.1	0.2
q	Hill coefficient	0	0
x_B	Intrinsic decay rate of Blue	0.5	0.25
x_R	Intrinsic decay rate of Red	0.5	0.5
x_G	Intrinsic decay rate of Green	0.25	0.25
$x_{\min R}$	Minimum decay rate of Red	0.25	0.25
$x_{\min G}$	Minimum decay rate of Green	0.125	0.125
$x_{\max G}$	Maximum decay rate of Green	0.75	0.75
α	Blue recruitment surge	1	1

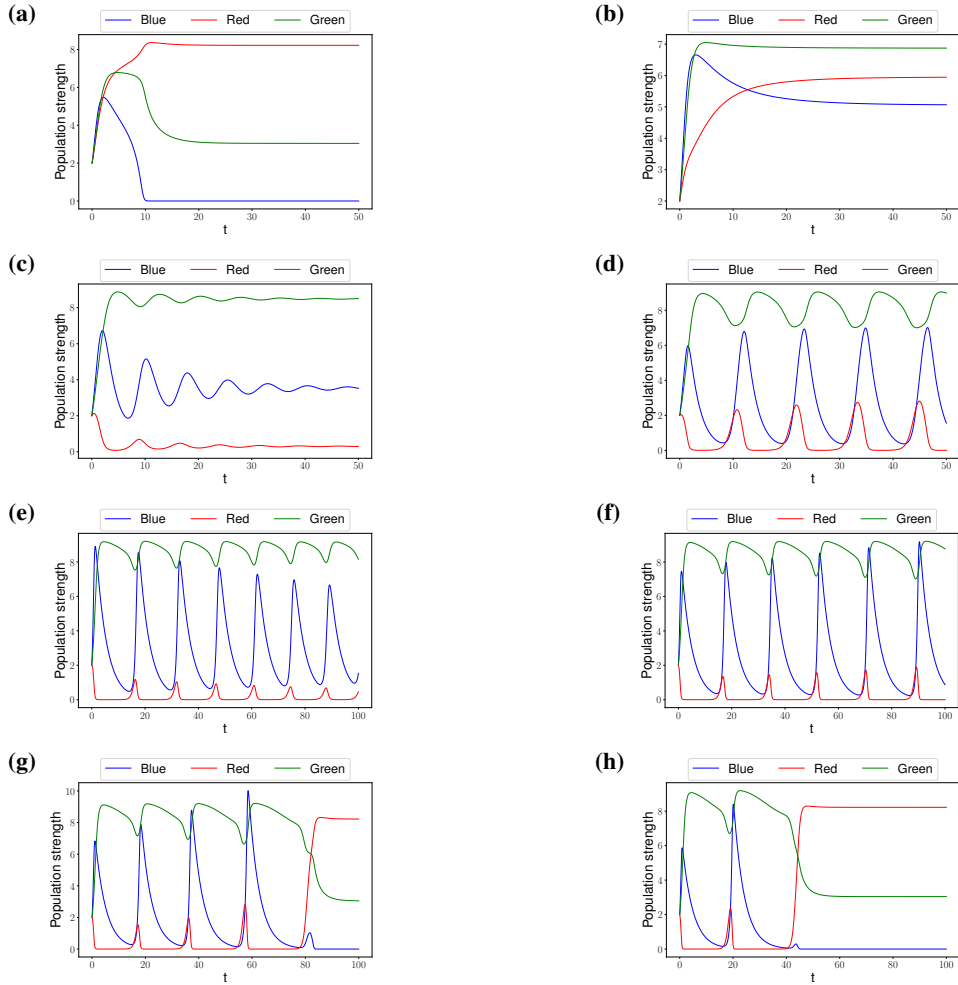


Figure 1. (a) - (d): Scenario 1. Population strengths for Blue, Green and Red over time for parametrically equal forces with a varying Blue to Red fire rate b_{BR} where (a): $b_{BR} = 0.1$. (b): $b_{BR} = 1$. (c): $b_{BR} = 4$. (d): $b_{BR} = 7$. (e) - (h): Scenario 2. Population strengths for Blue, Green and Red over time for a parametrically superior Blue force with a varying Blue to Red fire rate b_{BR} where (e): $b_{BR} = 1$. (f): $b_{BR} = 1.6$. (g): $b_{BR} = 2$. (h): $b_{BR} = 3$.

To display the variety of solution types present in our model, including the discrete cases shown so far, in Figure 2 we present a preliminary bifurcation analysis obtained through the software XPPAUT [Ermentrout, 2002] of the system where we set b_{BR} as our bifurcation parameter. In these plots, stable fixed points are denoted by red lines, stable periodic are solid green circles, unstable fixed points are black lines, and unstable periodic are open blue circles. We only show these plots for B as the qualitative behaviour of the system is sufficiently captured by this plot since the turning and bifurcation points occur for the same values of b_{BR} in the cases of R and G . In this plot, the emergence of stable and unstable limit cycles can be observed as b_{BR} varies, consistent with the behaviours seen thus far. In addition to the limit cycles, we note that in the case of Scenario 1, there exists a small region of bi-stability around $b_{BR} \approx 1.7$ where two stable fixed points (red curves) co-exist at the same value of b_{BR} . In order to determine which of the two non-trivial or trivial stable equilibrium states Blue eventually settles to, we require knowledge of the basin of attraction which is beyond the current scope of this paper. Significantly, in Scenario 2 with a superior Blue force, the region of Blue outright success is within a narrow range of b_{BR} values. Intuitively this is a consequence of two aspects: on the one hand, Blue's surge rate is slower than Red's (to reflect the logistic complexity of moving a technologically advanced force compared to indigenous fighters) and on the other the fact that, as we observed earlier, increasing fire-rate makes no impact given a finite target acquisition time. Thus Blue's superiority only provides limited success at lower fire-rates.

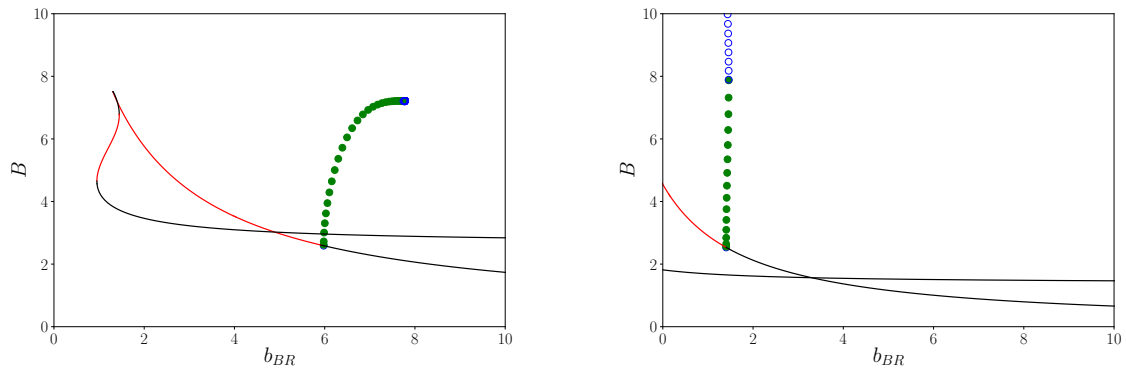


Figure 2. Bifurcation diagrams for Scenario 1 (left) and Scenario 2 (right). Here B is plotted against the Blue-Red fire-rate b_{BR} . Stable fixed points are denoted by red lines, stable periodic are solid green circles, unstable fixed points are black lines, and unstable periodic are open blue circles.

4 DISCUSSION AND CONCLUSION

Non-combatant groups, be they governmental or non-governmental, are an intrinsic part of modern conflicts where, despite the seeming unavoidability of violence, some effort at humanitarian support for the bystanders of the conflict is undertaken. In contrast to the typical exponential growth or decay solutions of the traditional Lanchester combat model, our model exhibits a richer set of solution types as demonstrated through our initial bifurcation analysis of the system on Figure 2. The manifestation of limit cycles as part of this solution set reflects behaviours recognisable in recent real world intervention engagements, where withdrawal of Nato/coalition forces after adversary destruction only below the level of visibility to western eyes results in “Losing by “Winning”” [Cordesman, 2018]. The implication of this unfortunate scenario is that even a negligible presence of Red in a combat theatre is sufficient to cause an eventual resurgence and thus Blue is required to maintain a constant presence (running the risk of appearing to be an occupation force) or undertake its own surges to avoid complete defeat. Our combat model with non-trophic interactions effectively captures the almost unavoidability of this tragic cycle. Analysis of the basin of attraction of the fixed points identified in the model can provide further bounds on the range of parameters for Blue’s success.

Generalisations of this global model will exploit recent developments in ecological modelling which use “multiplex” networks in networked non-trophic models Kéfi *et al.* [2016]; Pilosof *et al.* [2017]; Hutchinson *et al.* [2019], namely to develop a “multiplex” networked combat model. In separate work, some of us are also exploring extensions of the Lanchester model where the Red force draws upon members of the local population. The global model presented in this paper is thus a first step towards building a model of a complex Force operating in a genuinely complex environment that may contribute to force design considerations.

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