

# Stability test for semi-active suspensions with base disturbance

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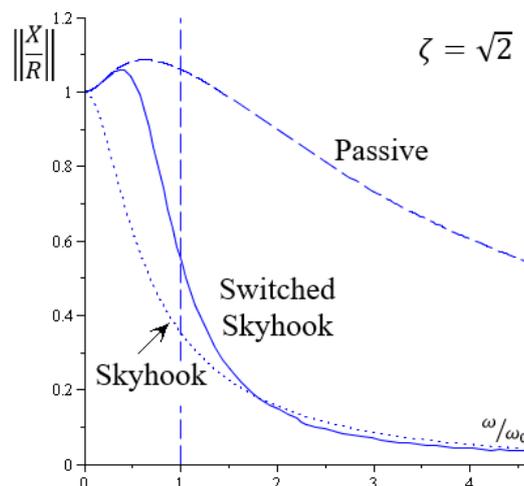
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**Abstract:** Previous investigations by the authors (Storey et al. 2006; Storey et al. 2009; Storey 2011) indicate that controlled suspensions are capable of being “soft” (have a low damping rate) under placid conditions, and “hard” (high damping rate) when required to improve “tracking” (staying close to the middle of and avoiding hitting suspension limits). The question arises then whether such suspensions are safe. On the one hand, vehicles have had relatively high mandated suspension damping rates after Ralph Nadar (1972) famously found that cars with soft suspensions were prone to rollover. On the other hand, improved isolation and tracking should make suspensions more resilient to destructive harmonics.

The authors were curious to perform frequency analyses to compare fundamental linear controls against a simple piecewise linear system with controlled damping. This should be a good comparison of these systems since the frequency analysis of the linear systems is well known and very well understood.

For our analysis we compared three controls, two linear and one piecewise linear. The two linear controls were the standard linear quarter-car model (here called the “linear passive” control) and the well-known skyhook control. It has been claimed by Reichert (Reichert 1997, pp. 12-3) that studies “indicate that the skyhook control is the optimal control policy in terms of its ability to isolate the suspended mass from the base excitations.”. Whether or not this is the case, it is clearly superior to the passive. The main disadvantage of the skyhook is that it cannot be implemented in full by a controlled damper in a semiactive suspension. In the experiments described below, however, we investigated the simplest damping control possible, in which the damper is either on or off. The damper turns off when the force from the damper would add to the magnitude of the chassis’ vertical velocity. This control was called the switched skyhook, for reasons explained below. Such a control should be a good test of stability. The advantage of this control over the skyhook is that it can be implemented by a controlled damper, such as the relatively cheap magnetorheological damper.

Results of our numerical experiments show that the switched skyhook greatly improves on the linear passive. Figure 1 shows an example of transmissibility against frequency (as a proportion of the natural frequency). In this example each control has a damper with the same damping rate,  $\zeta = \sqrt{2}$ . As shown in Figure 1, the switched skyhook can greatly reduce the response amplitude (much less than one) at the resonant frequency, even with relatively high damping rates. It also provides a much smoother response at higher frequencies.



**Figure 1.** Comparison of linear passive, skyhook and switched skyhook.

**Keywords:** Control, suspension, linear, skyhook, stability, frequency, damper

## 1. INTRODUCTION

Simple harmonic motion is generally introduced using models in which the damper is attached to a stationary point. However, in practical suspension systems the disturbing force is often communicated to the suspended mass from a moving base. This is the case for a vehicle subject to road disturbances, but it also applies to buildings, bridges, or even seismographs. It applies also to truly “suspended” masses hung from a vibrating frame, as were the original horse-drawn buggies. In the remainder of this article, the term “suspension system” refers to systems that are excited by a moving base, and the terms “chassis” and “road” will often be used in place of the suspended mass and the base disturbance, as this aids visualisation.

The important difference with a moving base suspension is that the damper contributes energy to the motion of the suspended mass for significant periods of time, sometimes it can add almost as much energy to the system as it dissipates. Passive suspensions have a wide range of frequencies at which there is no attenuation of road vibration, no matter how stiff the damper. This is easily seen from the transmissibility of the passive suspension, and the effects of this can be readily observed on a highway with vehicles moving up and down when encountering bumps causing the chassis to vibrate near the natural angular frequency of the main springs.

In a semi-active control the parameters of otherwise passive components can be varied on a moment-by-moment basis. While there has been a small amount of research into controlled springs, the overwhelming focus has been on controllable dampers, particularly magnetorheological dampers in vehicles. These tend to be much cheaper than active suspension systems, and they are easily integrated into a standard suspension.

A controlled damper in a semi-active suspension can be made entirely dissipative, potentially making a semi-active system more stable than the passive, (Reichert 1997; Song and Ahmadian 2004; Ahmadian et al. 2004; Storey et al. 2006). However, this has not been subject to the same kind of frequency analysis as is usual for passive suspensions.

For the controlled damper to be fully dissipative, its damping coefficient must be reduced to zero in some parts of the suspension travel. But zero damping is unstable. Furthermore, it is theoretically possible to make a semi-active system more unstable than the passive, if the control is badly designed. Finally, non-trivial semi-active systems are non-linear, and non-linear systems can show unexpected behaviour.

## 2. BACKGROUND

### 2.1. Suspension goals

Suspension design is a multi-objective problem with three main goals: isolation (smoothness), tracking (following the road movement and not hitting suspension travel limits), and stability/safety (keeping the system free from greatly disturbing movement, especially resonances). Controlled suspensions can improve on passive systems by being soft where there is no danger of hitting against the suspension travel limits, and by being hard when tracking is a priority (Storey et al. 2006; Storey 2011).

A control known as the “skyhook” has received a large amount of attention in the literature. Figure 2 compares the classic linear single degree of freedom (SDOF) passive suspension, Figure 2(a), with the skyhook basic linear SDOF skyhook suspension, Figure 2(b). Of course, the skyhook “is a purely fictional configuration, since for this to actually happen, the damper must be attached to a reference in the sky that remains fixed in the vertical direction” (Reichert 1997, p. 11). Modern controls routinely use accelerometer input, and the use of absolute position for control is not as unusual as it once was. The pure skyhook requires active components, such as hydraulic or electric actuators.

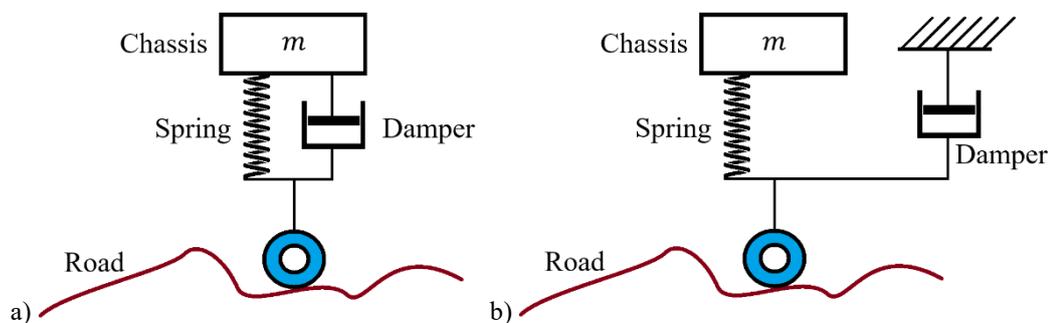


Figure 2. a) Passive suspension, b) skyhook suspension

It is claimed by Reichart that “most ... studies indicate that skyhook control is the optimal control policy in terms of its ability to isolate the suspended mass from the base excitations” (Reichert 1997, p. 12). Whether or not the skyhook is “optimal”, it is clear that its isolation is superior to the passive.

There have been many researchers (Song and Ahmadian 2004; Song et al. 2003; Ahmadian et al. 2004; Paddison et al. 1994; Sims and Stanway 2003) including Reichart who have examined semi-active variants of the skyhook. Paddison et al. (1994, p. 602) state that a semi-active control using “feedback of the absolute velocity is often referred to as ‘skyhook damping’”. A more general characterisation has to do with the energy of the suspended mass. It is claimed that, “since the semiactive damper does not add any energy into the system, the system is stable” (Song et al. 2003, p. 227).

The damper of a passive suspension can add energy to the vertical movement of a car chassis. Suppose, for instance, the sprung mass is moving upwards but the road height rate of change (base velocity) is moving upwards at a faster velocity than the chassis. The damping force will be upwards on the sprung mass, adding to its vertical velocity. To not add to the velocity of the sprung mass the damping rate must be set to zero. Similarly in the downward direction. Suppose we have a controllable damper with damping rate between a maximum value,  $c_M$ , and zero. Let  $x$  be the chassis height,  $r$  the road height, and  $d = x - r$  is the damper extension. The condition that the damper adds to the sprung mass velocity is whenever the chassis velocity,  $\dot{x}$ , and rate of change of damper length,  $\dot{d}$ , are of opposite sign,  $\text{sgn } \dot{x} \neq \text{sgn } \dot{d}$ . The damper would always dissipate vertical chassis kinetic energy if the damper rate was zero whenever  $\text{sgn } \dot{x} \neq \text{sgn } \dot{d}$ . And the damper would dissipate vertical chassis kinetic energy at the maximum rate, at any one time, using the control,

$$c = \begin{cases} c_M, & \text{sgn } \dot{x} = \text{sgn } \dot{d} \\ 0, & \text{sgn } \dot{x} \neq \text{sgn } \dot{d} \end{cases}$$

This represents a relatively simple piecewise-linear control that will here be termed the switching skyhook. It is hypothesised that this control could improve over the transmissibility of the passive suspension.

## 2.2. Transmissibility

To study the transmissibility of the sprung mass system we use the natural frequency (also known as the natural resonant angular frequency, or the undamped angular frequency),  $\omega_0$ , and the damping ratio,  $\zeta$ , as defined in (1) and (2), where  $k$  is the spring rate,  $m$  is the sprung mass, and  $c$  is the damping rate.

$$\omega_0 \triangleq \sqrt{k/m} \tag{1}$$

$$\zeta \triangleq \frac{c}{2\sqrt{km}} \tag{2}$$

The differential equation of motion then takes the form of (3), where a variable,  $p$ , allows us to represent the passive response,  $p = 1$ , and the skyhook,  $p = 0$ , in the one equation.

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2x = 2p\zeta\omega_0\dot{r} + \omega_0^2r \tag{3}$$

Let us represent road height  $r$  as a phasor,  $r = Re^{i\omega t}$ , and the chassis height is represented as  $x = Xe^{i\omega t}$ . The transmissibility is given by,

$$\frac{X}{R} = \frac{i2p\zeta\frac{\omega}{\omega_0} + 1}{1 + i2\zeta\frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2}$$

The transmissibility plots of Figure 3 show the response of the passive in Figure 3(a) and the response of the skyhook in Figure 3(b). These graphs show transmissibility as a function of the ratio of the road frequency over natural frequency,  $\omega/\omega_0$ . The damping ratios shown in this graph, from softest to hardest, are, 0.02, 0.2, 0.4,  $\sqrt{1/2}$ , 1,  $\sqrt{2}$ , 2.5, and 25. The  $x$ -axis represents frequency as a multiple of natural frequency, and the  $y$ -axis represents the amplitude of the chassis movement as a multiple of the road amplitude. The vertical dashed line shows the natural frequency.

Note that the passive amplifies the road movement for all damping rates at frequencies below  $\sqrt{2}$  times the natural frequency. Vehicles can be readily observed on highways heaving with displacement larger than a road disturbance at a frequency that is close to the natural frequency of its springs. “For most automobiles, the heave

natural frequency of the sprung mass is usually 1.0hz to 2.0hz” (Miller 1998, p. 2047). In the past, many cars were produced that set damping coefficients very low, giving the car a smooth, “luxurious” ride feel because they increased isolation at high frequencies, as can be seen in Figure 3(a). In fact, the cars tended to suffer from “tuck under”, wheel hop, and even rollover, especially when cornering. With the skyhook, there is no compromise between isolation at higher frequencies and isolation at lower frequencies, at least for damping rates above 0.707.

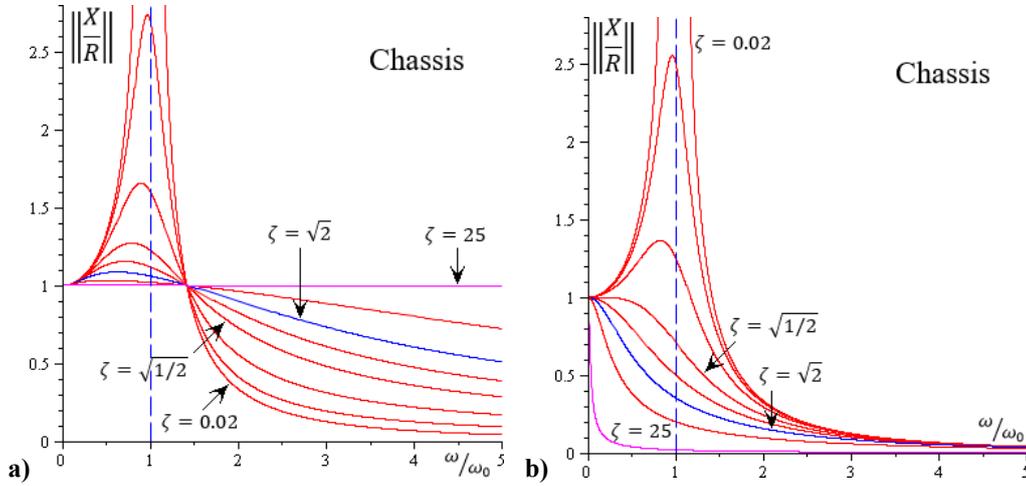


Figure 3. a) Passive suspension transmissibility, b) skyhook suspension transmissibility

Figure 3(b) shows that the skyhook “can isolate even at the resonance frequency” (Reichert 1997, p. 12). The skyhook clearly reduces dangerous movements around the natural frequency, even with relatively soft damping rates,  $\zeta=0.707$  ( $\sqrt{1/2}$ ). And its isolation at higher frequencies is clearly superior to the passive. “This is encouraging since we have removed the tradeoff associated with passive dampers” (Reichert 1997, p. 12)

### 2.3. Piecewise-Linear, Switching Skyhook

For a given mass and spring rate, the damping ratio,  $\zeta$ , is proportional to the damping rate,  $c$ , so the switched skyhook control can be expressed as (4). An example of a switched skyhook trajectory is shown below in Figure 4. Figure 4(a) shows the sinusoidal road height and chassis height. The chassis has been lifted by 1 to make the diagram less cluttered, and to make it correspond more intuitively with a vehicle body moving above a road.

$$\zeta(t) \triangleq \begin{cases} \zeta, & \text{sgn } \dot{x} = \text{sgn } \dot{d} \\ 0, & \text{sgn } \dot{x} \neq \text{sgn } \dot{d} \end{cases} \quad (4)$$

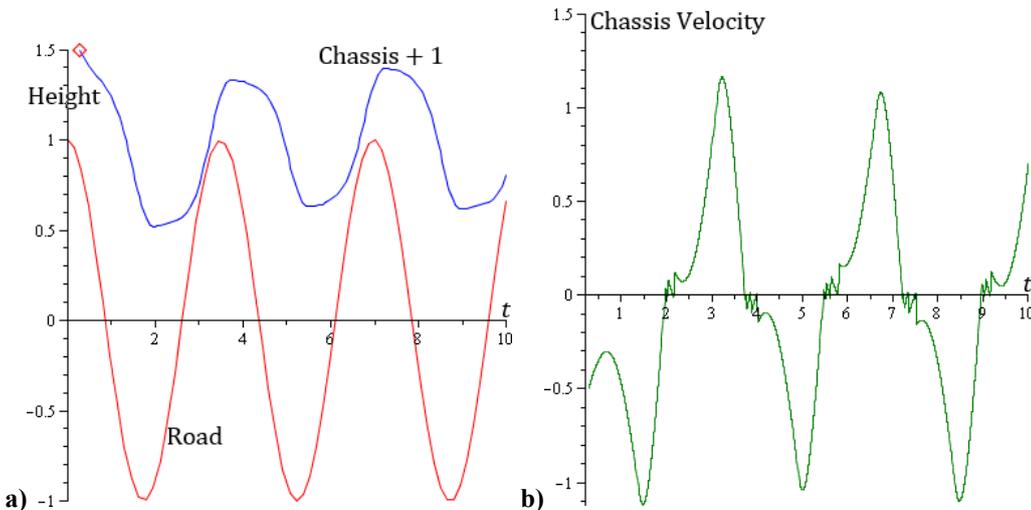
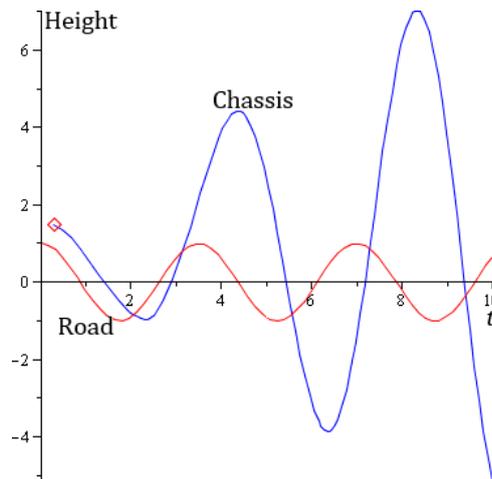


Figure 4. Switched skyhook: a) Chassis response and road. b) Chassis response and chassis velocity.

In this example,  $\zeta = 2$ . In Figure 4(b) the chassis velocity,  $\dot{x}$ , is shown for comparison. The other parameters of this example are as follows: the road amplitude is one with a phase shift of zero, the initial time is taken as  $t_0 = 0.2778$  seconds, and we have  $x(t_0) = 0.5$  and  $\dot{x}(t_0) = -0.5$ . Furthermore,  $\omega_0 = 1.5$ , and  $\omega = 1.8$ .

In the simulations exact piecewise linear solutions were found (using the Maple software package). The behaviour of the system is similar to the purely linear system in that there is an initial “transient”, and the system settles down to an approximate “steady state”. In initial simulations the authors attempted to determine the exact point at which the control would switch, however, when the chassis velocity was low,  $\dot{x} \approx 0$ , the control would switch rapidly, attempting to keep the velocity low, as seen in Figure 4(b). The authors experimented with various subtle controls that might “flatten out” this jitter, but it was decided to keep the experiments purely piecewise linear and use a very small step size.



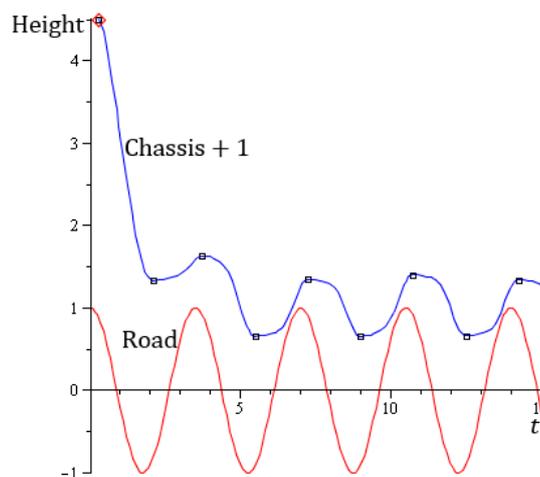
**Figure 5.** Inverse switched skyhook. Chassis movement is represented by the line of increasing amplitude.

It is relatively easy to show that it is not generally true that, “since the semiactive damper does not add any energy into the system, the system is stable.” (Song et al. 2003, p. 227). Figure 5 plots what could be referred to as the inverse switched skyhook. This uses the control of (5), which perversely increases the magnitude of chassis vertical velocity wherever possible, and switches to zero rather than decrease it.

$$\zeta(t) \triangleq \begin{cases} 0, & \text{sgn } \dot{x}(t) = \text{sgn } \dot{d}(t) \\ \zeta, & \text{sgn } \dot{x}(t) \neq \text{sgn } \dot{d}(t) \end{cases} \quad (5)$$

In this case the damper is never dissipative. It is then not strictly true that controlled damping always improves stability. However, it should be possible to analyse controls for stability by analysing the tendency of the damper to reduce vertical chassis velocity.

### 3. METHOD



**Figure 6.** Chassis movement with “transient”. The chassis is raised above the road for ease of interpretation.

To produce analogs of transmissibility plots for the switched skyhook, it is necessary to find the “steady state” response. For example, in Figure 6 the chassis movement settles on a repeated pattern after an initial period. The black boxes represent extremes of chassis height (determined algebraically in Maple 12). After reaching equilibrium the “transmissibility” was determined from these extremes. This was done for a number of damping rates and a range of frequency ratios. (The computations were performed in Maple using closed-form results where possible.)

To find the amplitude of the steady-state response numerically, the challenge is to estimate when the “transient” might die down. It was observed in numerical experiments that the “transient” time was of the same order as the transient for the larger damping rate. The transient has a time constant of the order of,

$$\tau \approx \begin{cases} \frac{1}{\zeta\omega_0}, & \zeta < 1 \\ \frac{1}{\omega_0(\zeta - \sqrt{\zeta^2 - 1})}, & \zeta \geq 1 \end{cases}$$

A simpler, more conservative approximation to these is given by,

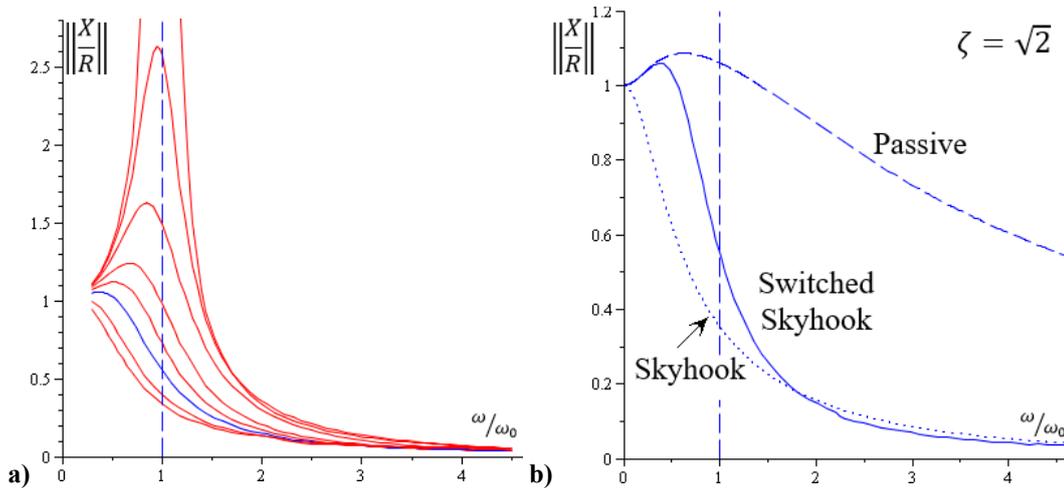
$$\tau \approx \begin{cases} \frac{1}{\zeta\omega_0}, & \zeta < \sqrt{1/2} \\ 2\zeta/\omega_0, & \zeta \geq \sqrt{1/2} \end{cases}$$

The numerical computations would then wait for several time periods for the “transient” to die down and then average the amplitude of a number of “steady state” oscillations.

#### 4. RESULTS

Various responses of the switched skyhook are plotted for different values of  $\zeta$  in Figure 6(a), giving curves that look similar to those of the linear skyhook. The amplitudes for very low road frequencies take an extremely long time to calculate, but it is clear that the curves converge at the point (0,1), as do the curves for the passive and the skyhook (compare with Figure 3).

Figure 7(a) shows that the switching skyhook has much superior isolation to the passive, both at the higher frequencies but, importantly, also around the natural frequency. The values used for  $\zeta$  here are the same as used earlier, except that the largest value is  $\zeta = 4$  rather than 25 (the numerical solution with larger values takes an exceedingly large amount of time).



**Figure 7.** Frequency response for the switched skyhook. a) Various values of  $\zeta$ . b) Comparison with passive and skyhook.

Figure 7(b) shows the switched skyhook compared against the linear passive and the linear skyhook, each with  $\zeta = \sqrt{2}$ . For the most part, the response lies between that of the passive and the skyhook. For even relatively moderate damping ratios, the switched skyhook does not resonate around the natural frequency and greatly decreases movement. It also has much better isolation at high frequencies.

## 5. CONCLUSION

The results show that the switched skyhook has the potential to improve suspension stability and safety using semi-active control. While semi-active controls have the potential to not only improve isolation and tracking they can also improve stability.

The jitter that is shown near the points that chassis velocity is zero, discussed above, could cause some discomfort because it is accompanied by sudden changes in force. A solution to this is to modify the control to anticipate such points and reduce the damping rate accordingly. Such a modification has been discussed previously by the authors (Storey et al. 2012, pp. 397-9). This period in the cycle is not generally critical in the reduction of chassis vertical velocity in any case.

Another possible simple modification to this control, especially for high-frequency road disturbances is to keep the damping rate low in relatively safe conditions, where  $r$ ,  $x$ ,  $d$  and their velocities are very low. Most road undulations and most disturbances are small at high frequencies (Cole 2001), although they can be associated with large accelerations and great discomfort. A suspension able to remain soft under these conditions will feel much more comfortable than the passive.

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