Optimization models to locate health care facilities

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Abstract: The rapid growth of population in cities and major regional areas, shorter length of stays in hospitals, ageing (and the desire of the elderly to stay longer in their homes), and traffic poses a challenge to health departments in meeting the demand for preventive, health center and emergency services. The changes in factors such as urbanization, demography and the rate of service utilization may affect the optimal distances or cost between patients and health care facilities. In addition, rapid population growth, increasing manmade and natural disasters seem to put increasing pressure on demands for timely health care.

There is a challenge in optimally locating health care facilities to enable the community to have good access to preventive, health center and emergency services. The location of health care facilities is an important aspect in health service delivery. It is therefore crucial for health care facilities to be located optimally to serve the community well. Facility location models have a greater importance when applied to the location of health care facilities because improper location will have a serious impact on the community.

The fundamental objectives of locating facilities can be summarized into three categories. The first category refers to those designed to cover demand within a specified time or distance. This objective gives rise to location problems which are known as the Location Set Covering Problem (LSCP) and the Maximal Covering Location Problem (MCLP). The LSCP seeks to locate the minimum number of facilities required to 'cover' all demand or population in an area. The MCLP is to locate a predetermined number of facilities to maximize the demand or population that is covered. The second category refers to those designed to minimize maximum distance. This results in a location problem known as the p-center problem which addresses the difficulty of minimizing the maximum distance that a demand or population is from its closet facility given that p facilities are to be located. The third category refers to those designed to minimize the average weighted distance or time. This objective leads to a location problem known as the p-median problem. The p-median problem finds the location of p facilities to minimize the demand weighted average or total distance between demand or population and their closest facility.

The objective of this study is to discuss the importance of the application of optimization models (maximal covering location and the p-median models) to locate health care facilities. We apply the p-median models and the maximal covering location models to real data from Mackay metropolitan area in Queensland, Australia. We compare the two models using the real data and with existing ambulance stations. The study shows that the p-median model gives a better solution than the maximal covering location model. We also noted that the results of the maximal covering location model depend on the pre-determined weighted coverage distance.

Keywords: *Optimization, models, healthcare, facilities*

1. INTRODUCTION

The location of facilities is important in a good health care delivery in a community. Health care services can be classified as preventive, health center and emergency service. The delivery of most public services involves direct contact between the service facility and the target population. The location of health care facilities is one of those public facilities in which the proximity to the target population is very important. Often communities in a service area desire a closer location of facilities such as hospitals, health centers, ambulance stations, fire stations, post offices, and schools in their service area. Due to scarcity of resources, a limited number of facilities tend to be located to serve larger settlement areas and this often leads to efficiency and effectiveness problems in terms of service delivery. The application of optimization models to locate facilities is therefore crucial in service delivery. A health care facility which is difficult to access leads to increase morbidity and mortality.

In the service industries, the location of health care facilities such as ambulance stations, emergency centers, clinics and hospitals impact significantly on the safety and well-being of the community. The performance of these services can be improved by either improving the existing location using optimization models or increasing the number of facilities. However, increasing the number of facilities is generally limited or impossible due to capital constraints. It is therefore important to locate health care facilities effectively and efficiently. The main concern of location problems is to place facilities to optimize some spatially dependent objectives such as minimize average travel time or distance between demand points and servers and minimize a cost function of travel or response time.

The LSCP, MCLP and the p-Median problem form the basis of models in health care application (Daskin and Dean, 2004). The LSCP minimizes the number of facilities selected so that all demand points are covered (Toregas et al., 1971). The cost of covering all demand nodes is often expensive. The MCLP maximizes the number of demands covered (Church and ReVelle, 1974). The p-Median problem minimizes the weighted distance between the demand and the closest facility (Hakimi, 1964,1965).

A number of authors have used optimization models to locate health care facilities. These include Dzator and Dzator (2018), Beheshtifar and Alimoahmmadi (2015), Fo and Mota (2012), Shariff et al. (2012), Griffin et al. (2008), Marianov and Taborga (2001). The detail of health care facilitity location is presented in Ahmadi-Javid et al. (2017) and Afshari and Peng (2014).

The objectives of this study are to discuss the important of discrete facility location models and to compare the p-median model with the maximal covering location model when applied to real data. We will also investigate the effect of a pre-determined coverage distance on the p-median and the maximal covering location models. The rest of paper is organized as follows. We briefly discuss two primary discrete location models in Section 2. We discuss Mackay metropolitan area and present Mackay data in Section 3. Computational results obtained from Mackay data are presented in Section 4 and conclusion in Section 5.

2. DISCRETE LOCATION MODELS

In this section we will present the Maximal Covering Location Problem (MCLP) and the p-Median Problem. The facility location problem has been modeled as maximal covering location problem; and the p-median problem. The general problem of locating facilities is to establish them to optimize some quantities such as distance, travel time, cost and demand satisfaction. We briefly outline below the models for two discrete location models.

We define the following notation:

$I = \{1,....,m\}$, the set of demand locations,

$J = \{1,....,n\}$, candidate sites for facilities,

$d_{ij} =$ the shortest distance between location i and location j,

$x_{ij} = 1$ if the customer at location i is allocated to facility at location j, 0 otherwise,

$y_j = 1$ if a facility is established at location j, 0 otherwise,

$p =$ the number of facilities to be established,

$x_j = 1$ if a server is stationed at j, 0 otherwise,

$y_i = 1$ if node i is covered, 0 otherwise,

$S =$ time or distance standard for coverage,

$N_i =$ the set of nodes j located within the standard distance or time from demand node i,

a_i = the population at demand node i

2.1.   Maximal Covering Location Problem (MCLP)

Church and Revelle (1974) introduce the maximal covering location problem to address a budgetary constraint of locating a fixed number of facilities in an area. The objective of the MCLP is to locate a predetermined number of facilities, p, in a way as to maximize the demand that is covered.

The maximal covering location problem (MCLP) can be formulated as follows:

$$\text{Maximize } \sum_{i \in I} a_i y_i \tag{1}$$

$$\text{subject to: } y_i \leq \sum_{j \in N_i} x_j, \forall i \in I \tag{2}$$

$$\sum_{j \in J} x_j = p \tag{3}$$

$$x_j, y_i \in \{0,1\}, \forall j \in J\, i \in I \tag{4}$$

In the above formulation, the objective stated in equation (1) maximizes the sum of covered demands. The expression (constraint) in (2) indicates that demand i can only be covered if at least one server is located within the time or distance standard S. Constraint (3) indicates that the total number of facilities that can be located is p. Constraint (4) are the integrality constraints on decision variables.

Megiddo *et al.* (1983) showed that the maximal covering location problem is *NP*-hard.

2.2.   The *p*-Median Problem

Hakimi (1964, 1965) introduced the *p*-median problem when he considered the optimal location of switching centers in a communication network. The objective of this problem is to find the locations of p facilities to minimize the demand weighted total distance (total cost) between each demand node and the nearest facility. For the *p*-median problem the cost of serving demands at node i is the product of the demand at node i and the distance between demand node i and the nearest facility to node i.

We formulate the *p*-median model as follows:

$$\text{Minimize } \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \tag{5}$$

$$\text{subject to: } \sum_{j \in J} x_{ij} = 1, \forall i \in I \tag{6}$$

$$\sum_{j \in J} y_j = p \tag{7}$$

$$x_{ij} \leq y_j, \forall i \in I, \forall j \in J \tag{8}$$

$$y_j \in \{0,1\}, x_{ij} \in \{0,1\} \tag{9}$$

The objective (5) is to minimize the total distance from customers or clients to their nearest facility. Constraint (6) shows that the demand of each customer or client must be met. From constraint (7), the number of facilities to be located is p. Constraint (8) shows that customers must be supplied from open facility. Constraint (9) present the problem as a binary integer programming. The above formulation assumes that the potential facility sites are nodes on the network. Hakimi (1964) however showed that allowing facilities to be located on the arcs of the network instead of the nodes would not reduce total travel cost. Garey and Johnson (1979) showed that the *p*-median problem is *NP*-hard for variable values of p.

3.   MACKAY METROPOLITAN AREA AND DATA

The Mackay metropolitan comprises of 24 suburbs with a total population of 77,137 in the 2016 census. It has two ambulance stations: Mackay Ambulance Station, which is located at Paget, and North Mackay Ambulance Station, located at Beaconsfield (shown in Figure 1). The suburbs are separated into those north of the river (North Mackay, Andergrove, Beaconsfield, Blacks Beach, Bucasia, Cremorne, Dolphin Heads, Eimeo, Erakala, Glenella (with Foulden), Mackay Harbour, Mount Pleasant, Richmond, Rural View (with Nindaroo), Shoal Point and Slade Point) and those south of the river (Mackay City, East Mackay, West Mackay, South Mackay, Oooralea, Paget, Racecourse and Te Kowai). There is one major public hospital located at West Mackay.

3.1. Data

For this study the distance matrix from node to node is determined by distance data using Google maps. This distance is the road network among the various suburbs in the Mackay metropolitan area. Hence the distance values are the shortest road travel distances (equivalent to distance on a road map) between the origin and the destination.

We weighted the distance by taking into consideration only the population of the origin suburb since we can also weigh by considering the nature of the road. The fact is that the better the road network the easier it is for a vehicle to move from one suburb to another. The weighted distance is thus the product of the population at the origin suburb and the distance between the origin suburb and the destination suburb. We note that the larger the population the larger the weight. This weighted distance is calculated for each of the twenty-four suburbs in the Mackay Metropolitan region. This calculation is done by noting the distance from a suburb to all other suburbs and each value is multiplied by the weight of the origin suburb. This is repeated for every suburb and values are recorded as a column matrix representing Mackay Metropolitan area. The population for the suburbs which was used for the weight for each node ranges from 33 for a suburb of Cremorne to 9419 which corresponds to Andergrove and this shown in Table 1. The minimum weighted distance for the Mackay Metropolitan area is 66 km and the maximum value is 178,068 km.

Figure 1. Ambulance Stations in Mackay. **Source:**
https://www.google.com/search?rlz=1C1GGRV_enAU751AU751&q=Ambulance+stations+in+Mackay

Table 1.   Names of suburbs in Mackay Metropolitan Area and their Population from 2016 Census Data

| Node	Suburb	Population	Node	Suburb	Population
1	Mackay City	3659	13	Erakala	547
2	East Mackay	3564	14	Glenella (with Foulden)	4519
3	West Mackay	6210	15	Mackay Harbour	555
4	North Mackay	5933	16	Mount Pleasant	4444
5	South Mackay	6706	17	Ooralea	3366
6	Andergrove	9419	18*	Paget	273
7*	Beaconsfield	5490	19	Racecourse	184
8	Blacks Beach	3861	20	Richmond	648
9	Bucasia	4771	21	Rural View (with Nindaroo)	4793
10	Cremorne	33	22	Shoal Point	977
11	Dolphin Heads	388	23	Slade Point	3349
12	Eimeo	3230	24	Te Kowai	218

4.   COMPUTATIONAL RESULTS

We used the SITATION software (Daskin; 1995) to determine the results for the *p*-median and the MCLP locations for different weighted coverage distance. The first two existing locations are known so we determined their objective values using the SITATION software. Facilities are then added optimally to existing locations using the SITATION software and we noted the objective value for the locations. We used the Lagrangian relaxation to determine the results for the *p*-median problem and MCLP. The detail of the method is discussed in Daskin (1995).

Table 2. The *p*-Median Location and Maximal Covering Location for 70km to 500km Weighted Coverage Distance

| Number of Facilities | Weighted Coverage Distance | | | | | | |
	p-Median – 70Km – 500Km	MCLP – 70km	MCLP – 100Km	MCLP – 200Km	MCLP – 300Km	MCLP – 400Km	MCLP – 500Km
2	6171	6237	6303	6237	6237	6303	6171
3	5643	5775	5808	6006	5775	5808	5643
4	5148	5313	5511	5676	5313	5346	5412
5	4686	4818	5049	5379	4818	4917	4917
6	4224	4587	4521	4917	4488	4389	4455
7	3795	4257	4389	4422	4158	3795	4257
8	3432	4125	4059	3960	3729	3597	4125
9	3102	4026	3927	3366	3135	3465	4059
10	2772	3828	3696	3003	2805	3234	3729
Average	**4330.33**	**4774**	**4807**	**4774**	**4495.33**	**4539.33**	**4752**

Table 3. Comparison of Existing Location with the *p*-Median Problem and MCLP

Number of Facilities	EL Cost	*p*-Median Cost	MCLP Cost (300 Km)	Cost Change Using *p*-Median	Cost Change Using MCLP	Cost Saving Using *p*-Median
2	82173	6171	6237	76002	75936	66
3	6237	5643	5775	594	462	132
4	5643	5148	5313	495	330	165
5	5115	4686	4818	429	297	132
6	4620	4224	4488	396	132	264
7	4158	3795	4158	363	0	363
8	3696	3432	3729	264	-33	297
9	3267	3102	3135	165	132	33
10	2904	2772	2805	132	99	33
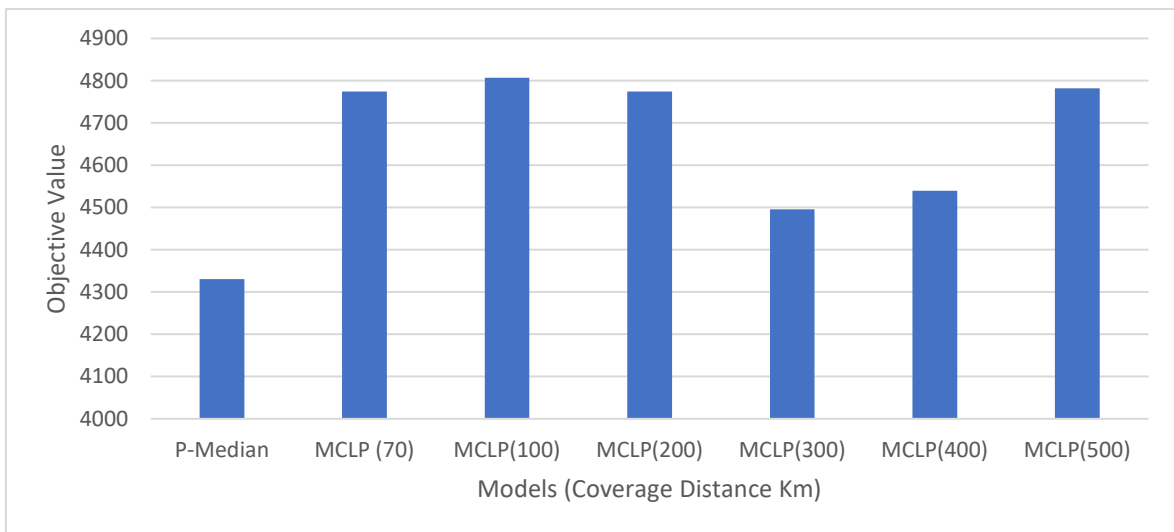
Figure 2. The Objective Value of the *p*-Median Problem and MCLP with different Coverage Distance

Table 2 shows the results of the *p*-median problem and the MCLP, The average of the objective values for two to ten facilities are as follows: *p*-median problem is 4330.33, MCLP with weighted coverage distance of 300km is 4495.33; MCLP with weighted coverage distance of 400km 4539.33, MCLP with weighted coverage distance of 500km is 4752, MCLP with weighted coverage distance of 70km is 4774, MCLP with weighted coverage distance of 100km is 4807.

Figure 2 clearly shows that the *p*-median problem gives the best result followed by the MCLP with weighted coverage distance of 300km, MCLP with weighted coverage distance of 400km and the worst result is given by MCLP with weighted coverage distance of 100km. We noted that as we change the coverage distance the results of the MCLP changes while that of the *p*-median problem remains the same for the facilities that are located.

In Table 3 the objective value of the existing locations is compared with the objective value of *p*-median problem and the MCLP. While the *p*-median problem peforms better in all cases when facilities are located the existing locations performs better than MCLP when eight facilities are located and is the same when seven facilitires are located. The cost saved if the *p*-median problem is used instead of MCLP when two to ten facilities are located ranges from 33 to 363.

5. CONCLUSION

In this paper we discussed two discrete location models namely the MCLP and the *p*-median problem and we outline the important of locating health care facilities using optimization models. We applied the models to real data obtained by calculating the weighted distance in Mackay metropolitan using 2016 population data. The results we obtained show that the *p*-median problem performs better than the MCLP. However, both models perform better than the existing locations. It therefore necessary to use optimization model to locate healthcare facilities for proper accessibility and cost savings.

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