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A stochastic weather generation method for temporal precipitation simulation

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Abstract: Disaggregation methods are designed to produce finer temporal scale data from coarser temporal scale data. An example of this need is the modification of monthly scale precipitation data to drive dynamic hydrological models operated at daily scale. Such issues occur for dynamic hydrological forecasts in which the General Circulation Models (GCMs) are used to generate the required precipitation data but the GCM outputs are only be reliable at coarser temporal scales. Stochastic weather generation methods together with simple adjustment of the total amounts provide a simple and efficient way for data disaggregation. However, the success of stochastic weather generation depends critically on whether the chain-dependent process can mimic the transient property between dry and wet days and whether the specified statistical distribution for amounts on wet days can capture the distributional properties of the amounts. Note that there are many small precipitation events within the catchment data which are treated differently for different purpose. Simple elimination by using different threshold values to define dry/wet days was frequently used in practice. A single statistical distribution may not be sufficient to capture the characteristics of amounts on positive data. Furthermore, the performance of dynamic hydrological model depends also on whether the specified statistical distribution for amounts can capture large precipitation events. Various proposals have been developed to better capture the precipitation pattern, including the use of high-order Markov Chain model for the occurrence pattern and the use of a mixed distribution for the amounts.

In this paper, a hybrid stochastic weather generation method is proposed to simulate daily precipitation data based on the monthly data at catchment scale. Firstly, a multiple state Markov chain model is used for occurrence pattern as two state model may not reflect different weather patterns. Secondly, a truncated Gamma distribution is used for smaller precipitation amounts as usual and a censored extended Burr XII distribution is employed for larger amounts as it can better capture the extreme values. By doing this, one needs not deal with discontinuity in the distribution of precipitation amounts and ensure that the state and its corresponding amount are well matched. In order to take account of seasonality, the models are constructed for individual months.

The proposed method is demonstrated by using catchment data from different climatic regions in Australia. Results show that

(1) The proposed multiple state Markov chain model can capture the states reasonable well;

(2) The seasonality is an important factor to be considered in the disaggregation;

(3) The proposed distributions for different states can mimic the amounts well.

As an regression based approach for the occurrences and distribution fitting approach for the amounts, the proposed method can be easily extended to include external predictors for generalization and/or improvements.

Keywords: *Censored Extended Burr XII distribution; Markov chain model; stochastic simulation; truncated Gamma distribution*

1. INTRODUCTION

Daily precipitation is a critical input for hydrological models, especially for dynamic hydrological models, as well as related models in other researches such as environment and agriculture. From a forecasting point of view, hydrological models rely heavily on precipitation at the same daily scale. While general circulation models (GCMs) are probably the only tools to generate the required precipitation data for using hydrological models to make streamflow forecasting, it is hard to produce reliable precipitation data at daily scale for the future seasons, mainly due to low skills in GCMs at daily scale, especially for precipitation. In order to use precipitation data with large temporal scale in hydrological models requiring daily data, the large temporal scale outputs need to be disaggregated into finer temporal scales.

Data disaggregation of precipitation series is a difficult task due to its unique characteristics, especially at daily and finer scales. As the main purpose of our disaggregation is to generate daily precipitation values for a given monthly value, we do not need to specify the (auto-)correlation structure in the monthly series but treat them as observed. Stochastic weather generation directly models the binary indicator variable of dry or wet event by well-known “chain-dependent process” model defined by transient matrix and precipitation amounts on wet day by a statistical distribution (see, for example, Richardson, 1981; Wilks, 1999). From statistical point of view, stochastic weather generator provides a simple and direct way to simulate the daily precipitation data.

The success of a stochastic weather generator depends critically on whether the chain-dependent process can mimic the transient property between dry and wet days and whether the specified statistical distribution can capture the distributional properties of the amounts. The use of first-order Markov Chain for the occurrence may not be enough to capture the transition pattern between dry and wet states (Wilks, 1999). Various proposals have been developed to better capture the occurrence pattern, including the use of high-order Markov Chain model (see, Lennartsson et al., 2008 for example) and the inclusion of external drivers and seasonality (Furrer and Katz, 2007).

In this paper, we propose a stochastic weather generation by using a multiple state Markov chain model for occurrences to handle large number of small amounts for catchment data and using truncated and censored distributions for the amounts in different rain states. The method considers seasonality by constructing individual models for different months and monthly variation by incorporating the low-frequency amounts as a model predictor. The proposed method is demonstrated by catchment data used in our project in Australia.

2. DATA AND PRELIMINARY ANALYSIS

Six catchments located in eastern Australia are investigated (as provided by the Bureau of Meteorology), covering different climate regions (summer rainfall, winter rainfall and non-seasonal rainfall) and having different sizes (from tens to thousand km2). Due to the page limit, we only present the results for the catchments numbered as “204041” (Orara River @ Bawden Bridge in New South Wales, covering 1637km^2, with summer rainfall pattern) as a representative for demonstration. The catchment rainfall data is determined by averaging the data from the Australian Water Availability Project (AWAP) 0.05 degree x 0.05 degree gridded dataset (Jones et al., 2009).

Unlike station data, catchment data are obtained by interpolation which causes a noticeable proportion of smaller values. There is no common agreement on the choice of threshold to define rainy days. Figure plots the proportions of the days with non-zero, above 2mm and between 0 and 2mm, respectively, for individual months. It can be seen from Figure 1 that there are large proportions of non-zeros and between 0-2mm. In fact even about 10% of days

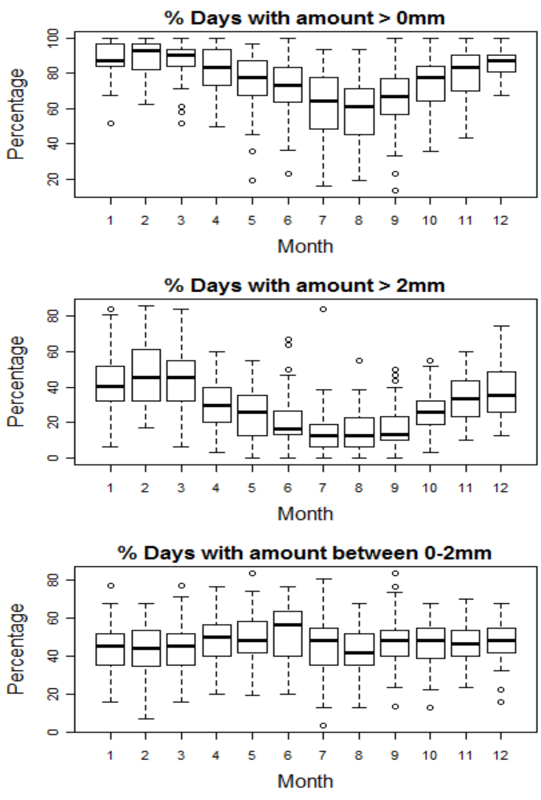

Figure 1. Boxplots of proportions for the days with different rain states by individual months.

have precipitation amounts between 0-1mm. It is obvious that different threshold values can affect statistical models significantly, especially the transition probabilities. Furthermore, a higher threshold value results in less number of values in the lower end, causing more emphasis on larger values which are frequently missed in simulation (see Furrer and Katz, 2007 and 2008 for discussions). Furthermore, clear seasonality can be viewed in the occurrence probability at different threshold values, confirming the importance of considering seasonality in both occurrence and amount models.

3. METHODOLOGY DEVELOPMENT

Throughout the paper, the daily precipitation series at catchment scale is measured in millimeter (mm) and denoted as z_t ($t = 1, 2, \cdots, n$) with *n* being the number of daily observations.

Model the occurrences of rain status

Instead of artificial definition of the threshold for wet/dry day and with intention to better capture the characteristics of amounts on wet day, we define a three-state process for the precipitation occurrence as

$$x_t = \begin{cases} 0 & \text{if } z_t \le 0, \\ 1 & \text{if } 0 < z_t \le c, \\ 2 & \text{if } z_t > c, \end{cases} \tag{1}$$

and name them as dry, moist and wet day, respectively. As usual, we treat *c* as pre-defined. The advantage of doing this is that we can model the distribution of precipitation amount in the moist day and has a better chance to simulate large amount in wet day. Let

$$J_{ti} = \begin{cases} 1, & \text{if the } i\text{-th category} \\ & \text{is observed at time } t, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Here only $m-1$ category indices J_{ti} ($i = 1, \cdots, m-1$) is needed with *m* being the number of category (which is 3 in the above setting). Unlike the traditional specification of Markov Chain defined by conditional probability of wet day, we use the following probability

$$\pi_{ti} = \Pr\{J_{ti} = 1 \mid x_{t-1}, x_{t-2}, \cdots\} \tag{3}$$

where the sequence of past occurrence values yet to be specified.
The multinomial logit model defined by Agresti (1990, Section 9.2), defined as

$$\pi_{ti}(\boldsymbol{\beta}) = \frac{\exp(\boldsymbol{\beta}_i' U_{t-1})}{1 + \sum_{l=1}^{m-1} \exp(\boldsymbol{\beta}_l' U_{t-1})}, \tag{4}$$

is frequently employed in the analysis of categorical data series. Here $\boldsymbol{\beta}_i = (\beta_{i,1}, \beta_{i,2}, \cdots, \beta_{i,q})'$ ($i = 1, \cdots, m-1$) are the vectors of regression coefficients with *q* yet to be determined and U_{t-1} is the collection of covariates and possible lag variables. In this paper, we specify the past events as the rain state in the previous day. The multinomial logit model is formed by linear relationships of

$$\log\{\pi_{ti} / \pi_{tm}\} = \boldsymbol{\beta}_l' U_{t-1}. \tag{5}$$

Note that our target is to disaggregate monthly data to daily and that the correlations between months are possibly dominated seasonality. Therefore, one can simply construct a model for each month by eliminate the smoothness between month. That is, we construct separate models for each month.

Model the amount distribution for moist days

Note that the precipitation amounts are bounded between 0 and *c*. The statistical distribution modeling the amounts should have a finite support and flexible enough to capture the distributional pattern. Following the traditional use of Gamma distribution, we propose the use of the called truncated Gamma distribution defined by density function

$$f_{tG}(x; \alpha_1, \alpha_2) = \begin{cases} \mathrm{K}_1^{-1}(\alpha_1, \alpha_2) e^{-x/\alpha_1} x^{\alpha_2 - 1}, & 0 \le x \le c, \\ 0, & \text{otherwise,} \end{cases}, \tag{6}$$

(Chapman, 1956) with $\alpha_1 > 0$ and $\alpha_2 > 0$ being the parameters, where

$$\mathrm{K}_1(\alpha_1, \alpha_2) = \int_0^c e^{-x/\alpha_1} x^{\alpha_2 - 1} dx = \alpha_1^{\alpha_2} \gamma(\alpha_2, c/\alpha_1). \tag{7}$$

with $\mathrm{K}_1(\alpha_1, \alpha_2) \to \alpha_1^{\alpha_2} \Gamma(\alpha_2)$ as $c \to \infty$. Here

$$\gamma(a,t) = \int_0^t e^{-x} x^{a-1} dx \quad (8)$$

is the truncated gamma function which can be evaluated directly in many software package. For a given c, the distributional parameters can be estimated by the maximum likelihood estimation.

Model the amount distribution for wet days

In order to capture the possible long-tail properties in precipitation amounts, we need a distribution with flexible tail properties. The Extended Burr XII distribution was proposed for modeling flood frequency distribution (Shao et al., 2004 and 2006) has been used for distribution modeling where the tail properties are important, including rainfall depth (Gargouri-Ellouze and Chebchoub 2008) and flow duration curve (Shao et al., 2009). We then propose the censored Extended Burr XII distribution with density function

$$f_{cEB12}(x;\alpha_3,\alpha_4,\alpha_5) = \begin{cases} \mathrm{K}_2^{-1}\left[\alpha_4\alpha_3^{-1}(x/\alpha_3)^{\alpha_4-1}\left\{1-\alpha_5(x/\alpha_3)^{\alpha_4}\right\}^{1/\alpha_5-1}\right], & x \ge c, \alpha_5 \ne 0, \\ \mathrm{K}_2^{-1}\left[\alpha_4\alpha_3^{-1}(x/\alpha_3)^{\alpha_4-1}\exp\left\{-(x/\alpha_3)^{\alpha_4}\right\}\right], & x \ge c, \alpha_5 = 0, \\ 0, & \text{otherwise,} \end{cases} \quad (9)$$

with

$$\mathrm{K}_2(\alpha_3,\alpha_4,\alpha_5) = \begin{cases} \left\{1-\alpha_5(c/\alpha_3)^{\alpha_4}\right\}^{1/\alpha_5}, & \alpha_5 \ne 0, \\ \exp\left\{-(c/\alpha_3)^{\alpha_4}\right\}, & \alpha_5 = 0. \end{cases} \quad (10)$$

The range of the censored EBXII distribution is $c \le x < \infty$ for $\alpha_5 \le 0$ and $c \le x \le \alpha_3 / \alpha_5^{1/\alpha_4}$ for $\alpha_5 > 0$. Note that EBXII distribution has the Pareto distribution as another embedded distribution (with $\alpha_4 \to +\infty$ and $\alpha_5 \to -\infty$), which however has a positive support and may leave a gap between the selected threshold and low bound of the Pareto distribution. To avoid this gap, we prefer to fix the parameter $\alpha_4 = 1$ as the special case which is indeed the Generalized Pareto distribution. Doing this specialization does not only reduce complexity of programming but also increase the flexibility.

For the censored EBXII distribution, it is important to determine if the estimated value of α_5 is positive, zero or negative. Let $(\hat{\alpha}_3, \hat{\alpha}_4, \hat{\alpha}_5)$ be the maximum likelihood estimates of EBXII distribution. Similar to the way for the extended Burr XII distribution, we need to derive a criterion to determine the sign of α_5, which is given by simple calcuations as

$$\Delta(x,\alpha_3,\alpha_4) = \frac{n}{2}(c/\alpha_3)^{2\alpha_4} + \sum_{i=1}^{n}\left\{(x_i/\alpha_3)^{\alpha_4} - \frac{1}{2}(x_i/\alpha_3)^{2\alpha_4}\right\}. \quad (11)$$

Let $(\alpha_3^{(w)}, \alpha_4^{(w)})$ be the maximum likelihood estimates by fitting the data with the Weibull distribution (the degenerated case of the EBXII distribution with $\alpha_5 = 0$). If $\Delta(x,\alpha_3^{(w)},\alpha_4^{(w)}) > 0$, then $\hat{\alpha}_5 > 0$, while $\Delta(x,\alpha_3^{(w)},\alpha_4^{(w)}) < 0$ implies $\hat{\alpha}_5 < 0$ and $\Delta(x,\alpha_3^{(w)},\alpha_4^{(w)}) = 0$ indicates the Weibill fitting with $\hat{\alpha}_5 = 0$.

Selection between GPD/Weibull and EBXII

Given that one prefers the simple GPD or Weibull to EBXII, a statistical support is needed for the use of simple models. The likelihood ratio test has been used widely for this purpose. Let

$$LRT = -2\log\left\{L_s(\hat{\theta}_s) / L_g(\hat{\theta}_g)\right\} \quad (12)$$

be the ratio of two likelihood functions: the sample distribution *s* (GPD or Weibull) having fewer parameters θ_S and the general distribution g (EBXII) having more parameters θ_g with $\theta_s \subset \theta_g$, where $\hat{\theta}_s$ and $\hat{\theta}_g$ are the maximum likelihood estimates for corresponding distributions, respectively. Asymptotically, the test statistic is distributed as a chi-square random variable with degree of freedom equal to the difference in the number of parameters between the two candidate distributions. Therefore, the p-value is given as $\Pr(\chi_1^2 > LRT)$ and the simple distribution will be accepted if the computed p-value is greater than the pre-determined confidence level (0.05 say).

4. RESULTS

Due to the space limitation, we do not report the results for two states Markov model here for comparison but simply state that the two state model varies dramatically according to different threshold values. The results for three state Markov model is given in Table 1, from which the seasonality is also clearly shown. In order to see the model performance, we conducted 1000 simulations for the three state model and summarized the results in Figure 2 by boxplot with the empirical probabilities from the observation. It is clearly seen that the model performance is quite good. The monthly variation is also clearly indicated from the empirical probabilities based on observations.

Table 1. The estimated logit regression coefficients with three precipitation states. The results are for individual months or for all data. The values in brackets are corresponding standard deviations.

Month	Log(Pr[Dry]/Pr[Wet]			Log(Pr[Moist]/Pr[Wet]		
	Intercept	Dry	Moist	Intercept	Dry	Moist
1	-3.550	5.405	2.857	-0.876	2.722	1.672
	(0.254)	(0.358)	(0.278)	(0.078)	(0.265)	(0.111)
2	3.526	5.037	2.688	-0.904	2.571	1.542
	(0.246)	(0.364)	(0.275)	(0.077)	(0.276)	(0.114)
3	-3.782	5.382	3.002	-0.830	2.570	1.500
	(0.280)	(0.372)	(0.302)	(0.076)	(0.254)	(0.109)
4	-3.373	5.687	3.068	-0.545	2.723	1.522
	(0.294)	(0.384)	(0.312)	(0.088)	(0.264)	(0.119)
5	-3.727	6.299	3.820	-0.405	2.513	1.581
	(0.382)	(0.441)	(0.396)	(0.093)	(0.244)	(0.127)
6	-3.077	6.018	3.402	-0.396	2.917	1.844
	(0.323)	(0.413)	(0.341)	(0.107)	(0.281)	(0.141)
7	-2.236	5.504	3.035	-0.415	3.001	2.081
	(0.255)	(0.350)	(0.280)	(0.126)	(0.275)	(0.164)
8	-2.970	6.150	3.793	-0.145	2.551	1.496
	(0.362)	(0.423)	(0.379)	(0.118)	(0.252)	(0.157)
9	-2.324	5.054	2.674	0.167	1.908	1.010
	(0.280)	(0.343)	(0.299)	(0.114)	(0.234)	(0.146)
10	-2.367	4.579	2.352	-0.288	2.101	1.178
	(0.201)	(0.278)	(0.224)	(0.090)	(0.216)	(0.123)
11	-2.804	4.831	2.417	-0.331	2.130	1.056
	(0.230)	(0.314)	(0.252)	(0.084)	(0.232)	(0.116)
12	-3.106	4.925	2.619	-0.700	2.757	1.521
	(0.223)	(0.338)	(0.247)	(0.080)	(0.263)	(0.113)
All	-3.119	5.616	3.033	-0.557	2.648	1.555
	(0.076)	(0.100)	(0.082)	(0.026)	(0.072)	(0.036)

For the rain amount modeling, Table 2 gives the estimated distributional parameter values for different threshold values when Gamma distribution is used to model the rain amount on rain day. It can be seen that the estimates change greatly for different threshold values. It again support that the small rain amounts should be treat carefully.

Based on three state model for occurrences with “moist” for amounts between (0, 2mm) and “wet” for amounts>2mm, and using the truncated Gamma distribution for moist day and censored extended Burr XII distribution for wet day, Table 3 gives the estimates of the distributional parameters.

For the censored extended Burr XII distribution, we also include the parameter estimation for the special cases of Weibull distribution (with α_5=0) and Generalized Pareto distribution (with α_4=1). The likelihood ratios test concludes that the censored extended Burr XII distribution should be used for all cases. For the truncated Gamma distribution, the shape parameter (α_2) is relatively stable while the scale parameter (α_2) varies from month to month. The parameter values for the censored extended Burr distribution change greatly because the distribution is used to capture the tail behavior which changes from month to month. The fitted density distributions together with the histograms from the observations for individual months are plotted in Figure 3 for the moist days and Figure 4 for the wet days. It can be seen that the fitted distributions capture the empirical ones very well for both states. The seasonality is also clearly shown in the fitting distributions for amounts.

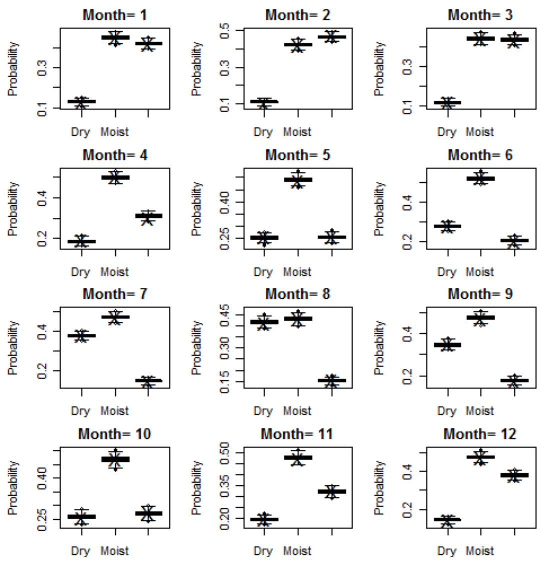

Figure 2. Boxplots of simulated states for three-state Markov model. The results are for individual months. The empirical values from observations are plotted as “X”.

Table 2. The estimated distributional parameters for Catchment 204041, represented by Gamma distribution, conditional on wet days with difference threshold defining dry day (less or equal to the threshold is marked as dry). The data were shifted to the support of $[0,\infty)$ by extracting the threshold values.

Month	Threshold=0mm		Threshold=0.2mm		Threshold=1mm		Threshold=2mm	
	Shape	Rate	Shape	Rate	Shape	Rate	Shape	Rate
1	0.370	0.055	0.521	0.063	0.626	0.062	0.671	0.060
2	0.391	0.053	0.537	0.061	0.627	0.060	0.657	0.056
3	0.382	0.057	0.537	0.066	0.618	0.064	0.632	0.058
4	0.316	0.060	0.492	0.067	0.587	0.062	0.635	0.058
5	0.301	0.066	0.457	0.069	0.549	0.061	0.580	0.055
6	0.259	0.059	0.409	0.055	0.498	0.047	0.533	0.042
7	0.250	0.073	0.429	0.070	0.496	0.057	0.583	0.052
8	0.270	0.083	0.458	0.084	0.563	0.072	0.616	0.066
9	0.333	0.135	0.526	0.138	0.673	0.130	0.680	0.112
10	0.343	0.086	0.512	0.095	0.630	0.091	0.645	0.081
11	0.366	0.075	0.529	0.084	0.629	0.077	0.749	0.081
12	0.375	0.073	0.563	0.085	0.702	0.087	0.786	0.087
All	0.321	0.064	0.494	0.071	0.597	0.067	0.643	0.063

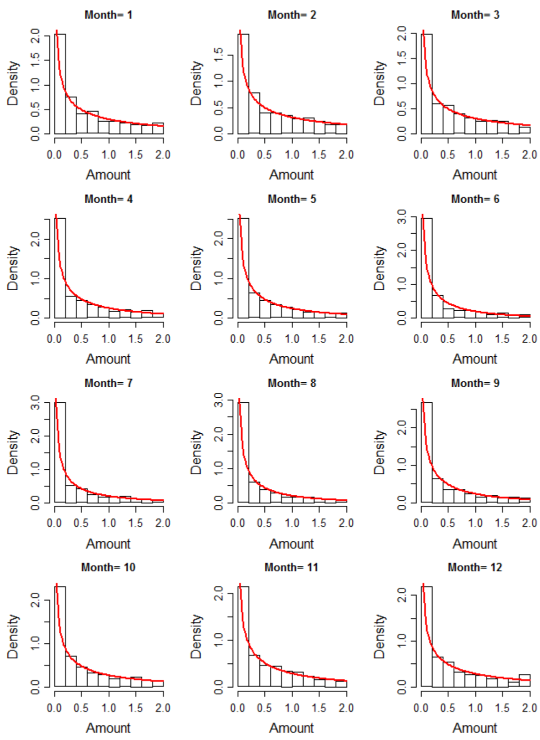

Figure 3. Histograms of observations for moist state with precipitation between 0 and 2 mm. The fitted densities are given by red line. The results are for all data with division to monthly.

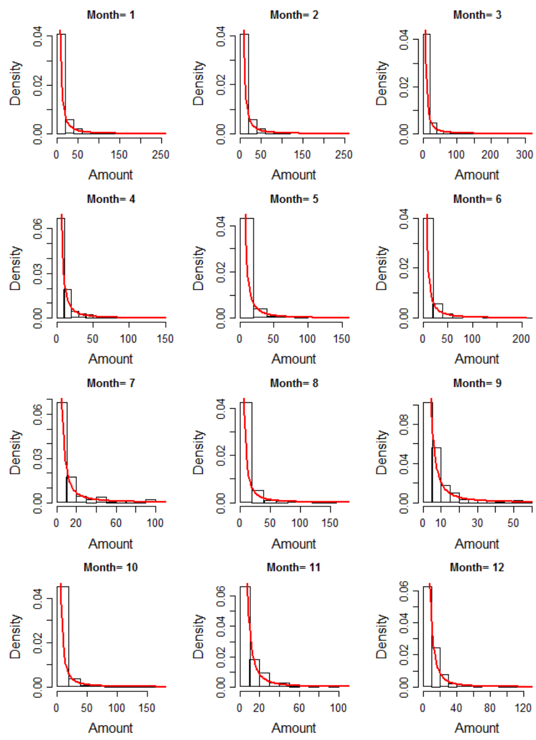

Figure 4. Histograms of observations for wet state with precipitation being greater than 2 mm. The fitted densities are given by red line. The results are for all data with division to monthly

5. CONCLUSIONS

In this paper, we developed a multi-state Markov model for rainfall status in order to better capture the transition from state to state. For moist state, we use the Gamma distribution to model the precipitation amounts as usual but in the sense of truncation due to its upper limit. For the wet state, we use the censored extended Burr XII distribution due to its flexibility capability to capture different tail behaviors. The examples demonstrate the necessity of monthly modeling in order to capture seasonality which is an inherit property in many cases.

Table 3. The estimated distribution parameters for Catchment 204041, for moist state (with precipitation amounts between 0 and 2 mm) by truncated Gamma and wet state (with precipitation amount>2mm) by censored Extended Burr XII distribution. The results are for individual months or for all data. The results for truncated GPD and truncated Weibull distributions are also given.

Month	Gamma		Burr			GPD		Weibull		
	α_1	α_2	α_3	α_4	α_5	α_3	α_5	α_3	α_4	Δ
1	3.837	0.450	4.213	4.757	-5.043	10.939	-0.170	13.26	0.999	-251
2	5.040	0.460	4.492	4.220	-4.369	10.996	-0.191	13.56	0.983	-237
3	4.167	0.460	9.934	21.02	-26.48	9.935	-0.213	12.53	0.959	-364
4	2.031	0.429	3.759	6.185	-6.395	9.321	-0.262	12.37	0.938	-200
5	1.857	0.420	2.932	11.21	-12.55	8.560	-0.302	11.78	0.903	-175
6	1.197	0.404	2.115	100	-136.0	9.645	-0.344	13.55	0.877	-99.7
7	1.464	0.359	2.346	40.947	-51.15	9.142	-0.304	12.48	0.913	-64.1
8	1.339	0.382	2.600	17.19	-19.88	8.761	-0.217	11.16	0.969	-110
9	1.391	0.446	2.135	134.79	-137.27	7.413	-0.080	8.512	1.129	-91.5
10	2.633	0.427	2.642	15.939	-17.02	8.084	-0.178	10.04	1.009	-247
11	2.247	0.478	5.222	3.370	-2.753	10.707	-0.052	11.87	1.121	-123
12	3.025	0.446	5.908	2.835	-1.999	10.540	-0.041	11.65	1.148	-208
All	2.216	0.421	3.047	9.906	-11.47	9.661	-0.201	12.10	0.980	-2347

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