SEVA: A non-linear mathematical framework for climate change vulnerability assessment

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Assessing vulnerability to climate change allows policymakers to prioritize policy interventions Abstract: and better allocate resources. Climate Change Vulnerability Assessment (CCVA) can play an important role in developing long-term environmental and infrastructure plans. Typically, a CCVA exercise combines knowledge from multiple disciplines (e.g., climate science, public health, social science, infrastructure systems, economics) in order to build models of the vulnerability to a climate-related stress of a valued attribute (e.g., health, prosperity, security) of a socio ecological system (SES) (e.g., local government area, councils, counties). In indicator-based vulnerability assessments (IBVA), indicators are adopted as proxy measures of processes generating vulnerability. However, some of the widely recognized challenges of IBVA have been the absence of a mathematically robust framework that can combine information from different knowledge domains, while taking into account the partial compensation of loss/gain between indicators, nonlinearities, and tipping points. In one of our previous papers we showed that even though their goals are fundamentally different, Multi-Criteria Decision Analysis (MCDA) and IBVA have the same structural features. Hence, we proposed an aggregation framework for IBVA using insights from the older and more mature field of MCDA (El-Zein and Tonmoy, 2013a). The goal of this paper is to extend this framework in order to define and deal with non-linear relationships and threshold effects commonly occurring in IBVA.

We first identify different sources of non-linearity present in the context of IBVA. We distinguish between a *fundamental nonlinearity* (dependence of vulnerability on magnitude of stress), a *deductive nonlinearity* (where a deductive or mechanistic model is required to identify the relationship between a set of indicators and vulnerability) and an *intuitive nonlinearity* (where the same relationship is characterized as non-linear through inductive arguments by stakeholders and/or experts). Second, we build a new framework called the Sydney Environmental Vulnerability Assessment (SEVA) by introducing harm as a concept that replaces or mediates the relationship between the indicator and the vulnerability it represents. Harm can be conceived of as a more concrete, less abstract form of vulnerability that is more amenable to quantification. A harm criterion then, like an indicator, acts as a proxy for a process generating vulnerability. However, a harm criterion allows us to achieve two key objectives: a) to relax the conditions concerning linearity, and b)



to separate deductive and intuitive nonlinearities in order to better deal with both of them.

SEVA conducts aggregation of harm criteria using an outranking framework. In a previous paper, we showed that outranking methods, developed in decision



science, are better suited to IBVA because their theoretical requirements are less stringent than multiattribute utility theory (MAUT) approaches, and do not require perfect knowledge of preference structures (El-Zein and Tonmoy, 2013a). Outranking procedures are especially powerful in dealing with partial compensation and fuzzy relationships and, through SEVA (Figure 1), we extend the capability of an outranking procedure (ELECTRE III) to deal with the nonlinearities defined above. We simulate various combinations of nonlinearities and partial compensation through specific definitions of *thresholds of differences* (which are basic parameters of fuzziness in outranking algorithms). We demonstrate the use of SEVA by applying it to a hypothetical model of vulnerability of beach residents to sea level rise and its associated processes.

Keywords: vulnerability assessment, climate change, aggregation, nonlinear, outranking, mathematical framework

1. INTRODUCTION

The Earth's climate system includes the natural spheres (e.g., atmosphere, biosphere, hydrosphere and geosphere), the anthropo-sphere (e.g., economy, society, culture), and their interactions (Schellnhuber, 1999). These interactions are one of the main sources of non-linear behaviour and our present inability to characterize them is a major source of uncertainty in our attempts to predict the effects of global environmental change (Rial et al., 2004). Climate change vulnerability assessment (CCVA) is an attempt to understand these interactions in a way that helps a systematic incorporation of climate futures in planning (Füssel, 2007). CCVA involves assessing the risk to populations, economies, and engineering infrastructures from exposure to increased climate hazards by combining the physical dimensions of risk exposure with the socio-economic and institutional processes that moderate the impacts of the hazards in question. A risk-hazard approach combines the Global Circulation Models (GCM) projections with non-linear mechanistic bio-physical or bio-chemical models (e.g., hydrological, epidemiological, atmospheric) to identify regional and local impacts. However, extending this approach to include the socioeconomic dimensions of risk and identify all important forms of nonlinearity (including those characterising the interaction between these two systems) remains a challenge for CCVA.

Indicator-based vulnerability assessments (IBVA) have been widely used because they offer a relatively simple way of quantifying different components of the risk, bio-physical, institutional and socio-economic (Füssel, 2007, Hinkel, 2011). However, IBVA, as typically conducted, suffer from multiple theoretical and methodological shortcomings. First, our imperfect knowledge of the interaction of bio-physical and socioeconomic systems makes it difficult to identify and use measurable proxy indicators that can represent all the significant processes generating vulnerability. Second, developing aggregation principles for indicators is challenging because of the multiplicity of knowledge domains (e.g., climatic, social, economic, engineering, institutional), data types (continuous, discrete and ordinal variables), forms of relationships between indicators and vulnerability (linear and non-linear), as well as different possible relationships of compensation and non-compensation between the indicators, present in these analyses (El-Zein and Tonmoy, 2013a). A majority of the IBVA literature uses simple aggregation approaches that are based on Multiple Attribute Utility Theory (MAUT) (e.g., simple additive or multiplicative aggregation). Although MAUTbased additive aggregation is a powerful tool, its strict theoretical requirements (e.g., indicator additive independence, complete knowledge on the system etc) are hardly ever met in the context of IBVA (El-Zein and Tonmoy, 2013a). Moreover, a number of assumptions are typically made in IBVA studies that use MAUT—a linear, monotonic relationship between indicator and vulnerability; complete compensation between indicators-none of which usually hold in reality.

In a previous paper, we showed that even though their goals are fundamentally different, Multi Criteria Decision Analysis (MCDA) and IBVA have the same structural features and therefore we developed an aggregation framework for IBVA borrowing insights from MCDA (El-Zein and Tonmoy, 2013a). We showed that outranking methods built in decision science, are better suited for IBVA because they provide a more structured approach to the challenges mentioned above, and proposed an outranking based framework for IBVA (El-Zein and Tonmoy, 2013a, Tonmoy and El-Zein, 2012). Outranking procedures are especially powerful in dealing with compensation challenges and fuzzy relationships, but in their present form cannot accommodate different forms of non-linearity that might occur in the context of IBVA. In this paper, we define different types of non-linearities relevant to IBVA and propose a new outranking-based framework for IBVA that incorporates and represents these non-linearities.

2.0 DIFFERENT FORMS OF NON-LINEARITIES IN IBVA

We begin with a general mathematical framework for IBVA we developed earlier. This framework operates with a multi-dimensional definition of vulnerability and takes into account the impact of adaptive events. Vulnerability assessment aims to develop some measure, quantitative or qualitative, of the susceptibility to damage of, or damage likely to be inflicted on the valued attribute of an SES, as a result of its exposure to one or more climate stresses. For the purpose of the discussion below, damage is denoted by D, vulnerability by V, and the magnitude of the climate stress in question by M. It is reasonable to assume that as the magnitude M of the climate stress of does the damage D. This framework defines vulnerability as the ratio of damage to magnitude, i.e. as the marginal rate of damage relative to the magnitude of the stress (El-Zein and Tonmoy, 2013b), hence:

 $D = \mathbf{V}M$

(1)

Tonmoy and El-Zein, A non-linear mathematical framework for climate change vulnerability assessment

where V is a positive number (for clarity, D and M are represented in italics and the slope connecting them, i.e. vulnerability, in bold-faced font, throughout). In some cases V is largely independent of M and (1) simply reflects a linear relationship between D and M. For example, within a given range, the extent of physical damage inflicted on houses in a "do nothing" scenario may be roughly proportional to the level of sea rise that caused it, i.e. V does not depend on M. In reality such relationships are seldom linear because more often than not, D is a non-linear function of M. Rivers bursting their banks and sea waves breaching beach fortifications are examples in which a threshold effect generates a non-linear relationship between D and M (Table 1). It is possible to represent such non-linearity by introducing a dependence of V on M:

$$D=V(\mathbf{M})M$$

Hence, it is now possible to speak about assessing vulnerability to a given *magnitude* of stress, i.e. developing some measure of V(M) at a given M. We call this non-linearity (i.e. dependence of V on M), the *fundamental non-linearity*, to distinguish it from other forms of non-linearities that will be introduced below. Provided D is differentiable over M, it is possible to generalise from equation (2) and define vulnerability as:

$$\mathbf{V}(\mathbf{M}) = \frac{\partial \mathbf{D}}{\partial \mathbf{M}} \tag{3}$$

For more details on this mathematical framework please refer to El-Zein and Tonmoy (2013b).

Let $s_k = \{s_1, ..., s_n\}$ be a set of n comparable SESs which need to be ranked according to the vulnerability of a valued attribute (e.g., health, economic well-being, productivity etc.) to one or more climate hazards (e.g., increase in average temperatures, rise in sea level, increased frequency of flooding etc.). IBVA expresses the vulnerability as functions of measurable indicators. Hence, above equations applied to a specific socio-economic system (SES) k becomes:

$$\overrightarrow{\mathbf{V}_{\mathbf{k}}}(\mathbf{M}) = f_{\mathbf{k}}(I_{1\mathbf{k}}, I_{2\mathbf{k}}, I_{3\mathbf{k}}, \dots)$$

In IBVA, each indicator is ideally chosen so as to represent a process generating vulnerability, based on intuitive or deductive reasoning. Typically, our degree of knowledge of the relationship between an indicator and the vulnerability it represents is *highly variable* and as a minimum, can be summed up by:

 $\mathbf{f_k} \propto \mathbf{I_{1k}}, \mathbf{I_{2k}}, \mathbf{I_{3k}}, \dots$

 $f_k = \sum w_j \bar{I}_{jk}$

However, this simply reiterates the reason for which these indicators were selected in the first place. A simple, if dangerous, way out of this impasse is to make the following assumption:

^{j=1}
the bar on variable
$$I_{jk}$$
 denotes normalised indicators; and w_j is a weight for the jth indicator. Normalisation
can be conducted in a number of different ways; commonly IBVA studies use a normalization method that
leads to a new variable which ranges between 0 and 1. Equation (6) essentially generates a function based on
multi-attribute utility theory (MAUT) as a form of aggregation of the indicators. This immediately presents
us with a problem related to three assumptions underlying equation (6): a) indicators are linearly
independent; b) all indicators are commensurable with each other, i.e. a deficiency in one indicator can be
made up for with an excess in any other indicator, with the exact rate of exchange between two indicators
determined by the choice of respective weights *(the indicators incommensurability* problem); and c)
vulnerability is a linear monotonic function of indicators *(the deductive non-linearity* problem).
Unfortunately, these assumptions do not usually hold in IBVA. Furthermore, in the absence of deductive
arguments for characterising the exact relationship between a set of indicators and vulnerability, stakeholders
and experts are sometimes able to build an *intuitive form of nonlinearity* based on their knowledge of the
system in question. An example of this form of nonlinearities is given in the last row of table 1. While this
approach is obviously less valid than a deductive one, it still carries information that the analysis ought to
take into consideration.

An alternative to equation (6) is a Condorcet approach which proceeds by pairwise comparisons of SESs, rather than building a global utility function. In an earlier publication, we showed that outranking methods, based such an approach, are better suited for dealing with the challenges of partial compensation and data uncertainty present in IBVA (El-Zein and Tonmoy, 2013a). Outranking methods have been used widely in the context of environmental decision making (Roy, 1968, Hokkanen and Salminen, 1997, El Hanandeh and El-Zein, 2010, El-Zein and Tonmoy, 2013a, Linkov et al., 2006). Hence, we proposed an IBVA framework

(5)

(4)

(2)

(6)

Tonmoy and El-Zein, A non-linear mathematical framework for climate change vulnerability assessment

which aggregates indicators by measuring the truth of the statements "a is more vulnerable than b", "b is more vulnerable than a" or "a and b are equally vulnerable", where a and b are two SESs.

In what follows, we extend this framework to deal with the different forms of nonlinearity identified above. We do so by introducing the concept of *harm*, as a mediator between indicators and the vulnerability they represent.

Types of non-linearity	Source	Example
Fundamental non-linearity	Dependence of V on M (Equation 1)	Rivers bursting their banks and sea waves breaching beach fortifications are examples in which a threshold effect generates a non-linear relationship between extent of damage and magnitude of hazard.
Deductive non-linearity	Mechanistically characterised relationship between a set of indicators and vulnerability	Infrastructures are often interdependent and disruption to one usually cascades through the network. As an example, water supply infrastructures are dependent on the power supply, and therefore disruption to the power supply infrastructure during a storm event may lead to disruption of water supply to houses. Therefore, vulnerability of households to a storm event depends on a complex interaction between a set of infrastructure parameters; this interaction can be mechanistically established using simulation models (Tonmoy and El-Zein, 2013).
Intuitive non- linearity	Weakly characterized relationship between an indicator and vulnerability	Based on empirical evidence from social sciences, it is assumed that the adaptive capacity of a community is partly reflected by its collective income and assets—the wealthier it is, the higher its adaptive capacity and the less vulnerable it is to the hazard in question. However, it is very difficult to characterise the exact relationship between wealth W and adaptive capacity A_c . On the other hand, it is reasonable to assume that small differences in wealth do not translate into differences in adaptive capacities or vulnerabilities (Reid et al., 2009)

Table 1: Types, sources and examples of non-linearity in IBVA

3.0 THE SYDNEY ENVIRONMENTAL VULNERABILITY APPROACH (SEVA)

Harm, as suggested by Hinkel (2011), can be conceived of as a more concrete, less abstract form of vulnerability that is more amenable to quantification. In conventional IBVA studies, an indicator selected to represent vulnerability is taken to satisfy the following three conditions:

a) it represents a process generating vulnerability; b) it holds a linear, monotonic relationship with vulnerability; c) it is either readily available or computable.

SEVA In we replace indicators with harm criteria. A harm criterion must satisfy the following conditions:

1. It represents a process generating vulnerability; 2. It holds either a linear, monotonic relationship or an intuitively non-linear

relationship with vulnerability;



c. Harm criterion-vulnerability relationship in SEVA when harm criterion is readily available

Figure 2: The harm concept

3. It is either readily available or computable; computable harm criteria may be the output of deductively non-linear relationships or models whose input is a set of readily available indicators.

A harm criterion then, similar to an indicator, acts as a proxy for a process generating vulnerability. However, a harm criterion, as opposed to an indicator, allows us to achieve two key objectives in the process of building a vulnerability model: a) to relax the conditions concerning linearity; b) to separate deductive and intuitive non-linearity in order to better deal with both of them. Figure 2 shows these features graphically. It highlights the fact that, in SEVA, when the harm criterion is readily available (and no deductive non-linearity is present), there is no need for indicators.

To illustrate how harm criteria can be used in vulnerability assessments, we present a simplified model that aims to rank the vulnerability of a number of coastal beaches to a rise in sea level. The model shown in table 2 is *not* a complete and accurate representation of vulnerability; it is only illustrative, used to demonstrate key relationships between harm criteria and vulnerability. H_1 is the damage cost (damaged private properties) due to a disaster event associated with sea level rise (SLR) (e.g. storm event), in a do nothing scenario. H_1 can be estimated through a hazard line study that identifies at-risk properties and infrastructure near the coast, based on mechanistic models (e.g., coastal hydrodynamic models, soil foundation mechanics, structural integrity). Similarly, H_2 is an estimate of the likely number of disrupted households as a result of an interruption to services of the public infrastructure that falls inside the hazard line. This can be measured using a system dynamics model that simulates infrastructure interdependency (Tonmoy and El-Zein, 2013). Both H_1 and H_2 are hence derived on the basis of deductive arguments and are the outcome of deductively non-linear relationships. On the other hand, H_3 , an income-based proxy measure of adaptive capacity, is usually readily available as primary data from demographic and population census databases.

Finally, the model assumes that H_2 is intuitively-linear as it holds a linear relationship to vulnerability—while H_1 and H_3 are intuitively non-linear. This is to say that, for example, small differences in H_3 (income) do NOT translate into differences in vulnerability and, beyond a given threshold, bigger differences in income no longer translate into bigger differences in vulnerability.

Table 2: Hypothetical 3-harm criteria model of vulnerability to sea level rise (D-NL: Deductive Non-Linearity; I-NL: Intuitive Non-Linearity; COMP: compensation)

Н	Description	Dir ^a	Process	D-NL	I-NL	COMP
H1	Damage cost of private properties		Densely developed and exposed beaches are more at risk	Yes	Yes	Total
H2	Total number of affected households due to disruption of public infrastructures during a disaster (e.g., storm event)		Disruption to public infrastructures affects its users (households) adversely	Yes	No	Partial
Н3	Weekly median income of the households	Ļ	Lack of access to adaptive resources leads to more risk	No	Yes	Partial

a :Dir= Direction: \uparrow (\downarrow) indicates that vulnerability increases (decreases) with increasing harm.

It is now possible to express equation (2) in terms of harm criteria rather than indicators:

$$\overrightarrow{\mathbf{V}_{\mathbf{k}}}(\mathbf{M}) = f_{\mathbf{k}}(H_{1\mathbf{k}}, H_{2\mathbf{k}}, H_{3\mathbf{k}}, \dots)$$

(7)

where H_{ik} is either given or can be computed as a non-linear function of a set of indicators I_{1k} , I_{2k} , I_{3k} or the outcome of a complex mechanistic model, which may contain thresholds and tipping points, with a set of indicators as input. By starting from equation (7), which we now take as fully representing the vulnerability of the system, for each pair of SESs a and b, we can define three different categories of relative vulnerability:

- 1. b is indifferent to a according to harm criterion H_i if and only if $|H_{ib} H_{ia}| \le q_i$, where $q_i \ge 0$ is the relative vulnerability indifference threshold for harm criterion H_i ;
- 2. b is strictly more vulnerable than a according to harm criterion H_i if and only if $H_{ib} H_{ia} \ge p_i$, where $p_i \ge 0$ is the *relative vulnerability threshold for harm criterion* H_i ($p_i \ge q_i$);
- 3. b is weakly or proportionately more vulnerable than a according to harm criterion H_i if and only if $q_i < H_{ib} H_{ia} < p_i$.
- b is at least as vulnerable as a, if one criterion exists for which H_{ib} − H_{ia} > v_i, regardless of the performances of a and b on all other harm criteria, where v_i≥p_i is called *dominance threshold* for harm criterion I_i.

Hence, v_i sets a limit beyond which a disparity in the values of a harm criterion for 2 SESs is so great that the resulting difference in vulnerability cannot be compensated for by reverse disparity in another harm criterion. In other words, compensation is either partial or completely absent. The values of each harm criterion for each SES are assembled in what we call a vulnerability matrix (each row representing a harm criterion and each column representing an SES). Converting the vulnerability matrix into a ranking of SESs according to their vulnerability to a climate change hazard can be made using any available outranking procedures. We use the ELECTRE-III outranking method (Roy, 1968, Tonmoy and El-Zein, 2012). Figure 1 shows the overall architecture of SEVA framework.

4.0 LIMIT CONDITIONS FOR THRESHOLDS OF DIFFERENCE

In outranking methods, thresholds of differences $(q_i, p_i \text{ and } v_i)$ dictate the extent of intuitive non-linearity and compensation through concordance and discordance matrices. In SEVA, we distinguish between eight possible types of harm criteria depending on the presence (or not) of deductive nonlinearity, intuitive

nonlinearity and partial compensation. Table 3 shows the limit conditions for thresholds of difference needed to generate the different types of harm criteria. For more details on the thresholds of differences and how they can be estimated, the reader is referred to El-Zein and Tonmoy (2013a), Tonmoy and El-Zein (2012).

	Intuitively Linear	Intuitively Non-Linear						
	and Fully-Compensating	Full	Full or Partial	Full, Partial or No				
	Tuny Compensating	Compensation	Compensation	Compensation				
	Type 1	Туре 2	Type 3	Туре 4				
	H _{ik}	H _{ik}	H _{ik}	H _{ik}				
Lincor	q _i =0	q _i ≥min H _{ik} -H _{ij} (k=1,n; j=1,n)	q _i ≥0	q _i ≥0; p _i ≥0				
Linear	max(H _{ik} -H _{ij}) (k=1,n;	$max(H_{ik}-H_{ij})$ (k=1,n;	p _i <max(h<sub>ik-H_{ij})(k=1,n;</max(h<sub>	v _i ≤max(H _{ik} -H _{ij})(k=1,n; j=1,n)				
	j=1,n)≤pi≤vi	j=1,n)≤pi≤vi	j=1,n)≤vi					
	Type 5	Туре б	Type 7	Туре 8				
Deduction	$H_i = f(I_1, I_2,)$	$H_i = f(I_1, I_2,)$	$H_i = f(I_1, I_2,)$	$H_i = f(I_1, I_2,)$				
Nen Lincer	$q_i = 0$	q _i ≥min H _{ik} -H _{ij} (k=1,n; j=1,n)	q _i ≥0	q _i ≥0; p _i ≥0				
Non-Linear	max(H _{ik} -H _{ij}) (k=1,n;	$max(H_{ik}-H_{ij})$ (k=1,n;	p _i <max(h<sub>ik-H_{ij})(k=1,n;</max(h<sub>	p _i <v<sub>i<max(h<sub>ik-H_{ij})(k=1,n;</max(h<sub></v<sub>				
	j=1,n)≤p _i ≤v _i	j=1,n)≤pi≤vi	j=1,n)≤v _i	j=1,n)				

Table 3: Limit cases of thresholds of difference for combinations of nonlinearity and partial compensation $(q_i \le p_i \le v_i)$

5. AN ILLUSTRATIVE EXAMPLE: VULNERABILITY TO SEA LEVEL RISE USING SEVA

Table 4 shows the results of SEVA analyses conducted on the model shown in Table 2. The SESs in this case are four hypothetical city districts of similar scales. We compare the vulnerabilities of the well being of residents of these districts to the effects of a rise in sea level and its associated processes (e.g., long term erosion, increased flooding etc.), assuming they are adequately reflected by the three harm criteria shown in Table 2. Starting from a scenario where all the relationships are linear and full compensation between harm criteria is available (case 1), we generate new scenarios (cases 2 to 4) by gradually introducing different forms of intuitive and deductive non-linearity and partial compensation. It can be seen in Table 4 that, in case 1, SES4 is least vulnerable primarily because of its low damage cost and smaller number of affected households. SES1, SES2 and SES3 are equally vulnerable as the differences between all three criteria for these SESs are in balance in the absence of any indifference thresholds (e.g., SES1 is most vulnerable based on H1, SES3 based on H2, and SES2 based on H3). In case 2, the introduction of an indifference threshold of q3=\$90 for H3 disturbs this balance because now any difference below q3 can be regarded as insignificant. This yields SES 1 as the most vulnerable. Case 3 shows the effect of introducing partial compensation for H3, with p3=\$100 and v3=\$110. Hence, the advantage that SES3 carries over SES2 in terms of median income has become decisive: SES3 can no longer be more vulnerable than SES2, regardless of its performance on other harm criteria. The interaction between infrastructure components is introduced into the model in case 4: it is assumed that power failure leads to water supply disruption which increases the total number of affected households. It is clear that omitting interaction underestimates the impact of power failure. Introducing the non-linear relationship made SES3 most vulnerable because it harbours the highest number of disrupted households.

bold cha	aracters; in all in	ntuitive	ely linea	r relatio	onship qi=	0 and	pi=max	(Hik-Hi	ij), k=	1,n; j=1,n)	; H= Harr	n criteria;	
Case	Description	Н	Units	Dir ¹	Type ²	qi	pi	Vi	Wi	SES1	SES2	SES3	SES4
Case 1	Intuitively,	H ₁	\$	↑	1	0	8000	N/A	1	35,000	29,000	38,000	25,000
	deductively	H ₂	count	↑	1	0	140	N/A	1	100	107	125	113
	linear and	H ₃	\$	\rightarrow	1	0	200	N/A	1	\$875	\$800	\$1000	\$955
	fully-				Vulnerability Ranking					1	1	1	4
	compensating												
	Intuitive non-	H_1	\$	1	1	0	8000	N/A	1	35,000	29,000	38,000	25,000
Case	linearity	H_2	count	↑	1	0	140	N/A	1	100	107	125	113
2	present only	H ₃	\$	\rightarrow	2	90	200	N/A	1	\$875	\$800	\$1000	\$955
	for H ₃				Vulnerability Ranking				1	3	2	4	
Case 3	Same as case	H ₁	\$	↑	1	0	8000	N/A	1	35,000	29,000	38,000	25,000
	2 with partial	H ₂	count	↑	1	0	140	N/A	1	100	107	125	113
	compensation	H ₃	\$	\rightarrow	4	90	100	110	1	\$875	\$800	\$1000	\$955
	only for H ₃				Vulnerability Ranking				1	2	2	4	
Case 4	Deductive	H ₁	\$	↑	1	0	8000	N/A	1	35,000	29,000	38,000	25,000
	non-linearity	H ₂	count	↑	1	0	140	N/A	1	430	480	520	380
	introduced for	H ₃	\$	\downarrow	5	0	200	N/A	1	\$875	\$800	\$1000	\$955
	H_2				Vulnerability Ranking				3	2	1	4	

Table 4: Four sample scenarios and effects of nonlinearity and degrees of compensation on rankings using SEVA (rankings: 1: most vulnerable; 4:least vulnerable; cases 2 and 3 are modifications of case 1 with the change highlighted in bold characters; in all intuitively linear relationship qi=0 and pi=max(Hik-Hij), k=1,n; i=1,n); H= Harm criteria;

¹Dir= Direction: \uparrow (\downarrow) indicates that vulnerability increases (decreases) with increasing harm.

²Type refers to the different relationships shown in Table 3

Tonmoy and El-Zein, A non-linear mathematical framework for climate change vulnerability assessment

6. CONCLUSION

We identified different sources of non-linearity prevalent in the context of IBVA. In the process, we differentiated between fundamental non-linearity, deductive non-linearity and intuitive non-linearity. We developed a multi-dimensional framework (SEVA) which allows a combination of non-linear and partial compensation effects to be incorporated in vulnerability assessments. An illustrative example of ranking vulnerability of four fictional beaches to sea level rise was presented to show how consideration of interdependency of infrastructure (which is a form of deductive non-linearity) can affect the final vulnerability ranking. We are currently applying this framework to a real-life vulnerability assessment exercise, namely the vulnerability of a set of coastal communities in Sydney to a rise in sea level (Tonmoy et al., 2012).

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