Modelling the term structure of Japanese bond yields with the Nelson-Siegel model

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Abstract: The Nelson-Siegel (1987) (NS) model has been credited for its high efficacy in the in-sample fitting and out-of-sample forecasting of the term structures of interest rates. The term structure of interest rates, popularly known as the yield curve, is a static function that relates the time-to-maturity to the yield-to-maturity for a sample of bonds at a given point in time. The conventional way of measuring the term structure is by means of the spot rate curve, or yield curve, on zero-coupon bonds. Yet in reality, the entire term structure is not directly observable, which gives rise to the need to estimate it using several approximation techniques. Over the last three decades, various methods to estimate term structures from bond prices have been proposed. In recent years most of the existing studies (as well as major central banks around the globe) have been employing the class of NS models to estimate and construct zero-coupon yield curves.

This paper aims to study the term structure of the Japanese bond yields by employing the NS model vs other non-NS models using five different sets of zero-coupon bond yield rates data obtained from the Bank of Japan covering the period spanning from January 2000 to November 2007. This period has been chosen because it clearly exhibits the liquidity trap problem, which forces all bond yields to remain close to zero for an extended period. We propose 18 different NS models, each with different decay components and time series appendages, against 14 other non-NS models ranging from the simple random-walk model to complicated specifications like the VAR and VECM models. A h-period(s)-ahead out-of-sample expanding window forecast is conducted for each of these 32 different models, using daily, weekly and monthly bond yields of 15 different maturities.

This study has demonstrated that due to the presence of liquidity trap in Japan, out-of-sample expanding window forecasts in general perform inferiorly vis-à-vis other non-NS models, and this is coupled with the other problem of obtaining negative yield forecasts for bonds with shorter maturities. Moreover, the results show that the NS class of models can be useful in forecasting shorter horizons like weeks and days, works better with a decay rate other than the conventional way of treating it as the value that maximizes the loading on the medium-term factor at exactly 30 months, and can work well with time series models such as GARCH and EGARCH in terms of volatility forecasting. It is also found that, when the NS models are used for yield forecasts, the NS-VAR model should be considered since it is up to par against the competitor models, even with liquidity trap at work.

Keywords: In-sample fitting and out-of-sample forecasting; Japanese bond yields; the Nelson-Siegel (NS) model

1. INTRODUCTION

The term structure of interest rates, popularly known as the yield curve, is a static function that relates the time-to-maturity to the yield-to-maturity for a sample of bonds at a given point in time. The conventional way of measuring the term structure is by means of the spot rate curve, or yield curve, on zero-coupon bonds. Yet in reality, the entire term structure is not directly observable, which gives rise to the need to estimate it using several approximation techniques. There are a wide variety of diverse yield models in the literature. The major goal of these models is to describe the future yield curve structure as best as possible. However, the modelling of a yield curve is more complicated than the modelling of a stock derivative, and the models for the yield curve structure alone are more complex as well. In recent years, the Nelson-Siegel (1987) (NS) model and its extended models have been credited for its high efficacy in the in-sample fitting and out-of-sample forecasting of the term structures of interest rates. Many existing studies (as well as some major central banks around the globe) have been employing the class of NS models including the Svensson-Soderlind model to estimate and construct zero-coupon yield curves.

The primary goal of this paper is to investigate the Nelson-Siegel and the extended NS models on their forecasting performances, and to examine the term structure of the Japanese bond yields by employing the NS model vs other non-NS models using five different sets of zero-coupon bond yield rates data obtained from the Bank of Japan covering the period spanning from January 2000 to November 2007. In particular, we propose 18 different NS models, each with different decay components and time series appendages, against 14 other non-NS models ranging from the simple random-walk model to complicated specifications like the VAR and VECM models. A h-period(s)-ahead out-of-sample expanding window forecast is conducted for each of these 32 different models, using daily, weekly and monthly bond yields of 15 different maturities. The sample period was chosen because it clearly exhibits the liquidity trap problem, which forces all bond yields to remain close to zero for an extended period.

This study has demonstrated that due to the presence of liquidity trap in Japan, out-of-sample expanding window forecasts in general perform inferiorly vis-à-vis other non-NS models, and this is coupled with the other problem of obtaining negative yield forecasts for bonds with shorter maturities. Moreover, the results show that the NS class of models can be useful in forecasting shorter horizons like weeks and days, works better with a decay rate other than the conventional way of treating it as the value that maximizes the loading on the medium-term factor at exactly 30 months, and can work well with time series models such as GARCH and EGARCH in terms of volatility forecasting. It is also found that, when the NS models are used for yield forecasts, the NS-VAR model should be considered since it is up to par against the competitor models, even with liquidity trap at work.

The rest of this study is organized as follows. Section 2 discusses the NS model and the extended NS models as well as the methodology used in this study. Section 3 analyzes the data sets, namely, the daily, weekly and monthly spot rates to be used in this study, and evaluates the various forecast results. The last section concludes with implication drawn from our findings.

2. METHODOLOGY AND THE MODEL

We discuss the Nelson and Siegel model in this section. Assume that spot rates are obtained from a secondorder differential equation with real and unequal roots. The instantaneous forward rate function $f_t(\tau)$ at maturity τ is defined as:

$$f_{t}(\tau) = b_{1,t} + b_{2,t}e^{-\tau/\lambda_{1,t}} + b_{3,t}e^{-\tau/\lambda_{2,t}}$$
(1)

Where $\lambda_{1,t}$ and $\lambda_{2,t}$ are time constants associated with the equation, and the parameters $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are determined by the initial conditions. Yet, this model was deemed to be over-parameterized by Nelson and Siegel (1987), and having too many parameters makes it difficult for any standard nonlinear estimation software to converge. For this reason, equation (1) was further streamlined by setting $\lambda_t = \lambda_{1,t} = \lambda_{2,t}$ and adding $\frac{r}{\lambda_t}$ to the factor loading on $b_{3,t}$, thereby transforming the forward rate curve into the following form:

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$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} e^{-\tau/\lambda_t} + \beta_{3,t} \left(\frac{\tau}{\lambda_t}\right) e^{-\tau/\lambda_t}$$
(2)

With this transformation, the forward rate curve can now be seen as a constant plus a Laguerre function, which is basically a product of a polynomial and an exponential decay term. Intuitively, this particular forward rate curve can be assumed to be the solution to a second-order differential equation with equal roots for spot rates. By taking the average of forward rates over different maturities, the corresponding spot rate or yield curve can then be obtained:

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left(\frac{1 - e^{-\tau/\lambda_t}}{\tau/\lambda_t} \right) + \beta_{3,t} \left(\frac{1 - e^{-\tau/\lambda_t}}{\tau/\lambda_t} - e^{-\tau/\lambda_t} \right)$$
(3)

As the NS yield curve corresponds to a discount curve that begins at one at zero maturity and approaches zero at infinite maturity, following Fabozzi et al. (2005), Diebold and Li (2006) and Diebold, Rudebusch, and Aruoba (2006b), we treat λ_{t} as a multiplier and hence transform equations (2) and (3) into:

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} e^{-\lambda_t \tau} + \beta_{3,t} \lambda_t \tau e^{-\lambda_t \tau}$$

$$\tag{4}$$

$$y_{t}(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda_{t}\tau}}{\lambda_{t}\tau}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda_{t}\tau}}{\lambda_{t}\tau} - e^{-\lambda_{t}\tau}\right)$$
(5)

which are used for all in-sample fitting and out-of-sample forecasting exercises in the study. In the NS model, the exponential decay rate (λ_t) controls the speed of decay for the NS yield function $y_t(\tau)$. Theoretically speaking, a smaller value of λ_t is supposed to produce slow decay and can thus better fit the yield curve at long maturities; whereas a greater λ_t exhibits the direct opposite of producing an accelerated decay which results in the better fitting of the curve at short maturities. However, a challenge faced is how to choose a suitable decay rate λ_t^* for each single point in time. As regards the three coefficients $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$, which are called the "latent level, slope and curvature factors" in Diebold and Li (2006), they each have their own idiosyncratic traits. The long-term factor, $\beta_{1,t}$, governs the yield curve level since an increase in this coefficient raises all short- and long-term yields equally, thereby changing the level of the yield curve. $\beta_{2,t}$ relates to the short-term factor and is closely associated with the yield curve slope. $\beta_{3,t}$ is closely linked to the yield curve curvature and it may be viewed as a medium-term factor, akin to its loading $\frac{1-e^{-\lambda_t \tau}}{\lambda_t - \tau} - e^{-\lambda_t \tau}$.

The NS model was late extended by researchers including Litterman and Scheinkman (1991), Björk and Christensen (1999), Bliss (1997) and Svensson (1994) to explore more flexible NS specifications, either through the use of additional factors, further decay parameters, or by a composite of both. Some of these extensions have been tested in other papers, though the results were mixed. It is generally found that extensions like Litterman-Scheinkman's (1991) two-factor model are overly simplistic and yield inaccurate results, whereas models such as the Svensson extension are currently welcomed by major banks around the globe.

We now turn to the discussion of general specification of the models. Let Y_t be a $N \times 1$ vector of observed spot rates or yields which is dependent on N different maturities, such that $Y_t = [y_t(\tau_1) \dots y_t(\tau_N)]'$, at any one point in time. In order to facilitate term structure forecasting, we incorporate dynamics to create timeseries models for the collected β_t factor estimates, and then assess the models performance. Models selected for the factor forecast include RW, AR(p), VAR(p), GARCH and EGARCH specifications. Given the Tsui et al., Modelling the term structure of Japanese bond yields with the Nelson-Siegel model

voluminous literature on ARCH, models built around this concept have become an indispensable tool for financial analysts, bankers and fund managers throughout the world. McAleer (2005) gives a recent exposition of a wide range of univariate and multivariate, conditional and stochastic models of volatility, and other recent surveys have examined multivariate extensions of the original GARCH framework (Bauwens et al., 2006; Maasoumi and McAleer, 2006; and Caporin and McAleer, 2012). Unlike Diebold and Li (2006) and de Pooter (2007) who conveniently assumed first orders for all their time series models, in this study we adopt a more cautious approach by first finding the respective optimum orders with the help of various information criteria. Thus, we define the general specification of the NS model as follows:

$$Y_t = X_t \beta_t + \varepsilon_t \tag{6}$$

$$\beta_t = \mu + \Phi \beta_{t-1} + \nu_t \tag{7}$$

where X_t is a $(N \times K)$ matrix of factor loadings which are potentially time-varying if the decay parameter(s) are estimated together with the factors, β_t is a $(K \times 1)$ vector of factors and its order of integration is assumed to be I(0) at this point in time, and ε_t is a $N \times 1$ vector of estimation errors and each component within it is assumed to be independent across maturities and have different variance terms $\sigma^2(\tau)$. μ is a $(K \times 1)$ vector, while Φ is a $(K \times K)$ square matrix, and v_t is also a $(K \times 1)$ vector of residuals for the time series model. It is also assumed that the measurement equation and state equation error vectors are both orthogonal and normally distributed, such that:

$$\begin{bmatrix} \mathcal{E}_t \\ V_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0_{N \times 1} \\ 0_{K \times 1} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & Q \end{bmatrix}\right)$$
(8)

where *H* represents a diagonal $(N \times N)$ measurement equation covariance matrix, and *Q* denotes a state equation covariance matrix, which may be assumed to be either a diagonal $(K \times K)$ matrix or a full matrix, depending on the estimation procedure. For instance, *Q* (and also Φ) are diagonal if we consider separate AR(*p*) models for each factor; on the contrary, *Q* (and Φ) are full matrices when a joint VAR(*p*) estimation is carried out instead. The NS GARCH (1,1) model used for forecast is specified as follows:

$$\Delta \hat{y}_{t+h} = \Delta \hat{\beta}_{1,t+h} + \Delta \hat{\beta}_{2,t+h} \left(\frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) + \Delta \hat{\beta}_{3,t+h} \left(\frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right)$$
(9)

$$\Delta \beta_{i,t+h} = \mu_i + \varepsilon_{t+h}, \quad \varepsilon_{t+h} \sim \mathcal{N}\left(0, \sigma_{\tau,t+h}^2\right), \quad i = 1, 2, 3$$
(10)

$$\sigma_{\tau,t+h}^2 = \omega + \alpha \varepsilon_{t+h-1}^2 + \beta \sigma_{\tau,t+h-1}^2$$
(11)

And the EGARCH(1,1,1) with AR(1) in the mean equation can be expressed as follows when combined with the NS model:

$$\Delta \hat{y}_{t+h} = \Delta \hat{\beta}_{1,t+h} + \Delta \hat{\beta}_{2,t+h} \left(\frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) + \Delta \hat{\beta}_{3,t+h} \left(\frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right)$$
(12)

$$\Delta \beta_{i,t+h} = \mu_i + \phi_i \Delta \beta_{i,t+h-1} + \varepsilon_{t+h}, \quad \varepsilon_{t+h} \sim \mathcal{N}\left(0, \sigma_{\tau,t+h}^2\right), \quad i = 1, 2, 3$$
(13)

$$\ln\left(\sigma_{\tau,t+h}^{2}\right) = \omega + \alpha \left|\frac{\varepsilon_{t+h-1}}{\sigma_{t+h-1}}\right| + \beta \ln\left(\sigma_{\tau,t+h-1}^{2}\right) + \gamma\left(\frac{\varepsilon_{t+h-1}}{\sigma_{t+h-1}}\right)$$
(14)

3. EMPIRICAL ANALYSIS

3.1 Data Description

For the purpose of in-sample fitting and out-ofsample forecasting, we collected five different sets of zero-coupon bond yield rates data from the Bank of Japan archives, including daily yield (1888 observations), weekly yield (408), and monthly yield (94), spanning almost eight years from January 2000 to November 2007. This period was chosen because it clearly exhibits the liquidity trap problem, which forces all bond yields to remain close to zero for an extended period. Similar to the United States, Japan applies a particular variant of the "smoothing splines"

method in the estimation of zero-coupon vield rates. It means that the instantaneous forward rate curves. expressed as a linear combination of cubic B-splines, are constructed from price quotes on selected risk-free fixed income assets: 3-, 6-, 120- and 240-month bonds. Following that, the forward rate curves are interpolated by using smoothing splines, after which the spot rates can then be computed by taking the average over the forward rates. The purpose for us to use the daily and weekly yield rates is to test the robustness of the NS model at the daily and weekly levels.





Table 1: Descriptive Statistics for Monthly Data

Maturity (Months)	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera Probability	ρ_1	$ ho_{12}$	$ ho_{24}$
3	0.129	0.010	0.650	0.001	0.201	1.358	3.334	0.000	0.892	0.108	0.090
6	0.141	0.011	0.700	0.001	0.214	1.327	3.295	0.000	0.905	0.109	0.095
12	0.181	0.044	0.785	0.008	0.230	1.186	2.955	0.000	0.899	0.118	0.094
24	0.319	0.153	1.025	0.030	0.296	0.951	2.445	0.000	0.927	0.123	0.116
36	0.444	0.302	1.124	0.077	0.307	0.685	2.002	0.004	0.934	0.100	0.093
48	0.597	0.482	1.294	0.128	0.328	0.505	1.869	0.011	0.935	0.087	0.087
60	0.760	0.652	1.478	0.164	0.355	0.314	1.834	0.032	0.935	0.073	0.083
72	0.911	0.859	1.595	0.224	0.353	0.145	1.951	0.098	0.930	0.031	0.084
84	1.081	1.062	1.752	0.268	0.355	-0.068	2.181	0.259	0.922	-0.032	0.094
96	1.238	1.235	1.855	0.379	0.341	-0.277	2.514	0.345	0.914	-0.086	0.065
108	1.360	1.376	1.924	0.464	0.324	-0.521	3.011	0.120	0.908	-0.148	0.115
120	1.456	1.472	1.930	0.540	0.300	-0.832	3.683	0.002	0.906	-0.080	0.082
180	1.668	1.671	2.219	0.654	0.314	-1.006	4.388	0.000	0.901	-0.072	0.110
240	1.987	2.040	2.500	0.904	0.292	-1.666	6.595	0.000	0.896	-0.130	0.035
360	2.261	2.342	2.995	1.014	0.337	-1.553	6.353	0.000	0.876	-0.154	-0.077

Also, utilizing yield rates on daily and weekly bases allow for a larger sample, which in turn makes the estimates more stationary. We also pool all the data sets into fifteen fixed maturities, i.e., 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months, which will greatly simplifies our estimation and forecasting sequences.

To save space, we show in Figure 1 the time-series of zero-coupon yield curves for 3-, 12-, 60-, 120- and 360-month bonds, based on monthly raw data from January 2000 to November 2007. Two observations can be noticed: there is a low-yield period between 24 to 36 months for all types of bonds from January 2000; and bonds with maturities of 12 months or less have yields close to zero from 12 months onwards until 72 months. The former characteristics may suggest a possible structural break, corresponding to the privatization of the Postal Savings System as well as a complete overhaul of the existing financial structure in Japan. The latter is a classic example of "liquidity trap". Table 1 reports the summary of the descriptive statistics for monthly yields from January 2000 to November 2007. As it can be seen in Table 1, the mean values confirm that the average yield curve is characterized as upward sloping and concave, and the standard deviation reveals that it is most volatile for medium and long-term maturities, but more stable for the shorter-term maturities. One possible explanation is that the presence of liquidity trap forces short-term yields to converge towards zero, thereby "stabilizing" it. It is also noted that sample serial autocorrelations at a displacement of 1 month for all maturities, as well as pairwise correlations between yields that have close maturities are extremely high. In addition, the descriptive statistics on daily and weekly data lead to more or less the same conclusions, except for the fact that serial autocorrelations remained very high even up to a displacement of 24 months for all maturities, and the Jarque-Bera probability converges to zero as the number of observations gets larger.

3.2 Empirical Results

We conducted in-sample fitting exercise by estimating equation (5) using different values for λ_r . In particular, following Diebold and Li (2006) we performed OLS estimation of equation (5) by assigning λ_r

equal to 0.0598 and 0.0299, and also NLS estimation assuming λ_i is time varying with daily, weekly and monthly yield data from January 2000 to November 2007. The results (not reported due to space limitation but available upon request) show that in general the long-term factor ($\hat{\beta}_{1,t}^{oLS}$) and the medium-term factor ($\hat{\beta}_{3,t}^{oLS}$) are found more persistent than the short-term factor ($\hat{\beta}_{2,t}^{oLS}$), which is consistent with the stylized facts that yield dynamics are persistent and longer rates are more persistent than the short rates. It is also found that the pairwise correlation between $\hat{\beta}_{1,t}^{oLS}$ and $\hat{\beta}_{2,t}^{oLS}$ is the largest in all the three cases and also significantly negative.

We have conducted the h-period(s)-ahead expanding window forecasting for both the NS and non-NS models, but report in Table 2 the results for one-week ahead expanding window only due to space limitation. The rest of the results are available upon request.

Yield Forecasting

		-			1-MOIIUI-PUI	eau Porecast				_			3-MOHUIS-94H	eau Porecast			
MODEL		ADMODE	TOMODE	RMSPE					ADMODE	TOMODE	RMSPE						
_		ARMOPE	TRMSPE	3m	6m	12m	60m	120m	360m	ARMSPE	TRABPE	3m	6m	12m	60m	120m	360m
MODELS	NS-OLS(λ=0.0598)-RW-L	0.14086	0.14924	0.11727	0.04839	0.12531	0.12892	0.12793	0.28353	0.19736	0.20345	0.13161	0.08952	0.16220	0.20258	0.18835	0.31250
	NS-OLS(λ=0.0598)-AR-C	0.14222	0.15093	0.11807	0.04677	0.12687	0.12853	0.12972	0.28906	0.20283	0.20943	0.13510	0.08834	0.16791	0.20397	0.19342	0.32826
	NS-OLS(λ=0.0598)-VAR-C	0.10228	0.10674	0.04858	0.04880	0.05742	0.11523	0.12452	0.12946	0.17821	0.18453	0.08869	0.08996	0.11298	0.21172	0.20045	0.17892
2	NS-OLS(λ=0.0299)-RW-L	0.10758	0.11245	0.06501	0.05178	0.04951	0.11654	0.13058	0.13449	0.17418	0.18000	0.10768	0.09302	0.08772	0.20617	0.19945	0.17753
5	NS-OLS(λ=0.0299)-AR-C	0.11155	0.11704	0.06262	0.05007	0.04722	0.12310	0.13880	0.14031	0.18256	0.18916	0.10590	0.09311	0.09154	0.21652	0.21379	0.18867
ı.	NS-OLS(λ=0.0299)-VAR-C	0.10282	0.10740	0.04821	0.04856	0.05737	0.11563	0.12552	0.13001	0.17893	0.18543	0.08774	0.08922	0.11290	0.21251	0.20188	0.17938
ő	NS-NLS(λ=0.0330)-RW-L	0.22636	0.24245	0.05696	0.06443	0.10891	0.28108	0.28874	0.19302	0.25456	0.26799	0.09593	0.10071	0.13217	0.31625	0.30670	0.22055
NELS	NS-NLS(λ=0.0330)-AR-C	0.22659	0.24245	0.05782	0.06692	0.10931	0.28094	0.28822	0.19312	0.25834	0.27194	0.09582	0.10202	0.13572	0.32015	0.31103	0.22368
	NS-NLS(\alpha=0.0330)-VAR-C	0.10819	0.11350	0.05018	0.04807	0.06101	0.11771	0.13016	0.15657	0.19553	0.20415	0.08835	0.08758	0.11169	0.21855	0.22620	0.25298
	RW-L	0.09812	0.10196	0.04982	0.04847	0.05963	0.11002	0.11679	0.12643	0.16925	0.17470	0.08834	0.09054	0.11127	0.19917	0.18684	0.17102
ŝ	SR	0.09929	0.10317	0.04982	0.04771	0.06114	0.11237	0.11785	0.12575	0.18340	0.18720	0.08834	0.18058	0.12828	0.21000	0.19319	0.17744
E E	AR-L	0.09943	0.10302	0.05156	0.04978	0.06194	0.11273	0.11672	0.12418	0.17420	0.17933	0.09406	0.09419	0.11778	0.20995	0.18678	0.16711
RIVAL MOD	AR-C	0.10042	0.10436	0.04980	0.05080	0.05993	0.11229	0.11875	0.12936	0.17558	0.18180	0.08581	0.08726	0.11354	0.20827	0.19447	0.17806
	VAR-L	0.11164	0.11651	0.05162	0.05300	0.06182	0.13838	0.13058	0.14286	0.25237	0.26550	0.10470	0.11247	0.12033	0.31291	0.29217	0.30624
	VAR-C	0.10071	0.10364	0.05746	0.05502	0.07101	0.11293	0.11838	0.13192	0.18273	0.18791	0.10257	0.10250	0.12334	0.21159	0.20225	0.19205
	VECM: 1 Common Trend	0.91934	1.13405	0.05481	0.04594	0.08236	0.75262	1.40151	2.20065	6.15185	7.56024	0.17677	0.12925	0.48929	5.18180	9.32125	14.40202
_	VECM: 2 Common Trends	0.91977	1.13424	0.05530	0.05540	0.08229	0.75102	1.40187	2.20284	6.10571	7.47118	0.29485	0.30329	0.49242	5.13801	9.18780	14.21447
Vield Volatility Forecasting																	
Yi	eld Volatility Forecasting	2															
Yi	eld Volatility Forecasting				1-Month-Ah	ead Forecast							3-Months-Ah	ead Forecast			
Yi	eld Volatility Forecasting	2			1-Month-Ah	ead Forecast RM	SPE						3-Months-Ah	ead Forecast RM	SPE		
Yi	eld Volatility Forecasting MODEL	ARMSPE	TRMSPE	3m	1-Month-Ah 6m	ead Forecast RM 12m	SPE 60m	120m	360m	ARMSPE	TRMSPE	3m	3-Months-Ah 6m	ead Forecast RM 12m	SPE 60m	120m	360m
Yi	MODEL NS-OLS(A=0.0998)-GARCH-C	ARMSPE 0.03901	TRMSPE 0.04009	3m 0.05642	1-Month-Ah 6m 0.05407	ead Forecast RM 12m 0.05119	SPE 60m 0.03664	120m	360m 0.03312	ARMSPE 0.03905	TRMSPE 0.04037	3m 0.05713	3-Months-Ah 6m 0.05514	ead Forecast RM 12m 0.05289	SPE 60m 0.03689	120m 0.03005	360m
Yi	MODEL NS-OLS(A=0.0598)-GARCH-C NS-OLS(A=0.0598)-GARCH-AR-C	ARMSPE 0.03901 0.08717	TRMSPE 0.04009 0.08827	3m 0.05642 0.07988	1-Month-Ah 6m 0.05407 0.06491	ead Forecast RM 12m 0.05119 0.05419	5PE 60m 0.03664 0.09433	120m 0.03098 0.09403	360m 0.03312 0.10442	ARMSPE 0.03905 0.08264	TRMSPE 0.04037 0.08371	3m 0.05713 0.08022	3-Months-Ah 6m 0.05514 0.06527	ead Forecast RM 12m 0.05289 0.05153	SPE 60m 0.03689 0.08603	120m 0.03005 0.08994	360m 0.03092 0.10610
Yi	MODEL NS-OLS(A=0.0998)-GARCH-C NS-OLS(A=0.0998)-GARCH-AR-C NS-OLS(A=0.0998)-EGARCH-C	ARMSPE 0.03901 0.06717 0.05063	TRMSPE 0.04009 0.08827 0.05110	3m 0.05642 0.07988 0.04507	1-Month-Ah 6m 0.05407 0.06491 0.04225	ead Forecast RM 12m 0.05119 0.05419 0.03803	60m 0.03664 0.09433 0.05311	120m 0.03098 0.09403 0.05550	360m 0.03312 0.10442 0.05741	ARMSPE 0.03905 0.08264 0.06062	TRMSPE 0.04037 0.08371 0.06141	3m 0.05713 0.08022 0.05368	3-Months-Ah 6m 0.05514 0.06527 0.05276	ead Forecast RM 12m 0.05289 0.05153 0.04542	SPE 60m 0.03689 0.08603 0.06009	120m 0.03005 0.08994 0.06943	360m 0.03092 0.10610 0.07412
Yie	MODEL MODEL NS-OLS(A=0.0598)-GARCH-C NS-OLS(A=0.0598)-GARCH-AR-C NS-OLS(A=0.0598)-GARCH-C NS-OLS(A=0.0599)-GARCH-C	ARMSPE 0.03901 0.08717 0.05063 0.05331	TRMSPE 0.04009 0.08827 0.05110 0.05474	3m 0.05642 0.07988 0.04507 0.07525	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261	ead Forecast RM 0.05119 0.05419 0.03803 0.06776	5PE 60m 0.03664 0.09433 0.05311 0.05502	120m 0.03098 0.09403 0.05550 0.04362	360m 0.03312 0.10442 0.05741 0.03640	ARMSPE 0.03905 0.08264 0.06062 0.05592	TRMSPE 0.04037 0.08371 0.06141 0.05739	3m 0.05713 0.08022 0.05368 0.07869	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607	ead Forecast RM 12m 0.05289 0.05153 0.04542 0.07136	SPE 60m 0.03689 0.08603 0.06009 0.05732	120m 0.03005 0.08994 0.06943 0.04567	360m 0.03092 0.10610 0.07412 0.03837
Yie	MODEL MODEL NS-OLS(A=0.0598)-GARCH-C NS-OLS(A=0.0598)-GARCH-AR-C NS-OLS(A=0.0598)-EGARCH-C NS-OLS(A=0.0299)-GARCH-C NS-OLS(A=0.0299)-GARCH-AR-C	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328	3m 0.05642 0.07988 0.04507 0.07525 0.04647	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565	ead Forecast RM 0.05119 0.05419 0.03803 0.06776 0.04811	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.05584	120m 0.03098 0.09403 0.05550 0.04362 0.04362	360m 0.03312 0.10442 0.05741 0.03640 0.09495	ARMSPE 0.03905 0.08264 0.06062 0.05592 0.07315	TRMSPE 0.04037 0.08371 0.06141 0.05739 0.07474	3m 0.05713 0.08022 0.05368 0.07869 0.05053	3-Months-Ah 6m 0.05514 0.05276 0.05276 0.07607 0.04968	ead Forecast RM 0.05289 0.05153 0.04542 0.07136 0.05075	SPE 60m 0.03689 0.08603 0.06009 0.05732 0.07500	120m 0.03005 0.08994 0.06943 0.04667 0.04520	360m 0.03092 0.10610 0.07412 0.03837 0.09644
Yie Steder woders	NODEL NS-OLS(J=0.098);GARCH-C NS-OLS(J=0.098);GARCH-AR-C NS-OLS(J=0.098);GARCH-C NS-OLS(J=0.098);GARCH-C NS-OLS(J=0.098);GARCH-C NS-OLS(J=0.099);GARCH-C NS-OLS(J=0.099);GARCH-C NS-OLS(J=0.099);GARCH-C NS-OLS(J=0.099);GARCH-C	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459	3m 0.05642 0.07988 0.04507 0.04507 0.04507 0.04647 0.06240	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.05961	ead Forecast RM 12m 0.05119 0.05419 0.03803 0.06776 0.04811 0.05844	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.05884 0.08187	120m 0.03098 0.09403 0.05550 0.04362 0.06874 0.08206	360m 0.03312 0.10442 0.05741 0.03640 0.09495 0.07295	ARMSPE 0.03905 0.08264 0.06062 0.05592 0.07315 0.07315	TRMSPE 0.04037 0.08371 0.06141 0.05739 0.07474 0.07665	3m 0.05713 0.06022 0.06368 0.07869 0.05053 0.06146	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607 0.04968 0.05882	ead Forecast RM 12m 0.05289 0.05153 0.04542 0.07136 0.05075 0.05697	SPE 60m 0.03689 0.06603 0.06009 0.06732 0.07500 0.08333	120m 0.03005 0.08994 0.06943 0.06567 0.08520 0.08326	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.06954
Yie ON-SIEGEL MODELS	NS-OLS(I=0.0999) GARCH-C NS-OLS(I=0.0999) GARCH-C NS-OLS(I=0.0999) GARCH-AR-C NS-OLS(I=0.099) GARCH-C NS-OLS(I=0.099) GARCH-R-C NS-OLS(I=0.099) GARCH-R-C NS-OLS(I=0.099) GARCH-C NS-OLS(I=0.099) GARCH-C NS-OLS(I=0.099) GARCH-C NS-OLS(I=0.099) GARCH-C NS-OLS(I=0.099) GARCH-C NS-OLS(I=0.099) GARCH-C	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372 0.04262	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459 0.04365	3m 0.05642 0.07988 0.04507 0.07525 0.04647 0.06240 0.06031	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.05961 0.05908	ead Forecast RM 0.05119 0.05419 0.03803 0.06776 0.04811 0.05844 0.05844	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.05884 0.08187 0.04283	120m 0.03098 0.09403 0.05550 0.04362 0.06874 0.08206 0.03503	360m 0.03312 0.10442 0.05741 0.03640 0.09495 0.07295 0.03147	ARMSPE 0.03905 0.08264 0.06562 0.07515 0.07542 0.04354	TRMSPE 0.04037 0.08371 0.06141 0.05739 0.07474 0.07665 0.04484	3m 0.05713 0.08022 0.05368 0.07869 0.05053 0.06146 0.06418	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607 0.04968 0.05882 0.06175	ead Forecast RM 0.05289 0.05153 0.04542 0.07136 0.05075 0.05697 0.05703	SPE 60m 0.03689 0.06003 0.06009 0.05732 0.07500 0.08333 0.04308	120m 0.03005 0.08994 0.06943 0.04567 0.08520 0.08326 0.03513	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.06954 0.03167
Yie SIN SIEGEL MODELS	NODEL NS-OLS/N=0.099)-GARCH-C NS-OLS/N=0.099)-GARCH-AR-C NS-OLS/N=0.099)-GARCH-C NS-OLS/N=0.099)-GARCH-AR-C NS-OLS/N=0.099)-GARCH-AR-C NS-OLS/N=0.099)-GARCH-AR-C NS-OLS/N=0.099)-GARCH-AR-C	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372 0.04262 0.07527	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459 0.04365 0.07924	3m 0.05642 0.07988 0.04507 0.07525 0.04647 0.06240 0.06031 0.04806	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.05961 0.05808 0.04504	ead Forecast RM 0.05119 0.05419 0.03803 0.06776 0.04811 0.05844 0.05844 0.05384 0.04332	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.05884 0.08187 0.04283 0.06914	120m 0.03098 0.09403 0.05550 0.04362 0.06874 0.08206 0.03503 0.09596	360m 0.03312 0.10442 0.05741 0.03640 0.09495 0.07295 0.03147 0.11907	ARMSPE 0.03905 0.08264 0.06062 0.07315 0.07542 0.04354 0.09065	TRMSPE 0.04037 0.08371 0.06141 0.05739 0.07474 0.07665 0.04484 0.09656	3m 0.05713 0.06022 0.05368 0.07869 0.06053 0.06146 0.06418 0.06150	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607 0.04968 0.05882 0.06175 0.04829	ead Forecast RM 0.05289 0.05153 0.04542 0.07136 0.05075 0.05697 0.05703 0.04397	SPE 60m 0.03689 0.06003 0.06732 0.07500 0.06333 0.04308 0.08888	120m 0.03005 0.08994 0.06943 0.04567 0.08520 0.08526 0.03513 0.12285	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.03167 0.13711
Yie Norsieger woders	NODEL NS-0.52(n-0.0998)-GARCH-AR-C	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372 0.04262 0.07527 0.31430	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459 0.04365 0.07924 0.31759	3m 0.05642 0.07968 0.04507 0.07525 0.04647 0.06240 0.06031 0.04806 0.30144	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.05961 0.05808 0.04504 0.29759	ead Forecast RM 0.05119 0.05419 0.03803 0.06776 0.04811 0.05844 0.05384 0.06384 0.04332 0.30747	SPE 60m 0.03664 0.09433 0.06311 0.05502 0.05884 0.08187 0.04283 0.06914 0.35717	120m 0.03098 0.09403 0.05550 0.04362 0.06874 0.08206 0.03503 0.09596 0.31783	360m 0.03312 0.10442 0.05741 0.03640 0.02455 0.07295 0.03147 0.11907 0.21626	ARMSPE 0.03905 0.08264 0.06562 0.07515 0.07542 0.04354 0.09065 0.30304	TRMSPE 0.04037 0.08371 0.06141 0.05739 0.07474 0.07665 0.04484 0.08656 0.30558	3m 0.06713 0.06022 0.06368 0.07869 0.06053 0.06146 0.06418 0.06150 0.31440	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607 0.04968 0.05882 0.05882 0.06175 0.04829 0.30570	ead Forecast RM 0.05289 0.05153 0.04542 0.07136 0.05075 0.05697 0.05703 0.04397 0.31634	SPE 60m 0.03689 0.08603 0.06009 0.05732 0.07500 0.08333 0.04308 0.08888 0.38640	120m 0.03005 0.08994 0.06943 0.04567 0.08520 0.08326 0.03513 0.12285 0.30037	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.06954 0.03167 0.13711 0.21234
Yie Nerson-Sieger WODERS	NOCE NS.015(J.019805) GARCH-2 GARCH-1	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372 0.04262 0.07527 0.31430 0.19076	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459 0.04365 0.07924 0.31759 0.24619	3m 0.05642 0.07988 0.04507 0.07525 0.04647 0.06240 0.06031 0.04806 0.30144 0.68775	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.059061 0.05808 0.04504 0.29759 0.4454	ead Forecast RM 0.05119 0.05419 0.0803 0.06776 0.04811 0.05844 0.05384 0.05384 0.04332 0.30747 0.07963	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.05884 0.08187 0.04283 0.06914 0.35717 0.16939	120m 0.03098 0.09403 0.05550 0.04362 0.06874 0.08206 0.03503 0.09596 0.31783 0.10280	360m 0.03312 0.10442 0.05741 0.03640 0.03495 0.07147 0.11907 0.21626 0.12874	ARMSPE 0.03905 0.08264 0.06062 0.05592 0.07315 0.07542 0.04354 0.09065 0.30304 0.78637	TRMSPE 0.04037 0.08371 0.06141 0.05739 0.07474 0.07665 0.04484 0.09656 0.30558 1.92708	3m 0.05713 0.08022 0.05368 0.07869 0.06053 0.06146 0.06418 0.06150 0.31440 7.16089	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607 0.04968 0.05882 0.06175 0.04829 0.30570 1.94978	ead Forecast RM 12m 0.05153 0.04542 0.07136 0.05075 0.05697 0.05703 0.04397 0.31634 0.20939	SPE 60m 0.08693 0.06009 0.05732 0.07500 0.08333 0.04308 0.08888 0.38640 0.24232	120m 0.03005 0.08994 0.06943 0.04567 0.08520 0.08520 0.03513 0.12285 0.30037 0.14630	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.03664 0.03167 0.13711 0.21234 0.17474
Yie Nerson-Sieger WODERS	NOCE NS-0.52(ve3.088)-0.48C+h.C OARCH-L OARCH-L OARCH-L	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372 0.04262 0.07527 0.31430 0.19076 0.19076 0.03373	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459 0.04365 0.07924 0.31759 0.24819 0.06147	3m 0.05642 0.07988 0.04507 0.07525 0.04647 0.06240 0.06031 0.04806 0.30144 0.68775 0.22468	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.05961 0.05808 0.04504 0.29759 0.42454 0.02152	ead Forecast RM 12m 0.05119 0.05419 0.0803 0.06776 0.04811 0.05844 0.04332 0.30747 0.07963 0.00849	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.05884 0.08187 0.04283 0.09914 0.35717 0.16939 0.02099	120m 0.03098 0.09403 0.05550 0.04362 0.06874 0.06206 0.03503 0.09596 0.31783 0.10280 0.02255	360m 0.03312 0.10442 0.05741 0.03640 0.0295 0.03147 0.11907 0.21626 0.12874 0.03322	ARMSPE 0.03905 0.08264 0.05592 0.07315 0.07542 0.04354 0.09065 0.30304 0.78637 0.39573	TRMSPE 0.04037 0.08371 0.05739 0.07474 0.07665 0.04484 0.06656 0.30558 1.92708 1.42550	3m 0.05713 0.08022 0.05368 0.07869 0.06053 0.06146 0.06448 0.06150 0.31440 7.16089 5.51825	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607 0.04968 0.05882 0.06175 0.04829 0.30570 1.94978 0.15436	ead Forecast RM 0.05289 0.05153 0.04542 0.07136 0.05075 0.05697 0.05703 0.04397 0.31634 0.20939 0.00964	SPE 60m 0.08689 0.06603 0.06009 0.05732 0.07500 0.08333 0.04308 0.08888 0.33640 0.24232 0.02122	120m 0.03005 0.06994 0.06943 0.04567 0.08520 0.03513 0.12285 0.30037 0.14630 0.02251	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.06954 0.03167 0.13711 0.21234 0.17474 0.03457
YIE NODELS NELSON-SIEGEL MODELS	NOCE NS.015(J.40 508) GARCH-RC MS.015(J.40 508) GARCH-RC MS.015(J.40 508) GARCH-RC GARCH-RC	ARMSPE 0.03901 0.06717 0.05063 0.065331 0.06189 0.07372 0.04262 0.07527 0.31430 0.19076 0.03373 0.06806	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459 0.04365 0.07924 0.31759 0.24619 0.06147 0.17781	3m 0.05642 0.07988 0.04507 0.07525 0.04647 0.06240 0.06031 0.04806 0.30144 0.68775 0.22468 0.68140	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.05961 0.04504 0.04504 0.29759 0.42454 0.02152 0.04769	ead Forecast RM 0.06119 0.05419 0.05419 0.03803 0.06776 0.04811 0.05844 0.05384 0.04332 0.30747 0.07963 0.00849 0.05166	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.05884 0.08187 0.04283 0.06914 0.35717 0.16939 0.02099 0.02146	120m 0.03098 0.09403 0.08550 0.04362 0.08574 0.08206 0.03503 0.09596 0.31783 0.10280 0.02255 0.02230	360m 0.03312 0.10442 0.05741 0.03640 0.04455 0.07285 0.03147 0.11907 0.21626 0.12874 0.03222 0.0322	ARMSPE 0.03905 0.08264 0.06062 0.07515 0.077542 0.04354 0.09065 0.30304 0.78637 0.39673 0.81987	TRMSPE 0.04037 0.08371 0.05141 0.07665 0.07474 0.07665 0.0484 0.08656 0.30558 1.92708 1.42550 2.96435	3m 0.06713 0.08022 0.05368 0.07869 0.06053 0.06146 0.06148 0.06150 0.31440 7.16089 5.51825 11.47280	3-Months-Ah 6m 0.05514 0.05276 0.07607 0.04968 0.05882 0.06175 0.04829 0.30570 1.94978 0.15436 0.36489	ead Forecast RM 0.05289 0.05153 0.04542 0.07136 0.05075 0.05697 0.05703 0.04397 0.31634 0.29939 0.00964 0.21743	SPE 60m 0.03689 0.06003 0.06009 0.06732 0.07500 0.06333 0.04308 0.06888 0.33640 0.24232 0.02122 0.02219	120m 0.03005 0.08994 0.06943 0.04567 0.08520 0.08326 0.03513 0.12285 0.30037 0.14630 0.02251 0.02251	360m 0.03092 0.10610 0.07412 0.03837 0.09844 0.06954 0.03167 0.13711 0.21234 0.17474 0.03457 0.03059
T WODERS NETS ON-SIEGER WODERS	NODEL NS-DE2(NA-0598)GARCH-E NS-DE3(NA-0598)GARCH-E MACH-E GARCH-RA-L GARCH-RA-L GARCH-RA-L	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372 0.04262 0.07527 0.31430 0.19076 0.03373 0.06806 0.02147	TRMSPE 0.04009 0.08827 0.05110 0.05474 0.06328 0.07459 0.04365 0.07924 0.31759 0.24619 0.26419 0.26419 0.26419 0.26419 0.26419 0.26419	3m 0.05642 0.07968 0.04507 0.0525 0.06240 0.06031 0.04606 0.06031 0.04006 0.30144 0.68775 0.22468 0.68140 0.68745	1-Month-Ah 6m 0.05407 0.06491 0.04225 0.07261 0.04565 0.05961 0.05961 0.05961 0.05961 0.05961 0.04504 0.29759 0.42454 0.02152 0.04769 0.04769 0.04769	ead Forecast RM 0.05119 0.05419 0.03803 0.06776 0.04811 0.05844 0.05844 0.05844 0.05844 0.05844 0.05844 0.05844 0.05844 0.05844 0.05844 0.05844 0.05963 0.00649 0.06166 0.00793	SPE 60m 0.03664 0.09433 0.06311 0.05502 0.08884 0.08187 0.04283 0.00914 0.35717 0.16939 0.02099 0.02146 0.02146 0.01971	120m 0.03098 0.09403 0.05550 0.04362 0.08874 0.08206 0.03503 0.09596 0.31783 0.10280 0.02255 0.02230 0.022264	360m 0.03312 0.10442 0.05741 0.03640 0.07285 0.03147 0.11907 0.21626 0.112874 0.03322 0.02867 0.03862	ARMSPE 0.03905 0.08264 0.06062 0.05592 0.07315 0.07542 0.04554 0.09065 0.30304 0.78637 0.39573 0.31967 0.33119	TRMSPE 0.04037 0.08371 0.06141 0.07685 0.04484 0.09656 0.30558 1.92708 1.42550 2.96435 0.04728	3m 0.05713 0.08022 0.05368 0.07869 0.05053 0.06146 0.06146 0.06148 0.05150 0.31440 7.16089 5.51825 11.47280 0.15964	3-Months-Ah 6m 0.05514 0.06527 0.05276 0.07607 0.04968 0.05882 0.06175 0.04829 0.30570 1.94978 0.15436 0.36489 0.064257	ead Forecast 12m 0.05289 0.05153 0.04542 0.07136 0.05697 0.05703 0.04597 0.05703 0.04594 0.20309 0.00964 0.21743 0.00910	SPE 60m 0.03689 0.06009 0.06732 0.07500 0.06333 0.04308 0.04888 0.33640 0.24232 0.02122 0.02219 0.02012	120m 0.03005 0.08994 0.06943 0.04567 0.08520 0.08526 0.30037 0.14630 0.02251 0.02052 0.02051	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.03167 0.13711 0.21234 0.17474 0.03457 0.03059 0.04151
YIII STAL MODELS NELSON-SIEGEL MODELS	NODE NS-0126/002989-047CH-42 NS-0126/002989-047CH-42 NS-0126/002989-047CH-42 NS-0126/002989-047CH-42 NS-0126/002989-047CH-42 NS-0126/00299-047CH-42 NS-0126/00299-047CH-42 NS-0126/00299-047CH-42 NS-0126/00299-047CH-42 NS-0126/00299-047CH-42 NS-0126/0029-047CH-42 NS-0126/0029-047CH-42 O47CH-42 O47CH-42 O47CH-42 O47CH-42 O47CH-42 O47CH-42	ARMSPE 0.03901 0.08717 0.05063 0.05331 0.06189 0.07372 0.04262 0.07527 0.31430 0.19076 0.03373 0.06806 0.02147 0.02282	TRMSPE 0.04009 0.0827 0.05110 0.05474 0.06328 0.07924 0.3759 0.24619 0.24619 0.24619 0.246147 0.071781 0.02459	3m 0.06642 0.07988 0.04507 0.07525 0.04647 0.06240 0.04806 0.30144 0.68775 0.22468 0.68140 0.04214 0.04214	1-Month-Ah 6m 0.05407 0.04641 0.04225 0.07261 0.04565 0.05961 0.05508 0.04504 0.25759 0.42454 0.02152 0.04769 0.021815 0.04451	ead Forecast RM 12m 0.05119 0.05419 0.05419 0.05016 0.04811 0.05844 0.04332 0.07963 0.07963 0.00646 0.00793 0.01234	SPE 60m 0.03664 0.09433 0.06512 0.05502 0.05864 0.04283 0.06914 0.35717 0.16339 0.02146 0.02146 0.02146 0.02146	120m 0.03098 0.09403 0.04362 0.04362 0.04362 0.04362 0.04560 0.03503 0.03503 0.03503 0.03503 0.03556 0.03574 0.02556 0.02250 0.02264 0.02264	360m 0.03312 0.10442 0.05741 0.03640 0.03640 0.03495 0.07285 0.03147 0.11907 0.21626 0.12874 0.0322 0.02867 0.03566	ARMSPE 0.03905 0.08264 0.06062 0.05592 0.07315 0.07542 0.04354 0.09065 0.30304 0.78637 0.399673 0.399673 0.81987 0.033119 0.10541	TRMSPE 0.04037 0.06371 0.06371 0.05739 0.07474 0.07685 0.04484 0.09656 0.30558 1.92708 1.42550 2.96435 0.04728 0.30035	3m 0.05713 0.08022 0.05368 0.05053 0.06146 0.05150 0.05150 0.05150 7.16099 5.51825 11.47280 0.15964 1.14228	3-Months-Ah 6m 0.05514 0.05276 0.07607 0.04968 0.04829 0.04829 0.04829 0.04829 0.04829 0.04829 0.04829 0.04829 0.04829 0.04829 0.04825 0.04257	ead Forecast RM 12m 0.05299 0.05153 0.05542 0.05075 0.05697 0.0575 0.05697 0.0575 0.04397 0.31634 0.20939 0.00964 0.21743 0.00910 0.01352	SPE 60m 0.03689 0.06603 0.06009 0.06732 0.07500 0.06333 0.04308 0.33640 0.24232 0.02219 0.02219 0.02012 0.0212	120m 0.03005 0.08994 0.04567 0.08520 0.03573 0.12285 0.30037 0.14630 0.02251 0.02251 0.02262 0.02211	360m 0.03092 0.10610 0.07412 0.03837 0.09644 0.03654 0.03167 0.03167 0.03167 0.13711 0.21234 0.13711 0.21234 0.13711 0.21234 0.13711 0.03457 0.03059 0.04151 0.02253
VIAL MODELS NELSON-SIEGEL MODELS	NODEL NS-DE2(NR-05089)-GARCH-6 NS-DE3(NR-05089)-GARCH-6 MALL GARCH-6 GARCH-RA-1 GARCH-RA-2 EGARCH-1 EGARCH-1	ARMSPE 0.03901 0.06717 0.05063 0.05033 0.05033 0.05331 0.06331 0.07372 0.04282 0.07372 0.04282 0.07527 0.31430 0.19076 0.03973 0.06806 0.02147 0.02282 0.34191	TRMSPE 0.04009 0.0810 0.0511 0.05474 0.0328 0.07499 0.4365 0.07924 0.31759 0.24619 0.07429 0.24619 0.02459 1.21580	3m 0.05642 0.07525 0.04507 0.07525 0.06240 0.06204 0.06204 0.06204 0.06031 0.04047 0.06214 0.68140 0.04214 0.03707 4.70578	1-Month-Ah 6m 0.05407 0.0425 0.07261 0.04255 0.07261 0.04265 0.04504 0.28759 0.42454 0.02162 0.04769 0.42454 0.04765 0.04765	ead Forecast RM 12m 0.05119 0.05419 0.03803 0.06776 0.04811 0.05844 0.04534 0.05844 0.04532 0.007963 0.007963 0.007963 0.007963 0.007973	SPE 60m 0.03664 0.09433 0.05311 0.05502 0.06841 0.09177 0.04283 0.0914 0.335717 0.16939 0.02161 0.02116 0.02116 0.02116	120m 0.03098 0.05550 0.04362 0.06574 0.08206 0.03503 0.08206 0.03503 0.01280 0.02255 0.02225 0.02225 0.02224 0.02254 0.02254	360m 0.03312 0.10442 0.05741 0.03640 0.07285 0.02345 0.03147 0.11907 0.21625 0.03147 0.12874 0.03222 0.02867 0.03662 0.03566	ARMSPE 0.03905 0.06264 0.06562 0.07542 0.07542 0.04354 0.09065 0.30304 0.78637 0.39573 0.81987 0.03119 0.10541 0.68158	TRMSPE 0.04037 0.081371 0.05739 0.07685 0.04484 0.09656 0.30558 1.92708 1.42550 2.96435 0.04728 0.30035	3m 0.06713 0.06022 0.05368 0.07869 0.06148 0.06148 0.06148 0.06140 7.16099 5.51825 11.47280 0.15964 1.14235 9.43302	3-Months-Ah 6m 0.05514 0.0527 0.05276 0.05276 0.05276 0.05276 0.05429 0.05882 0.05875 0.04529 0.30570 1.94978 0.16436 0.36479 0.04257 0.20668 0.50470	ead Forecast RM 12m 0.05289 0.05153 0.04542 0.0575 0.05703 0.05703 0.05703 0.04542 0.05075 0.05703 0.04543 0.0573 0.04543 0.05909 0.00964 0.021743 0.00910 0.01362 0.00886	SPE 60m 0.03689 0.06603 0.06732 0.07500 0.04308 0.04308 0.04308 0.33640 0.24232 0.02122 0.02212 0.02212 0.02212 0.0212 0.02012 0.0217 0.02017 0.02017	120m 0.03005 0.09944 0.04567 0.04567 0.03265 0.03513 0.12285 0.30037 0.14630 0.022251 0.02251 0.02251 0.022311 0.02261	360m 0.03092 0.10610 0.03837 0.06944 0.03167 0.13711 0.21234 0.17474 0.03457 0.03059 0.04151 0.02253 0.05320

As regards the yield forecasting, the RW model seems work exceptionally well in most cases, and both the AR and VAR models produced almost similar and relatively fine forecasts. There is also a tendency for these two models to converge towards the RW model, which means that, when the selection criteria of significant coefficients becomes more stringent, many of those coefficients on the lagged regressors as well as the constant term were found to be insignificant and the model ultimately becomes a RW. In contrast, for the weekly and daily forecasts, the NS models (except for NS-VAR) generally performed worse than the non-NS models (except VECM). For monthly forecasts, the NS models appear to function well when it comes to longer horizons, particularly for 12-months-ahead forecasts. On top of that, the NS-VAR model performed roughly the same for all three variations of decay rates. It is also interesting to note that the NS-RW and NS-AR specifications when $\lambda = 0.0299$ yields the lowest TRMSPE among the three decay rates, which indicates that Diebold's choice of λ is not necessarily the best one available.

On the other hand, the results indicate that the non-NS models outperform the NS models in volatility forecasts with the presence of liquidity trap. This finding actually is not a surprise as liquidity trap has impelled bond yields towards zero and less volatile. As regards the various decay rates, it appears that the lowest ARMSPEs and TRMSPEs are attained when $\lambda = 0.0598$ in most cases, while the NLS-estimated model with λ set at 0.033 performed a lot worse. This result confirms that a faster decay rate allows forecasted conditional volatility to converge quickly to zero. As liquidity trap has induced actual volatility close to zero, the NS model with the largest λ is likely to stand out from the rest.

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4. CONCLUDING REMARKS

We have demonstrated that due to the presence of liquidity trap in Japan, out-of-sample expanding window forecasts for the NS models in general perform inferiorly vis-à-vis the non-NS models, and this is coupled with the other problem of obtaining negative yield forecasts for bonds with shorter maturities. Moreover, the results show that the NS class of models can be useful in forecasting shorter horizons like weeks and days, works better with a decay rate other than the conventional way of treating it as the value that maximizes the loading on the medium-term factor at exactly 30 months, and can work well with time series models such as GARCH and EGARCH in terms of volatility forecasting. It is also found that, when the NS models are used for yield forecasts, the NS-VAR model should be considered since it is up to par against the competitor models, even with liquidity trap at work

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