

Evaluating Extremal Dependence in Stock Markets Using Extreme Value Theory

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Abstract: Estimation of tail dependence between financial assets plays a vital role in various aspects of financial risk modelling including portfolio theory and hedging amongst others. Extreme Value Theory (EVT) that provides well established methods for univariate and multivariate tail distributions which are useful for forecasting financial risk or modelling the tail dependence of risky assets. This paper uses nonparametric measures based on bivariate EVT to investigate asymptotic dependence and estimate the degree of tail dependence of the ASX-All Ordinaries daily returns with four other international markets, viz., the S&P-500, Nikkei-225, DAX-30 and Heng-Seng for both right and left tails of the return distribution in extreme quantiles. It is investigated whether the asymptotic dependence between these markets is related to the heteroskedasticity present in the logarithmic return series using GARCH filters. The empirical evidence from bivariate EVT methods show that the asymptotic dependence between the extreme tails of the stock markets does not necessarily exist and rather can be associated with the heteroskedasticity present in the financial time series of the various stock markets.

Keywords: Tail dependence, Extreme Value Theory, heteroskedasticity, GARCH.

1 INTRODUCTION

Measuring extreme tail dependence of different financial markets or assets is one of the most important subjects in financial risk modelling and plays a vital role in quantification of codependent risk. It has become evident from the recent GFC that extreme shocks in one financial market for a given country may affect other financial markets around the globe. Hence the modelling of extremal dependence and the exploration of whether the tails of stock markets are asymptotically dependent becomes a research question of great interest.

The simplest measure of dependence, the Pearson correlation coefficient is only useful in detecting linear dependence between return series in financial applications. Because it is based on deviations from the mean of the distribution it assigns equal weights to all the instances in a random variable and hence is not a suitable measure for extreme tail dependence. To study tail dependence one of the most sophisticated statistical tools of interest is EVT which provides well established methods for tail based distributions and hence provides better univariate or multivariate tail estimates.

The contribution of EVT based techniques in financial applications has increased in recent years. Not only has univariate EVT been implemented in the quantification of extreme risk measures (Diebold, Schuermann et al. 1998, Neftci, 2000 Zhang, 2005; Mancini and Trojani, 2011; Onour, 2010; Gilli and K ellezi, 2006) but there are also now applications of multivariate EVT in the modelling of tail dependence and other financial risk related research. Multivariate extreme value methods for stock market return series have been evaluated in Longin and Solnik (2001), Longin (2000), Bouy e (2002), for VaR and portfolio risk modelling. Cole, Heffernan and Tawn (2000) outline the dependence measures for extremal events.

Poon, Rockinger and Tawn (2003) implemented bivariate EVT based dependence measures to investigate the asymptotic dependence in various international stock markets and how it relates to the presence of heteroskedasticity in financial return series. Fern andez (2003) implemented EVT methods for the quantification of VaR and dependence in world markets. Chen, Giles and Feng (2010) study the Chinese stock market and its dependence with other major international stock markets. Schich (2004) studies dependence in European stock markets for large price changes with multivariate EVT methods. Hilal, Poon and Tawn (2011) use the methods of Heffernan and Tawn (2004) to model extremal dependence between financial time series and demonstrate how it can be effectively used in hedging. Dupuis and Jones (2006) illustrate the use of multivariate EVT in actuarial applications.

Poon, Rockinger and Tawn (2003, 2004) found in their study that stock markets do not show statistically significant asymptotic tail dependence for return series filtered for heteroskedasticity. We conduct the empirical investigation for the left tail of the return series data, emphasising on the losses in financial markets. The asymptotic dependence between these markets is investigated using simple EVT measures and daily log return series. GARCH based filters are used as a filter for heteroskedasticity in the return series to test the filtered return series for tail dependence and asymptotic tail dependence.

The rest of the paper is designed as follows; section-2 discusses the two tail dependence measures, χ and $\bar{\chi}$, followed by section-3 which outlines the empirical exercise. Results are discussed in section-4 which is followed by conclusions in section-5.

2 EXTREME VALUE THEORY AND TAIL DEPENDENCE

The dependence relationships in a multivariate setting can be classified into four types: independence, perfect dependence, asymptotic dependence and asymptotic independence. The extreme values in an asymptotically dependent setting occur simultaneously whereas they occur at different times in an asymptotically independent structure. Heffernan (2000), Coles, Heffernan and Tawn (2000), Poon, Rockinger and Tawn (2003, 2004) developed two measures χ and $\bar{\chi}$ as a measure of extremal dependence particularly helpful in quantifying the asymptotic dependence between two sets of random data variables. χ quantifies the asymptotic extreme dependence if it exists which is measured by $\bar{\chi}$ with the advantage of having a non-parametric estimation method. Where there is no asymptotic dependence, $\bar{\chi}$ provides the measure of tail dependence.

2.1 Measures of Tail Dependence

According to *Sklar's Theorem* (Sklar, 1959) each joint distribution can be decomposed into its marginal distributions and its dependence structure (also called Copula C). If the marginal aspects of a joint distribution are removed by some transformation of data, the remaining differences between the distributions are then purely due to dependence (Embrechts, McNeil and Strautman, 2002). With bivariate returns (X, Y) the marginal aspects of the joint distribution can be removed by transforming them to standard Fréchet marginals (S, T) as follows:

$$S = -\frac{1}{\log F_X(X)} \text{ and } T = -\frac{1}{\log F_Y(Y)} \quad (2.1)$$

where F_X and F_Y are marginal distribution functions for X and Y respectively. In practical applications F_X and F_Y are empirical distribution functions of separate variables. The Fréchet transformation is used as risk asset returns tend to exhibit fat tails (Loretan and Phillips, 1994). Here $P(S > s) = P(T > s) \sim s^{-1}$ as $s \rightarrow \infty$ and S and T have the same dependence structure as (X, Y) .

Asymptotic Dependence-The Conventional Approach. For an extreme threshold s (s can also be a quantile q) the events $S > s$ and $T > s$ are equally extreme for both variables. The first nonparametric measure of dependence χ (Coles et. al, 2000; Poon et. al, 2003;2004) is given by :

$$\begin{aligned} \chi &= \lim_{s \rightarrow \infty} P(T > s | S > s), \\ &= \lim_{s \rightarrow \infty} \frac{P(T > s, S > s)}{P(S > s)} \end{aligned} \quad (2.2)$$

and $0 \leq \chi \leq 1$. Here χ can be used to investigate the dependence between S and T . If $\chi > 0$, S and T are asymptotically dependent, and perfect dependence occurs if $\chi = 1$. If $\chi = 0$, S and T are asymptotically independent.

Asymptotic Independence- An Alternative Measure of Dependence. The measure $(\bar{\chi})$ can be used to measure dependence for asymptotically independent variables. As per Coles, Heffernan and Tawn (2000), $\bar{\chi}$ is defined as

$$\bar{\chi} = \lim_{s \rightarrow \infty} \frac{2 \log P(S > s)}{\log P(S > s, T > s)} - 1, \quad (2.3)$$

where $-1 < \bar{\chi} \leq 1$. The measure $\bar{\chi}$ is a measure of the rate of $P(T > s | S > s) \rightarrow 0$.

The variables S and T have perfect dependence for $\bar{\chi} = 1$ and independence for $\bar{\chi} = 0$. The two cases where $\bar{\chi} > 0$ and $\bar{\chi} < 0$ indicates positive and negative association respectively.

With these two dependence measures $(\chi, \bar{\chi})$, all the information needed to characterise the degree and type of extremal dependence can be obtained. In practice first the bivariate distribution is tested for $\bar{\chi} = 1$, before quantifying the bivariate asymptotic dependence using the estimate χ . If variables are not asymptotically dependent, i.e. $\bar{\chi} \neq 1$, the degree of dependence is given by the value of $\bar{\chi}$. In case of asymptotic dependence ($\bar{\chi} = 1$) the degree of dependence is quantified by the value of χ .

2.2 The Hill Estimator

For the case of the tail of a univariate heavy tailed variable Z above a high threshold u

$$P(Z > z) = \mathcal{L}(z)z^{-1/\xi} \text{ for } z > u, \quad (2.4)$$

where ξ gives the shape parameter which is also known as tail index, and $\mathcal{L}(z)$ is a slowly varying function of z given by

$$\lim_{u \rightarrow \infty} \frac{\mathcal{L}(uz)}{\mathcal{L}(u)} = 1 \text{ for all } z > 0. \quad (2.5)$$

Assuming the observations to be *i.i.d* and taking $\mathcal{L}(z) = d$ as constant for all $z > u$ the maximum likelihood estimators of ξ , also known as Hill's estimator (Hill, 1975), and $\mathcal{L}(z)$ are

$$\hat{\xi} = \frac{1}{n} \sum_{j=1}^{n_u} \log \left(\frac{z_{(j)}}{u} \right), \quad (2.6)$$

$$\widehat{\mathcal{L}}(z) = \frac{n_u}{n} u^{1/\hat{\xi}}, \quad (2.7)$$

where $z_{(1)}, \dots, z_{(n_u)}$ are the n_u observations above a threshold u in a variable Z .

2.3 Estimating χ and $\bar{\chi}$ - Nonparametric Method

To estimate $\bar{\chi}$ and χ , Ledford and Tawn (1996, 1998) established that under weak conditions,

$$P(S > s, T > s) \sim \mathcal{L}(s)s^{-1/\eta} \text{ as } s \rightarrow \infty, \quad (2.8)$$

where $0 < \eta \leq 1$ is a constant called the *coefficient of tail dependence* and $\mathcal{L}(s)$ is a slowly varying function. It follows that

$$\bar{\chi} = 2\eta - 1 \quad (2.9)$$

and

$$\chi = \begin{cases} c & \text{if } \bar{\chi} = 1 \text{ and } \mathcal{L}(s) \rightarrow c > 0, \text{ as } s \rightarrow \infty, \\ 0 & \text{if } \bar{\chi} = 1 \text{ and } \mathcal{L}(s) \rightarrow 0, \text{ as } s \rightarrow \infty, \\ 0 & \text{if } \bar{\chi} < 1 \end{cases} \quad (2.10)$$

If $\bar{\chi} = 1$ as $\eta = 1$, then $\chi = \lim_{s \rightarrow \infty} \mathcal{L}(s) = c$ which is the case of asymptotic dependence. If $0 < \eta < 1$ or if $\eta = 1$ and $\lim_{s \rightarrow \infty} \mathcal{L}(s) = 0$ (boundary case), then $\chi = 0$ and the variables are asymptotically independent with dependence given by $\bar{\chi} = 2\eta - 1$.

The degree of dependence between large values (extremes) of X and Y is determined by η and \mathcal{L} , an increasing value of η indicates a stronger association and for a given η the relative strength of dependence is given by \mathcal{L} .

If there is evidence to prove that $\bar{\chi}$ is significantly less than 1 that is $\bar{\chi} + 1.96\sqrt{\text{Var}(\bar{\chi})} < 1$ (Poon et. al., 2003) than it is inferred that variables are asymptotically independent and hence $\chi = 0$. In case there is no significant result to reject $\bar{\chi} = 1$, χ is estimated.

Therefore estimation of η and $\lim_{s \rightarrow \infty} \mathcal{L}(s)$ are required for estimating χ and $\bar{\chi}$. We will omit the further estimation details for the sake of brevity as they are well explained in the original paper by Poon et. al (2003, 2004). These two estimates of tail dependence are used to investigate the asymptotic tail dependence and quantify the extremal dependence between the stock markets in our dataset.

3 DATA AND METHODOLOGY

The objective of this empirical study is to investigate the bivariate extremal dependence structure between ASX-All Ordinaries and other four international markets, viz., the S&P-500, Nikkei-225, DAX-30 and Heng-Seng using bivariate EVT based dependence measures. The data period used is from 01/01/1978 to 22/03/2011, which generates a daily return data series with 8645 log returns for all five markets. This data period also includes the turbulent times of the 1987 crash and the recent GFC (2007-2008) which gives

important extreme observations needed to examine extreme tail dependence. The data is downloaded from Reuters Datastream and the prices are in US dollars.

Bivariate EVT based measures are used to evaluate extremal dependence in financial markets. The method uses a threshold based approach where the extremal dependence in left and right tails of bivariate (X, Y) distribution of two stock markets is tested. Here the dependence of Australian stock market (ASX-All Ordinaries) is checked with four other international stock markets, S&P-500, Nikkei-225, DAX-30, Heng-Seng. The analysis is conducted for the 5% and 10% quantile for the left tail of the distribution¹. The GARCH(1,1) and Asymmetric GARCH (1,1) or AGARCH(1,1) models are used as a filter for heteroskedasticity.

4 RESULTS

Table 1: $\bar{\chi}$ Estimates for Left Tail Extremal Dependence.

	90% Quantile				95% Quantile			
	Lower	$\bar{\chi}$	Upper	Std. Error	Lower	$\bar{\chi}$	Upper	Sd. Error
Returns not filtered for heteroskedasticity								
ASX-S&P	0.9242	1.0616	1.1990	0.0701	0.9368	1.1382	1.3396	0.1028
ASX-Nikkei	0.8050	0.9339	1.0627	0.0658	0.8979	1.0953	1.2926	0.1007
ASX-DAX	0.6761	0.7958	0.9154	0.0611	0.8398	1.0311	1.2225	0.0976
ASX-Heng Seng	0.7584	0.8839	1.0095	0.0641	0.7261	0.9056	1.0851	0.0916
GARCH(1,1) filtered returns								
ASX-S&P	0.5714	0.6836	0.7958	0.0572	0.5893	0.7545	0.9198	0.0508
ASX-Nikkei	0.4542	0.5580	0.6618	0.0530	0.4293	0.5780	0.7266	0.0758
ASX-DAX	0.2750	0.3660	0.4571	0.0464	0.3324	0.4709	0.6095	0.0707
ASX-Heng Seng	0.6521	0.7701	0.8880	0.0602	0.5102	0.6673	0.8243	0.0801
AGARCH(1,1) filtered returns								
ASX-S&P	0.4992	0.6063	0.7133	0.0546	0.6007	0.7671	0.9336	0.0849
ASX-Nikkei	0.5156	0.6239	0.7321	0.0552	0.4506	0.6015	0.7523	0.0770
ASX-DAX	0.3312	0.4262	0.5213	0.0485	0.3854	0.5295	0.6736	0.0735
ASX-Heng Seng	0.6521	0.7701	0.8880	0.0602	0.5178	0.6756	0.8334	0.0805

The empirical analysis is conducted for two different extreme quantile levels, 90% and 95%, for measuring the tail dependence between our four pairs, viz. Australia-USA, Australia-Germany, Australia-Japan and Australia-Hong Kong. To conduct the analysis for left tail the returns are transformed into their negative i.e. $R_t = -R_t$.

A two step analysis is conducted to examine the extremal tail dependence between the stock market pairs. The first step calculates $\bar{\chi}$ using the coefficient of tail dependence η . If there is no statistical evidence to reject $\bar{\chi} = 1$ then the second step calculates the measure of asymptotic dependence χ .

Table-1 gives the interval (95% confidence) and point estimates of $\bar{\chi}$ with their standard errors for two extreme quantile levels (left tail). The results show that the case of asymptotic dependence ($\bar{\chi} = 1$) only occurs in the return series which is not filtered for heteroskedasticity. Results from both the filtered series suggest no asymptotic dependence in our market pairs. In the case of asymptotic dependence, all four pairs show positive asymptotic dependence or association which suggests a dependent structure in which there is more frequent occurrence of observations for which both X and Y exceed a threshold s than exact independence. The results from the filtered series also infer that the highest degree of dependence is shown between Australian and US stock markets, which comes as no surprise. After US the dependence in decreasing order or magnitude is shown by the Heng Seng of Hong Kong, Nikkei-225 of Japan and

¹This study is also conducted for the right tail in a more comprehensive version of this paper.

the least by the DAX-30 of Germany. This shows that the Australian stock market has the least extremal dependence with the stock market of Germany and in Asian markets it is most associated with Hong Kong and then Japan. The major result of the extremal dependence analysis shows that within the tested quantiles there is no statistical evidence to prove asymptotic dependence in the four market pairs. As no statistically significant results are obtained to prove asymptotic dependence in the filtered series the second step of analysis which calculates χ is omitted for the left tail.

5 CONCLUSION

We used two non-parametric measures of extremal dependence in this analysis to evaluate the tail dependence of five major international stock markets. The results demonstrate how portfolio (bivariate in the present case) extreme tail dependence can be modelled using multivariate EVT models. The analysis shows that the stock markets in this study do not necessarily show asymptotic dependence and are rather asymptotically independent. The asymptotic dependence shown by the original return series is contradicted by the results from the return series filtered for heteroskedasticity which shows that heteroskedasticity is responsible for associated asymptotic dependence and the use of unfiltered return series can lead to over calculation of the dependence.

The results show that in extreme market conditions better estimates of the dependence between stock markets can be obtained which can lead to better risk management. The two measures ($\tilde{\chi}, \chi$) used in this analysis prove vital in modelling financial dependence when the assets are asymptotically independent in which case the more usual measure of correlation can provide inaccurate estimates. This study generates possibilities for extending tail dependence modelling in efficient portfolio management, hedging practices and option pricing.

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