

Mathematical modelling in health care

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Abstract: We will describe several cases studies that illustrate how we have used modelling to improve the delivery of health care. The studies cover various settings in health care, utilise a variety of mathematical modelling techniques, and include descriptions of the impact of the model on the health care system. The studies can be described briefly as follows.

In the financial year 2006/2007, emergency departments (EDs) in Australian hospitals dealt with 5,287,451 presentations. Statistical models have been used to forecast the number of patients served by an ED each month, and to evaluate the impact of changes and innovations introduced to the ED.

The operation of an acute care hospital medical service has been described well by a double compartment model. Modification to the basic model facilitates the incorporation of occupancy fluctuations, such as winter peaks, across the year. These models can improve strategic decision making in relation to hospital beds.

At a more aggregated level, a cohort Markov model was used to identify the most cost-effective screening programme for cervical cancer. The model describes the development of precancerous lesions and progression via multiple stages to the advanced form of cervical cancer, and subsequent death. Cancer may be diagnosed at any time, via screening or clinical presentation with symptoms, at which point treatment may be initiated that will alter the natural history of the disease. The model was used to evaluate a large number of screening options, for which it would be infeasible to conduct clinical studies.

Transition care is a form of health care for hospital patients who have finished their stay in an acute care setting but are not be able to return home. Queueing theory has been used to estimate the number of places required for a new transition care facility in a hospital. This example demonstrates how modelling has been used in applying for funds to support new developments in a hospital.

These examples illustrate the potential for applying mathematical modelling and simulation in health care.

Keywords: *health services, hospital, time series, cohort Markov model, cervical cancer, queueing theory, compartment model*

1. INTRODUCTION

“The enjoyment of the highest attainable standard of health is one of the fundamental rights of every human being without distinction of race, religion, political belief, economic or social condition” (World Health Organisation (1946)). Thus, in its Constitution, the WHO places substantial responsibilities on the States that are signatories to ensure the effective delivery of health care services in their countries.

This paper summarises several case studies, in which the authors have been involved, that illustrate how modelling and simulation can assist governments to carry out these responsibilities. The examples are set in a variety of contexts, namely, a hospital emergency department, patient flow in an acute hospital, a national screening program for cervical cancer, and aged care. They employ a range of models including, time series, compartmental models, Markov chains and queueing models. In presenting each case study, we will give a brief introduction that sets the scene.

Collectively, these short stories show the potential for modelling in a wide range of problems in health care.

2. EMERGENCY MODELS

Overcrowding in the emergency department (ED) of a hospital is a common occurrence. Most of us have attended an ED either as a patient or carer and there is widespread appreciation of the problems and issues. Patients and carers are distressed—it is after all an emergency. Doctors and nurses seem to be run off their feet, perhaps at the end of a long shift. Decisions are often made quickly.

A creative writer may see the makings of a TV program in all this. A mathematically minded observer may see the ED as a fertile source of opportunities for modelling and simulation.

This case study deals with two applications of modelling in the ED of an Australian hospital. First, time series models were used to forecast the demand for emergency services at the hospital. Second, regression models were used to evaluate a new approach to managing patient flow in the ED. Both projects were undertaken by multidisciplinary research teams from universities and the hospital.

Forecasting

We cannot foresee falling over and breaking an arm, or cutting our hand in the kitchen while preparing a meal, or a child waking up in the night screaming with stomach pain. Emergencies are unpredictable. However, the number of patients who present themselves to the ED is quite predictable as a function of time. This thesis was the basis of the research reported in Champion *et al.* (2007).

A research team of academics and clinicians set about exploring patterns of presentations at a hospital ED over several years. By regarding the number of presentations over time as a time series, one can apply models from time series analysis to the data. Although time series models provide an obvious approach to exploring ED data, a review of the literature showed that these models have not been used often in this context.

The main findings were as follows. The average number of presentations per day over a month did not vary much with the month, although the variation was statistically significant. There are more obvious patterns in the number of presentations per day over the seven days of the week. And there are clear patterns in the number of presentations per hour for the 24 hours over the day.

One practical outcome of this work is that the hospital modified its staffing rosters in light of the arrival patterns discovered in the course of the project. The time series models led to accurate forecasts of the number of patients on a monthly basis. In the course of the study, the project threw up many more questions, and the team members developed a deeper appreciation of the data and models.

Evaluation

Hospitals are continually exploring new ways to improve their services, and striving to develop innovations that are based on evidence from research. Innovations need to be evaluated and a typical approach is summarised by the following steps.

1. Decide on a key performance indicator (KPI) that will reflect the quality of the service.
2. Measure the KPI before the innovation is introduced.
3. Measure it again some time after the innovation has been introduced.
4. Compare the values of the KPI before and after the introduction of the innovation.

This approach, common though it may be, has some obvious flaws. There is no control group against which to compare the change in the experimental situation. In reality, it is very difficult to find a suitable control

group when evaluating an innovation in a hospital. Also, it is often difficult to attribute the change in an indicator to the innovation because lots of things may be changing while the innovation is being considered.

The following case study suggests that time series models are useful tools for evaluating innovations in health care.

When a patient arrives at the ED, the first stop is at the triage desk. The triage nurse takes the patient's details, assesses the urgency of the case, and assigns a triage rating in accordance with the Australian Triage Scale. Then, patients are seen in order according to this triage rating.

At the hospital where this study was conducted, a new approach to managing patient flow, known as "streaming", was introduced. As usual, all patients receive a triage rating. In addition, the triage nurse would decide whether the patient required complex care or not. Patients who did not require complex care would be treated on a first-come-first-served basis; patients who required complex care would be treated according to their triage rating. It is similar to having a fast queue in a supermarket for people who do not buy many items.

Regression models were used to evaluate the streaming innovation (Kinsman *et al.* (2008)). The Victorian Department of Human Services has defined certain KPIs by which EDs are assessed. These KPIs were selected as the response variables in the regression models. A dummy variable indicated whether the observations were from a month before the innovation or a month after the innovation.

This work demonstrated clearly to the hospital that streaming had a positive effect on the KPIs involved. There was considerable interest in these findings at the hospital because there are financial implications for the hospital in meeting these government requirements. Furthermore, those associated with the project had a better understanding of the role that mathematical models can play in evaluating innovations.

Summary

These examples in this first case study show the potential for applying mathematical modelling and simulation in improving our understanding of patient flow in the ED of a hospital. The projects had practical outcomes that were of benefit to the hospital, and, during the course of the projects, many more interesting questions and possible projects were uncovered.

3. COMPARTMENTAL MODELS

This case study will describe the application of compartmental models in describing bed occupancy in an acute hospital.

The increasing proportion of older people in Australia presents a range of challenges for communities and governments, including a period of greater expenditure on health and other social services. This challenge is made even greater when combined with the shift in disease prevalence from acute infectious disease to one of chronic disease (Generational Health Review, 2003; Productivity Commission, 2005).

Health workers are also ageing and many will reach the end of their working lives within the next 8 years. The expected large number of retirements comes at a time of forecast high demand for services, a reduction of available workers and following a period of insufficient succession planning. Thus, there is a widening gap in the ability to supply services, both in terms of capital infrastructure and workforce.

Hospital occupancy levels have also increased in recent years. The ability to provide services to the growing wave of baby boomers that are expected will not exist, *ceterus paribus*, unless capacity is increased. Additional capacity, in terms of capital infrastructure, can be created relatively quickly. There is, however, little point in doing so if there are insufficient staff available to provide services to patients.

The forecast in future demand is of such concern that, in 2006, the South Australian Minister for Health reported that there was solid evidence that if the State continued to provide health care using current models of service delivery and care strategies, by 2043 the entire state budget will be required to meet the costs of providing health care.

Given the increasing demands being placed upon the health services and the likelihood of significant staff shortages, there are serious consequences in both economic resource allocation and patient (and population) health outcomes if decisions about future health service structures are incorrect. Given the recent advances in computing power and the need to improve decision making, there has never been a more opportune time to apply modelling to facilitate improved decision making in the health care sector. One aspect where modelling will become increasingly important in the health sector is in relation to modelling decisions around hospital beds.

In practice, decision making relating to hospital beds has often been based on either some “back of the envelope” calculation or a “rule of thumb”. And these approaches have tended to rely on the average length of stay (ALOS) metric. The modelling of hospital beds and patient length of stay, which are intertwined, is not new; see, for example, Yates (1982), Pendergast and Vogel (1988), and Sorensen (1996). It has been recognised that the use of the ALOS for modelling hospital bed issues is flawed; see, Farmer and Emami (1990), Harrison and Millard (1999), Mackay and Millard (1999), and Costa *et al.* (2003). There are numerical and practical reasons that using the ALOS is inappropriate for use in the development of models, most notably that the typical length of stay distribution is highly skewed distribution and not well summarised by its mean value.

Compartmental models offer an alternative approach. A compartmental model describes the flow of something, such as patients, through a system, where the system is comprised of a finite number of homogeneous subsystems known as compartments. Godfrey (1983) is a standard text on these models. Harrison and Millard (1991) introduced compartmental models to describe patient flow. The hospital bed compartmental flow model can be represented as shown in the Figure 1.

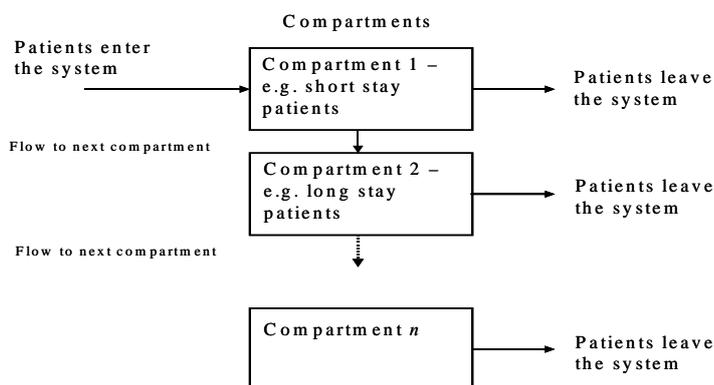


Figure 1: A representation of the flow of patients through compartments. The compartments may be virtual or real - the patients may not actually change location within the physical hospital (Mackay and Lee, 2005).

Work to date has focused on two or three compartment bed occupancy flow models to describe the patient stay profile within the hospital with additional compartments being added to incorporate the community. See Harrison (1994), Mackay (2001), McClean and Millard (1994, 1995, 1998), Taylor, McClean and Millard (1996), Harrison (2001).

An application in Australia and New Zealand

Recently, compartmental models have been applied to the acute care sector in Australia and New Zealand by Mackay with others. See Mackay and Millard (1999), Millard, Mackay, Vasilakis and Christodoulou (2000), Mackay (2001), Mackay and Lee (2004a, 2004b, 2005), Mackay, Lee, Millard and Rae (2004), Harrison, Shafer and Mackay (2005), Mackay (2006).

In his PhD thesis, Mackay (2007) explored whether the deterministic compartmental flow models of bed occupancy, described by Harrison and Millard (1991) in an aged-care facility in the UK, could be adapted, and enhanced, for application in Australia and New Zealand. The research sought to answer the following questions.

- Can the compartmental flow models be successfully applied to the acute care data?
- Can the models be used for other purposes, such as forecasting, evaluation of service change, and the effects of altering funding policy?
- Can bed occupancy compartmental flow model could be further developed to enable a better fit of the data?
- Could sensitivity and simulation techniques be used to incorporate uncertainty?

Administrative data were obtained from an Australian and a New Zealand hospital containing patient date of admission and discharge. The occupancy data for a large period (e.g., a year) were summarised using the average and standard deviation. The number of data points was reduced to a matrix of 2 (average and standard deviation) by approximately 100 (maximum time since admission) or 200 data points. Compartmental models were fitted to the data with the number of compartments varying between one and seven. The method of maximum likelihood was used to optimize the fit. The Bayesian information criterion provided information about the level of fit and complexity; the absolute error only provided information about model fit to the data.

Results

The acute care data from Australia and New Zealand was found to be well described by the compartmental model. The question of model complexity was considered and when the issue of the value of additional information about long-stay patients was taken into account, a two-compartment model of occupancy was found to describe the data sufficiently well.

Harrison and Millard's model was further developed to include other variables such as patient age, seasonality, and vacancy and to enable the application of simulation. Figure 2 summarises these developments.

Discussion

There is a continuum upon which decision making occurs. Strategic decision making is concerned with decisions that will occur in a longer time frame whereas operational decision making relates to decisions that come into effect in a short space of time. Strategic and operational decision making criteria may share some common inputs, but this will not always be the case. The weight placed upon the common factors may be different. Thus, it is likely that the models that can assist decision makers for strategic and tactical decision making will be different.

Compartmental models have the potential to contribute to improve decision making at the strategic level. They have the added advantage that they are easily interpreted by clinicians and decision makers.

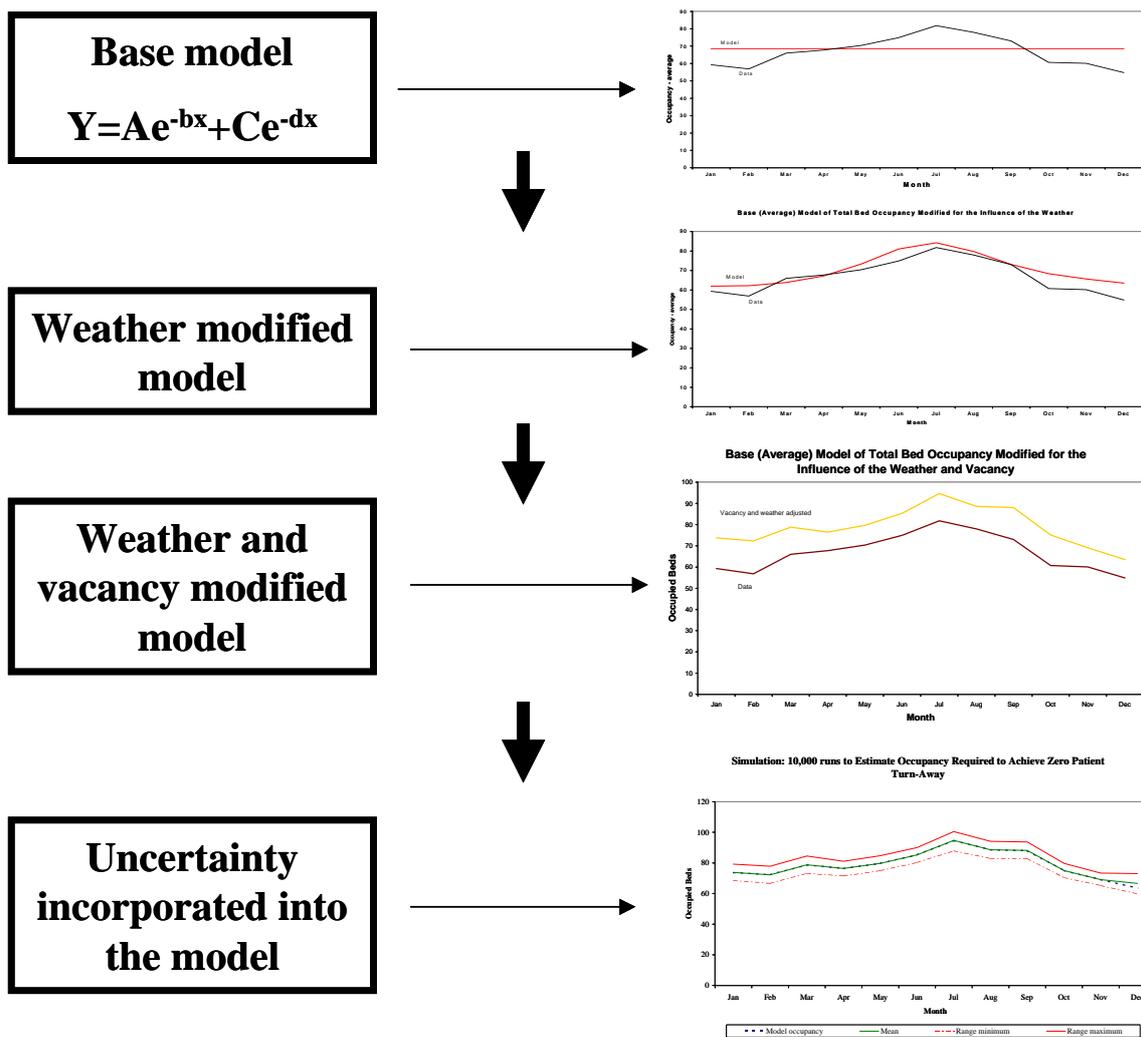


Figure 2: Development of the original deterministic compartmental model to incorporate seasonal variation and variability of patient arrivals.

4. SCREENING FOR CERVICAL CANCER

Most developed countries to establish some form of publicly funded health care system. A common problem in such systems is that there is more demand for health care than could possibly be funded from the public purse, and so choices have to be made about what interventions are provided to which patients. Given the need to distribute scarce resources, publicly funded health care systems generally have an objective of maximising the health benefits produced by the system (efficiency), subject to consideration of the equitable provision of health care.

Economic evaluation or cost-effectiveness analysis is a technique that estimates the costs and benefits of alternative health care interventions to inform the efficient allocation of resources. Individual economic evaluations involve comparisons of alternative interventions for the same class of patients, for example, patients with early breast cancer. The results of individual evaluations are compared to determine the best use of resources across diseases.

A simplified example of the resource allocation process is provided by a health care system in which there are two diseases, with two treatment options for each disease. The respective costs and benefits (represented as units of health gain) are presented in Table 1. Treatments are not divisible, so that all patients in each disease group receive the same treatment. If the budget is \$15,000, it is clear that health gains are maximised by allocating resources to Treatments 1A and 2B (17 units of health gain compared to 14). More generally,

we can assess the relative value of the two more effective treatments (1B and 2B) by comparing their incremental cost-effectiveness ratios (ICERs) where *ICER* is defined by:

$$ICER = \frac{Cost_B - Cost_A}{Health.Gain_B - Health.Gain_A}$$

In the example, the move from treatment 2A to 2B gains units of health at a cost of \$500, whilst 1B gains units at a cost of \$800, so 2B is more cost-effective than 1B. In a real health care system, where there are many thousands of separate conditions to be treated, it is not possible to list the costs and benefits of all possible treatments. Instead, we define a threshold value for the ICER that is assumed to represent the ICER of treatments that would be displaced in order to free-up funds to provide the treatment being evaluated. If the ICER of the treatment being evaluated is under the threshold value, then it is defined as being cost-effective.

In order to provide a common basis for comparisons between treatment areas it is necessary to estimate differences in costs and benefits in a similar manner across evaluations. In practice, this means using a generic measure of outcome (quality adjusted life years (QALYs)) and estimating costs and benefits over the remaining lifetime of the patient population. QALYs are estimated by applying utility weights to patients' survival that represents their health-related quality of life, where 0 and 1 are equivalent to death and perfect health, respectively. As an example, a patient living for 5 years in a health state equivalent to 80% as good as perfect health, followed by 5 years in a state 60% as good as perfect health is alive for 10 years, but gains 7 QALYs.

Clinical effectiveness is commonly established via randomised controlled clinical trials, but such studies are subject to limited follow-up periods that generally preclude the estimation of lifetime costs and benefits. In other cases it is either not practical or ethical to undertake clinical trials. In both cases, it is necessary for economic evaluations to synthesise data to predict the long terms costs and consequences of health care interventions (Briggs *et al.* (2006)). The most common framework for such syntheses is a Markov model, though simulation techniques are also used for more complex scenarios.

The model describes disease progression in a cohort of patients, represented as transitions between discrete health states from the point at which treatment is implemented. Cost and utility weights are applied to each health state, and summed over the time horizon of the model to estimate total costs and QALYs. The impact of treatment is described in terms of how it alters the pathway of progression between the defined health states.

The following section provides a brief description of an economic evaluation of screening for cervical cancer, which informed the allocation of resources at a national level in the United Kingdom. Full details of this evaluation are in a published monograph by Karnon *et al.* (2004).

Model development

The research question was “What is the effectiveness and cost-effectiveness of liquid-based cytology for cervical screening compared with conventional smear testing?”

A Markov process was used to describe the life experience of a cohort of women followed from age 18 to 95 years, with respect to the incidence and progression of cervical cancer. Pre-invasive cancer was defined as cervical intraepithelial neoplasia (CIN), which may be of three levels - CIN1, CIN2, and CIN3. In the absence of intervention, the disease is assumed to progress through each pre-invasive stage and from CIN3 to invasive cancer, with the proviso that regression to a disease-free state may occur from CIN1 only. There is some evidence that the higher grades of CIN may also regress (Sherlaw-Johnson *et al.* (1994)), and this possibility was explored in sensitivity analyses.

The model calculates state transitions at intervals of six months, which were informed by an earlier stochastic modelling study that had calibrated transition probabilities to observed estimates of the incidence of invasive cervical cancer (Sherlaw-Johnson *et al.* (1994)). Within any six-month interval, CIN lesions may progress to the next immediate state, though a proportion of fast growing CIN1 lesions may directly progress to CIN3. Age-specific incidence rates for CIN1 were applied, but disease progression was assumed to be non-age-specific.

A constant risk was assumed for mortality from invasive cancer, based on an average life expectancy with invasive cancer in an unscreened population of approximately 10 years (Obralic *et al.* (1997)) and a mean duration pre-diagnosis of approximately 5 years. The model incorporated age-specific other cause mortality

using life tables produced by the Government Actuary's Department (Government Statistical Service Review of the Registrar General).

Upon the confirmed detection of a lesion (either through clinical presentation or via the screening program), treatment can be initiated with the intention of removing pre-invasive lesions or slowing the rate of progress of invasive lesions. Screening can be implemented at different time intervals, and the alternative screening tests have different levels of accuracy for correctly detecting women with a cervical lesion (sensitivity) and identifying women with no lesions (specificity). Thus, improvements in test sensitivity increase detection rates, leading to earlier treatment, reduced disease progression, and improved survival and quality of life.

The uptake rate of a screening programme was informed by a pilot screening study, and it was assumed that women either attend screening at regular intervals or not at all. Both screening procedures produce a proportion of inadequate screens that require rescreening, which have cost consequences. Confirmation of positive screens involves a colposcopy, which was estimated to be 100% sensitive and specific. It was further assumed that all abnormalities found at colposcopy are treated.

Total direct costs of screening, diagnosis and treatment are included within the model and estimated from the following unit costs: conventional smear test, liquid-based cytology techniques, colposcopies, treatment of pre-invasive lesions, and treatment of invasive cancer.

Model validation

Reported incidence of invasive cervical cancers across all ages is 12 per 100,000 per annum (Office of National Statistics (1993)) which is comparable with the predicted incidence of 11.64 by the baseline model. The age-specific incidence figures predicted by the current model were compared with those predicted by an alternative model (Myers *et al.* (2000) which showed that the two models predict virtually the same pattern of incidence over a lifetime.

Results

Table 2 presents the cost and benefits (measured in years of life gained) for a cohort of 100,000 women aged 18 years, over their remaining life time. The results show that when the cost-effectiveness ratios are re-estimated to exclude dominated options, screening at a regular interval of three years using liquid based cytology is cost-effective, whilst screening at 2-year intervals approaches a reasonable level of cost-effectiveness.

A wide range of sensitivity analyses were undertaken that investigated the impact of altering the values of incidence rates for CIN1; disease progression rates; screening test sensitivity; rates of inadequate cervical smears; costs of screening; and discount rates. The impact of assuming regression from CIN2 and CIN3 lesions was also tested. Finally, the impact of quality adjusting survival was also tested by applying utility decrements to time spent with invasive cancer, waiting for confirmation of borderline screening tests, and undergoing a colposcopy. Liquid based cytology remained cost-effective over all of these analyses, though the optimal time period between screens varied.

Discussion

The case study model was developed as part of the independent process of health technology assessment that is used to inform resource allocation decisions in the UK. The National Institute for Clinical Excellence (NICE) commissioned our research group at the University of Sheffield to undertake the study, which was then subjected to a process of external peer review. The revised report, National Institute for Clinical Excellence (2000), was then presented to the NICE Technology Appraisal Committee, who "considered that, taking into account a number of factors – including the potential for increased sensitivity, reduction of inadequate smears and probable improvements in laboratory efficiency – the LBC method was likely to be cost effective compared with the Pap smear, despite its higher associated cost" (p. 14). They further stated that the health service should "develop implementation plans for the adoption of LBC as the primary means of collecting and processing samples" (p.16).

This case study provides one example of the many model-based economic evaluations that have been used by NICE and other reimbursement committees, such as the Pharmaceuticals Benefits Advisory Committee in Australia and the Canadian Expert Drug Advisory Committee. Modelling is an integral part of the resource allocation process in these countries. Moreover, the need for such models will expand as policymakers realise that explicit cost-effectiveness analyses should inform disinvestment decisions, as well as investment decisions.

In addition, the increased interest in public health interventions that aim to prevent, rather than cure, disease will require the development of new and more complex cost-effectiveness models. An example is an obesity model that we are currently developing that will link risk factors to obesity to the health consequences of obesity at an individual level, and then combine individuals to describe the population effects of obesity, and the cost-effectiveness of population-based interventions to combat obesity.

The future of health care is expensive, and will require tough decisions. Mathematical and simulation modelling are important tools that can better inform these decisions.

Treatment options	Total costs	Units of health gain	ICER*
Disease 1			
Treatment 1A	\$1,000	5	
Treatment 1B	\$5,000	10	\$800
Disease 2			
Treatment 2A	\$10,000	4	
Treatment 2B	\$14,000	12	\$500

Table 1: Costs and benefits in a two disease health care system

* incremental cost-effectiveness ratio (incremental cost per additional unit of health gained)

		Lifetime cost*	Incremental life years gained*	Incremental life years gained*	Average cost per life year gained†	Incremental cost per life year gained‡	Dominated options excluded¶
No screening		£315,139					
Screening 5 yearly	Conventional	£5,226,157	15,610	15610	-	£315	
	Liquid-based	£5,296,879	407	407	£174	£174	£311
Screening 3 yearly	Conventional	£7,918,977	173	173	£4,643	£15,195	
	Liquid-based	£8,026,471	159	159	£3,789	£677	£8,241
Screening 2 yearly	Conventional	£11,242,895	23	23	£7,896	Dominated	
	Liquid-based	£11,390,271	69	69	£7,418	£2,140	£36,719

Table 2: Cost Per Life Year Gained of Cervical Cancer Screening Interventions (Costs discounted at 6%, life years discounted at 1.5%)

* per 100,000 women (uptake rate 85%).

† Compared to conventional pap smear testing at 5-yearly intervals

‡ Each screening option is compared to next less costly option, e.g. LBC screening every 5 years is compared to conventional pap smear testing every five years, conventional pap smear testing every 5 years is compared to no screening.

¶ Options are extendedly dominated if the following option has a lower incremental cost-effectiveness ratio (i.e. the next option would always be chosen if the dominated option were chosen).

5. AGED CARE

Transition care is a form of care for patients moving between hospital and either home or other suitable accommodation. Consider the case of an elderly woman who has broken her collarbone and received acute care in a hospital. It may be that, when the patient is ready for discharge from the acute care setting, it is not be appropriate for her to return home. She may need some low-intensity care to get back her mobility while

some changes are made to her home. It may be necessary to find alternative accommodation for her in a nursing home or hostel. In such cases, the patient may be eligible for transition care.

The Australian Government has a formal definition of transition care which states, in part: “Transition Care provides short-term support and active management for older people at the interface of the acute/sub-acute and residential aged care sectors.” (Australian Government Department of Health and Ageing (2005), p. 17).

In planning to establish a transition care in a hospital, the manager has to consider the question: How many beds should we provide for the transition care unit? Too many beds will lead to waste of resources, too few beds will not satisfy demand. The manager needs a methodical approach to considering this question. In this case study, we summarise one approach to answering this question using ideas from queueing theory. Further details can be found in Crombie *et al.* (2008).

Each week, a certain number of inpatients require a place in the transition care unit. If a place is available, then the patient is transferred to the unit; otherwise, the patient must wait until a place becomes available. Once in the unit, the patient is cared for until discharge.

This situation fits a classic queueing model. The six basic characteristics of a queueing system can be interpreted as follows (Gross *et al.*, 2008, Chapter 1).

1. If an inpatient requires a place in the unit, then we say that a patient has arrived for service. We assume that the number of patients who arrive for service each week is a random variable. We can go further. In any week, there is a large number (n) of inpatients in the hospital but only a small proportion (p) of them require transition care. Thus, in any week, the number of new arrivals is a random variable which has, approximately, a Poisson distribution with mean $\lambda=np$.
 2. The service time is the length of time, in days, that a patient spends in transition care. We assume that the service times of patients are independent and identically distributed random variables, and that the distribution does not depend on the number of patients waiting. Let μ denote the expected service time.
 3. We assume that a patient is served on a first-come-first-served basis. This assumption is the least reliable of the assumptions made in this model. Clinical staff may apply some sort of priority system based on the needs of patients waiting. The “queue discipline” may vary from hospital to hospital.
 4. The system’s capacity is the number of inpatients who are waiting for a place in transition care plus the number being served.
 5. The number of service channels in the system is the number of places or beds in the transition care unit. The transition care unit can serve many patients simultaneously. We will denote the number of service channels by c .
 6. There is only one stage of service in the model, namely, caring for a patient in the transition care unit.
- A knowledge of these characteristics is important for being able to analyse an application of a queueing model.

In light of the potential complexity associated with these six parameters of a queueing model, the following result is remarkable in its simplicity. Suppose that $c=\infty$. This means that the transition care unit has infinitely many beds and there would be no waiting. At any time, t , the number of patients in the system would be the number of patients in the transition care unit. Let us denote this random variable by $N(t)$. Then we have the following result (Gross *et al.* (2008)):

$$\lim_{t \rightarrow \infty} P(N(t) = k) = \frac{\exp(-\lambda/\mu)(\lambda/\mu)^k}{k!}, (k = 0, 1, \dots).$$

In other words, in the long term, the random variable $N(t)$ has a Poisson distribution with mean λ/μ . The remarkable aspect of this result is that it does not depend on the distribution of service time—and it is usually difficult to estimate this distribution in health care applications.

A manager can use this result as follows. Suppose that we know λ and μ . If the transition care has been established for some time (t is large), then we can find the approximate distribution of $N(t)$. Suppose that we find that $P(N(t) \leq 15) \approx 0.90$. Then we know that 90% of the time, 15 beds will suffice and, in 10% of weeks will there be no room for new patients to enter transition care. By experimenting with different values of λ and μ , the manager can use these sort of results to explore options and eventually decide on an appropriate number of beds. Although the approach does not give the manager one specific answer, it does allow the hospital to get a feel for the issues.

This case study leads to interesting problems. When is t large enough to apply this limiting result? Could one create a more realistic model by adapting the result to deal with a priority queue (Jaiswal 1968)? How can the

manager apply this result in planning the introduction of transition without any historical data to indicate the values of λ and μ ? This point is discussed in Crombie *et al.* (2008).

This general approach could be applied in many different situations in planning for resources for community services.

6. CONCLUSIONS

Many aspects of the management of health care systems are quantitative. The examples above are drawn from systems that are hospital based, state-wide, or national. In each case, the mathematical researcher has been able to add to the expertise of the clinicians and policy makers. They illustrate that modelling and simulation can play an important role in improving health care systems at all levels.

Many ideas connected with modelling in health care can be applied to other community services. Housing, the justice system, and child care are just three examples where one finds congestion, or bottle-necks, or waiting lists, or innovation, or data. Their presence suggests opportunities for modelling and simulation.

Often, when one encounters a modelling problem in health care, there are corresponding problems associated with information systems. For example, alongside the issue of modelling patient flow in a hospital are the many, large problems associated with capturing data that can be used in modelling and planning.

How can we increase the use of modelling in the health sector? Alliance with powerful champions who can promote the use of modelling within the sector may be a short-term mechanism to overcoming barriers.

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