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Integrated urban drainage modeling: simplified versus detailed modeling approach for receiving water quality assessment

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Abstract: Nowadays, urban water quality management often requires more use of numerical models allowing for the evaluation of the cause-effective relationship between the input(s) (i.e. rainfall, pollutant concentrations on catchment surface and in sewer system) and the resulting water quality response.

The conventional approach to design and manage the system (i.e. sewer system, wastewater treatment plant and receiving water body), considering each component separately, does not enable the optimization of the whole system. However, due to recent gains in the understanding and modeling of the system, it is now possible to represent the system as a whole and to optimize its overall performance (Butler and Schutze, 2005). Indeed, integrated urban drainage modeling is of growing interest due to the need to dispose tools able to cope with the Water Framework Directive (WFD) requirements.

Despite the benefits that can be gained from a holistic approach there are some shortcomings that can hamper an effective application of such approach. One critical aspect of urban drainage integrated modeling is the computational time required by models for modeling of the whole system. Especially for detailed models, the whole computational time of the integrated model may be prohibitive for carrying out long term simulations of the entire system. More specifically, the bottle-neck from the computational point of view is the hydraulic equations, which describe flow propagation in sewer pipes and rivers, i.e. the de Saint-Venant equations (Meirlaen et al., 2002). Because such equations are non-linear partial differential equations, the solutions are often time consuming. These equations require complex numerical algorithms to solve, making the models slow and thus difficult to use for optimization studies. To cope with such problem, Meirlaen et al. (2002) suggested to use simplified models such as reservoir models that are characterized by reduced computational time compared to detailed models. However, an increase of the integrated model uncertainty may arise due to an over-simplification of the model approach.

In order to gain insight to the above problem, two different modeling approaches have been compared with respect to their uncertainty. In particular, the first urban drainage integrated model approach uses the de Saint-Venant equations and the second model consists on the simplified reservoir model approach. The analysis has been carried out employing a parsimonious home-made model developed in previous studies (Mannina et al., 2005). For the uncertainty analysis, the Generalized Likelyhood Uncertainty Estimation (GLUE) procedure of Beven and Binley (1992) was used, which requires a large number of Monte Carlo simulations where the random sampling of individual parameters from probability distributions is used to determine a set of parameter values. Following this approach, model reliability can be evaluated on the basis of their capacity of globally limiting the uncertainty. The models have been applied to an experimental catchment in Bologna (Italy) where quantity and quality data were available.

The results show that both models have a good capability to fit the experimental data giving the impression that all adopted approaches are equivalent both for the quantity and for the quality. The detailed model approach is more robust and presents less uncertainty in terms of uncertainty bands. On the other hand, simplified river water quality model approach behaves higher uncertainty and may be unsuitable for Receiving Water Body (RWB) quality state assessment. However, the model approach accuracy level is strictly connected with the research goal. Therefore, simplified model approach can be suitable to fulfill the study needs although less accurate in terms of uncertainty than the detailed model.

Keywords: *Monte Carlo simulation; sensitivity analysis; calibration; parameter estimation*

1. INTRODUCTION

The increasing sensitivity towards water quality environmental issues led to the setting up of water quality integrated approaches and the definition of water quality criteria that better represent the receiving water body quality status. More specifically, compared to the past, the tendency today is to design and manage the whole integrated urban drainage system, i.e. Sewer System (SS), Wastewater Treatment Plant (WWTP) and Receiving Water Body (RWB), considering each component not separately but jointly. This integrated approach is implicitly present in the Water Framework Directive (WFD) that introduces the stream-standard approach to river water quality analysis in contrast to the old emission-standard, and it enhances the importance of integrated design and management of urban drainage systems. The goal set in the directive is to reach a good ecological status of all water bodies throughout the catchment, rather than prescribing certain design rules specific to individual areas.

In order to put the integrated planning approach into practice, the engineer requires modelling tools for his work that allow to rebuild the cause-effect relationship between the input(s) (i.e. rainfall, pollutant concentrations on catchment surface and in sewer system) and the resulting water quality response. Today, several models are available to simulate single parts of the urban drainage system, but only a few of them can be adopted as reliable tools for integrated water-quality management. Therefore, one of the greatest challenges faced by researchers dealing with integrated modeling is the interconnection of these models and the definition of a full spectrum of modeling approaches that can suit the demands of specific applications.

Two different kinds of approaches are more often adopted: physically based detailed models and simplified conceptual models. Physically based models simulate the system by using algorithms which parameters have a clear physical meaning representing a specific characteristic of the simulated system (Freni et al., 2008). Conceptual models use simplified algorithms and their parameters do not necessarily have correlation with the real simulated systems. As a consequence of this simplification, different physical–chemical phenomena that take place during the pollution generation and its propagation are considered in an aggregated way. A simplified approach focuses on a reduced number of processes for which reliable information is more frequently available but, on the other hand, the processes parameters are more site specific and models need strong calibration. However, these latter models show the advantage of shorter calculation time, which may constitute an incentive when long-term simulation is necessary. Indeed, especially for detailed models the whole computational time of the integrated model may be prohibitive for carrying out long term simulations of the whole system. More specifically, the weak point from the computational point of view is the hydraulic equations, which describe flow propagation in sewer pipes and rivers, i.e. the de Saint-Venant equations (Meirlaen et al., 2002). Indeed, due to the fact that such equations are non-linear partial differential equations, the solutions are often time consuming, requiring complex numerical algorithms to solve, thus difficult to use for optimization studies. To cope with such problem, Meirlaen et al. (2002) suggested use of simplified models such as reservoir models that are characterized by reduced computational time compared to detailed ones. However, an increase of the integrated model uncertainty may arise due to an over-simplification of the model approach. Uncertainty of a model is stated by giving a range (or band) of values that are likely to embrace the true value of a specific simulated variable: stricter uncertainty bands demonstrate lower uncertainty, while larger bands are caused by highly uncertain models. Using the concept of uncertainty, the ‘‘better’’ model is the one able to correctly simulate a specific variable minimizing the width of uncertainty bands. Three main uncertainty sources are generally classified: uncertainty of the model input variables (input uncertainty), uncertainty of the model parameters values (parameter uncertainty) and uncertainty originating from the imperfect description of the physical reality by a limited number of mathematical relations (model structure uncertainty). Concerning the balance between sensitivity and model complexity, recently Lindenschmidt (2006), computed the error and the sensitivity for the river water quality modeling considering different complexities, and confirmed the hypothesis formulated by Snowling and Kramer (2001) stating that as a model becomes more complex in terms of increased number of parameters and variable, the error between simulations and measurements decreases and the overall model sensitivity increases. The aforementioned hypothesis has been tested using several types of models: river water quality (Lindenschmidt, 2006), transport in groundwater (Snowling and Kramer, 2001) and heavy metal transport in lotic waters of different scale (Lindenschmidt and Hesse, 2005).

In order to gain insights to the above problem, two different river water quality modeling approaches for the simulation of the RWB have been compared with respect to their uncertainty. The two approaches, addressed in the following as detailed and simplified river water quality modeling approach, respectively, Detailed River Water Quality (DRWQ) and Simplified River Water Quality (SRWQ), have been incorporated in an integrated homemade urban drainage model developed in previous studies (Mannina et al., 2005). The DRWQ approach is based on the de Saint-Venant equations for the quantity aspects and on the advection-dispersion equations for the quality ones. On the other hand, SRWQ approach is based on the reservoir modeling concept both for the quantity and for the quality aspects. For the uncertainty analysis, the Generalized Likelyhood Uncertainty Estimation (GLUE) procedure proposed by Beven and Binley (1992) has been employed. The GLUE procedure requires a large number of Monte Carlo simulations where the random sampling of individual parameters from probability distributions is used to determine a set of parameter values. Following this approach, model reliability was evaluated on the basis of their capacity of globally reducing the uncertainty (Beven and Binley, 1992). The models have been applied to an experimental catchment in Bologna (Italy) where quantity and quality data were available.

2. THE URBAN DRAINAGE INTEGRATED MODEL

In the present study, as discussed in the introduction, two river water quality modeling approaches for simulating RWB behavior have been compared with different complexity levels. Those approaches have been incorporated in an integrated homemade urban drainage model developed in previous studies (Mannina 2005). For the sake of conciseness, only the model structure will be discussed next, referring to earlier publications of the authors for further details (Mannina et al. 2004; Mannina, 2005). The model is able to estimate both the interactions between the different systems (SS, WWTP and RWB) and the modifications, in terms of quality, that urban stormwater causes inside the RWB. Such integrated SS-WWTP-RWB system is made up mainly of three sub-models (Figure 1):

- the rainfall-runoff and flow propagation sub-model, which is able to evaluate the quality - quantity features of SS outflows and simulates ancillary structures such as Combined Sewer Overflow (CSO) device and storm water tanks (SWT);
- the WWTP sub-model, which is representative of the treatment processes;
- the RWB sub-model that simulates the pollution transformations inside the river.

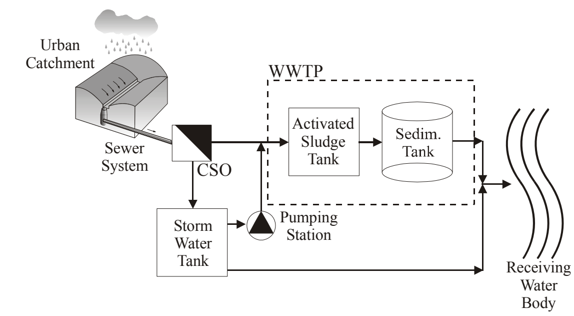

Figure 1. Schematic representation of the integrated urban drainage system.

The first sub-model, reproducing the physical phenomena which take place both in the catchments and in the sewers, allows determination of the hydrographs and pollutographs in the sewers. This sub-model is divided into two connected parts: a hydrological - hydraulic module, which calculates the hydrographs at the inlet and at the outlet of the sewer system, and a water quality module, which calculates the pollutographs at the outlet for three pollutants (TSS, BOD and COD). The hydrological - hydraulic module starts to evaluate the net rainfall, from the measured hyetograph, by a loss function (taking into account surface storage and soil infiltration). From the net rainfall, the model simulates the net rainfall-runoff transformation process and the flow propagation with a cascade of one linear reservoir and a linear channel (representing the catchment) and a linear reservoir (representing the sewer network). This simplified approach provided good results in several applications even when compared with more detailed approaches (Mannina et al. 2004; Mannina, 2005; Freni et al., 2008). The solid transfer module reproduces the build-up and wash-off of pollutants from the catchment and the propagation of solids in the sewer network considering also their sedimentation and re-suspension. CSO structures are simulated by means of the continuity equation and a rating curve equation describing the hydraulic behavior of the overflow (Mannina, 2005). The second sub-model is aimed at the analysis of WWTP during both dry and wet weather periods. The WWTP sub-model simulates the behavior of the part of the plant composed by an activated sludge tank and a secondary sedimentation tank. For the activated sludge tank model, mass balance equations derived from Monod's theory have been used in order to reproduce pollutant (BOD, COD, TSS) removal (Metcalf and Eddy, 1991). The sedimentation tank was simulated using the solid flux theory and the settling velocity function according to Takács et al. (1991). In particular, the solids concentration profile has been simulated by dividing the settler into a number horizontal layers. Within each layer the concentration is assumed to be constant and the dynamic update is performed by imposing a mass balance for each layer. The third sub-model examines the assessment of RWB. As aforementioned, two river water quality modeling approaches have been incorporated in the integrated home-made urban drainage model: a detailed model approach and a simplified one. The former is based on the completed form of the de Saint-Venant equation for the propagation of the flow along the river (quantity module) and the advection- dispersion equation for the assessment of the pollutant loads (BOD, O_2, NO_x) (quality module). The SRWQ is based on the reservoir modeling concept. As the detailed approach, the SRWQ is divided in two sub-modules: a quality and a quantity module. For the sake of conciseness, the descriptions of the model algorithms are remanded to literature (see, Mannina, 2005).

3. UNCERTAINTY ANALYSIS: THE GLUE METHODOLOGY

GLUE methodology has been used for the study purpose (Beven and Binley, 1992). Parameter sets with poor likelihood weights are classified as non-behavioural and they can be rejected. All other weights from behavioural or acceptable runs are retained and re-scaled so that their cumulative total sums is equal to 1. The GLUE procedure thus transforms the problem of searching for an optimum parameter set into a search for sets of parameter values that give reliable simulations. Following this approach, there is no requirement to minimize (or maximize) any objective function, but information about the performance of different parameter sets can be derived from some index of goodness-of-fit (likelihood measure). GLUE approach relies on the concept of equifinality, which maintains that, due to the errors inherent in the model structure, (e.g. due to simplification and aggregation) errors in observed data and the difficulty in determining an exact error model, it is inappropriate to perform calibration based on an optimum set of parameters. As likelihood measure, the Nash and Sutcliffe efficiency index (1970) has been used in the present study. Like other likelihood measures, the Nash – Sutcliffe index is equal or lower than zero for all simulations that are considered to exhibit behaviour dissimilar to the system under study, and it increases monotonically as the similarity in behaviour increases with a limit value equal to 1. Once defined a likelihood index, the likelihood value associated with a set of parameter values may be treated as a fuzzy measure that reflects the degree of belief of the modeller in that set of parameter values as a simulator of the system. The degree of confidence is derived from the predicted variables arising from that set of parameter values. Treating the distribution of likelihood values as a probabilistic weighting function for the predicted variables, an assessment of the uncertainty associated with the predictions is done, based on the definition of the likelihood function, input data and model structure used. A method of deriving predictive uncertainty bounds using the likelihood weights from the behavioural simulations has been shown by Beven and Binley (1992). The uncertainty bounds are calculated using the 5% and 95% percentiles of the predicted output likelihood weighted distribution. In the specific study, uncertainty connected with both quantitative and qualitative objective functions has been analysed and they will be described in the following paragraphs.

4. THE CASE STUDY

The integrated model and the uncertainty analysis have been applied to the catchment of the Savena river (Italy). The sewer system and the river studied in this work concern a part of the sewer network of Bologna, studied within the European Union research project INNOVATION 10340I (Artina et al., 1999). The studied river reach is about 6 km long and it receives discharge from 6 CSOs deriving from the Bologna sewer network. The sewer network is a part of the combined system serving the whole city of Bologna, which can be considered as hydraulically divided into many independent catchments, all connected to a WWTP. The part of Bologna connected to the studied river has an area of more than 450 ha, with an impervious percentage of about 66% and about 60,000 inhabitants. During experimental survey, carried out within the INNOVATION European Research Project, from December 1997 to July 1999, about 50 events have been recorded, but, for only 5 of these, water quality aspects have been analyzed regarding both RWB and SS. The monitoring infrastructure consisted of 3 raingauges, 8 sonic level gauges and 6 automatic 24-bottles sampler (3 in the sewer system and 3 in the river). The study has been focused on BOD_5, TSS, COD and DO, even if analogous considerations may be extended to other parameters. In this study, only a part of the Savena River has been simulated (400 meters downstream the CSO No. 6) because the contribution of this particular CSO to river pollution has been determined significant compared to all the others. The contribution of other polluting sources has been considered by monitoring river pollution load in the first cross-section upstream of CSO No. 6 and introducing this information as input in the models. Savena is an ephemeral river since there

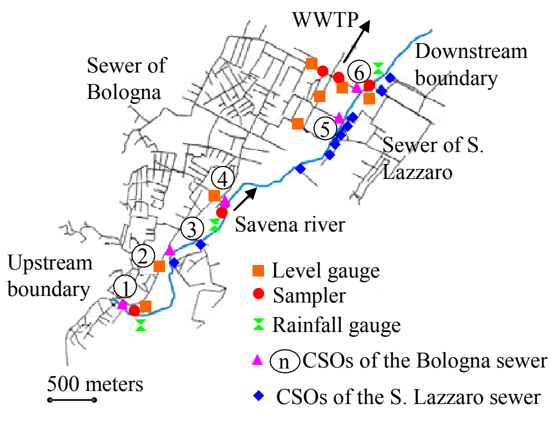

Figure 2. Savena catchment.

are wide flow variations during the different seasons and the river base flow is comparable with the CSO discharge. Further details on the monitoring campaign can be found in Artina et al. (1999).

5. RESULTS

As discussed above, two river water quality modeling approaches have been tested by incorporating them into an integrated home-made urban drainage model set in order to evaluate their uncertainty. This analysis helps gain insight about the level of accuracy that has to be provided for a correct assessment of the quality state of ephemeral rivers. In the following, for example, considerations will be based on graphs obtained for the rainfall event of 28 November 1998; similar behavior has been obtained for all the simulated events. The selected event is characterized by an ADWP of 3.8 days and rainfall duration of approximately 200 min. In order to compare the two different river quality modeling approach, 10,000 Monte Carlo simulations have been run for each approach varying both quantity and quality parameters of the RWB submodels. Conversely, the parameters of the other submodels (SS and WWTP) have been kept constant in order to focus only on the RWB modeling. Thereafter the uncertainty bands have been assessed for both modeling approaches enabling to highlight the differences among them. In particular, by means of the Monte Carlo simulations each parameter value has been drawn from ranges obtained by the calibration of the 5 fully monitored events. The model has been calibrated over single events: upstream sub-models parameters have been calibrated first and then kept constant during the calibration of downstream ones. Analogously, water quantity modules have been calibrated first and then kept constant during the calibration of the quality ones. The system geometry has been considered known and unaffected by errors. Parameter variation ranges used for the uncertainty analysis have been reported in Table 1. In order to better pin down the most sensitive model parameters a preliminary sensitivity analysis was performed. This analysis has been carried out generating 10,000 random sets of parameters considering their distribution uniform, without any prior knowledge about them and using these sets to perform model simulation. For each of these simulations a performance index has been evaluated in the form of Nash and Sutcliffe Efficiency Criterion (1970). Equifinality either indicates prediction insensitivity to parameters or that some parameters are interacting closely in producing behavioral models. In order to detect the above issue, a correlation matrix has been worked out for the model parameters. The correlations between most parameters are somewhat small. The weaker correlations in GLUE also indicate the phenomenon that the real response surface is flattened by GLUE. This is in accordance with other analyses of the GLUE methodology (Mantovan and Todini, 2006).

Table **1**. Parameter ranges of the employed models in the Monte Carlo sampling

	Parameter	Lower limit	Upper limit
SRWQ	Linear channel constant λ [s]	250	2000
	Quantity reservoir constant k [s]	150	300
	Quality reservoir constant k_c [s]	150	300
	Deoxigenation coefficient k_D[s^{-1}]	0.001	0.008
	Reaeration constant k_R[s^{-1}]	0.002	0.009
	Oxygen actual production Ph [$gO_2l^{-1}s^{-1}$]	0.02	0.045
	Respiration [$gO_2l^{-1}s^{-1}$]	0.02	0.045
DRWQ	River bed roughness ks [$m^{1/3}s^{-1}$]	20	100
	Longitudinal dispersion coefficient D_L [m^2 s^{-1}]	0.01	20
	Deoxigenation coefficient k_D [s^{-1}]	0.001	0.01
	Reaeration constant k_R [s^{-1}]	0.003	0.2
	Oxygen actual production Ph [$gO_2l^{-1}s^{-1}$]	0.02	0.045
	Respiration r [$gO_2l^{-1}s^{-1}$]	0.02	0.045

Figure 3 shows scatter plots for the likelihood (L) based on Nash and Sutcliffe for selected parameters sampled both for the DRWQ and for the SRWQ. Each dot represents one run of the model with different randomly chosen parameter values within the ranges of Table 1. The generation of the likelihood surface involves a decision about the criterion for model rejection; actually the uncertainty bounds associated with the retained simulations will depend on the choice of the likelihood measure and rejection criterion. Particularly, simulations that achieve a likelihood value less than zero are rejected as non-behavioral. The remaining simulations are rescaled between 0 to 1 in order to calculate the cumulative distribution of the predictive variables. The most sensitive parameters of the detailed modeling approach regards the ones connected to the processes of deoxigenation and reareation. Indeed, Figure 3c shows a strong sensitivity of the oxygen to the reaeration coefficient (K_R). Conversely, the processes which are related to the oxygen contribution due to photosynthesis phenomenon are less sensitive (Figure 3f). Such results are in agreement with the physics of the phenomenon. Indeed, during storm events, especially for an ephemeral river such as the Savena, the largest contribution of oxygen comes from the reaeration with the atmosphere due to the intense flow turbulence. This aspect is reflected in terms of reaeration coefficient values. In particular, these latter are some order of magnitude higher respect to the dry weather ones. This aspect confirms the important role played by the flow turbulence during storm weather. The intense turbulence is obviously caused by the high increment of the river flow respect to the dry weather flow. Such increment can be of some order of magnitude and it is due, especially for the ephemeral river, to the intermittent discharges coming from the urban sewer systems i.e. the CSOs. This is the case also of the selected case study (Figure 4) where the RWB flow rate during the wet weather rise up to 0.3 m^3/s so becoming approximately an order of magnitude higher respect to the dry weather one (0.02 m^3/s). Furthermore, these variations between dry and wet weather period require the recurrence to model approaches that employ dynamic models both for the quantity and for the quality aspects. These dynamic models should consider a narrow temporal time scale due to the fact that the involved phenomena occur in a short time (acute pollution, i.e. the effects last for a period comparable to that of the rainfall). Such requirement can not always be fulfilled by the available commercial models that generally consider daily time scale; indeed, lower time scale resolutions are not feasible due to large computational time.

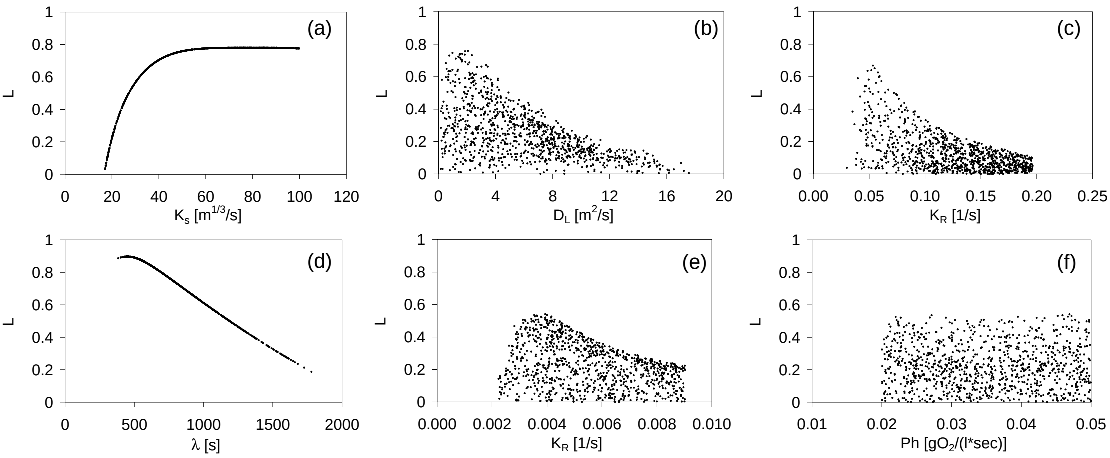

Figure 3. Scatter plots for some parameters of the detailed model approach [(a), (b), (c)] and of the simplified model approach [(d), (e), (f)].

Figure 4 shows the model results in terms of flow and concentrations (BOD and DO) for the two modeling approaches. More specifically, Figures 4a-c regard the detailed approach whereas Figures 4d-e refer to the simplified approach. As can be observed, both models show a good capability to fit the experimental data giving the impression that all adopted approaches are equivalent both for the quantity and for the quality. However, in terms of uncertainty bounds some differences can be addressed. The uncertainty bounds of the detailed model for the quantity module are wider than the correspondent simplified approach ones. On the other hand, the quality uncertainty bounds of the simplified approaches are wider respect to the detailed ones. Furthermore, for the simplified models the simulations of the BOD and DO are poorer. The detailed approach appears to be more robust respect to the simplified one showing generally narrower bounds and small discrepancies with the measured data. However, bearing in mind the uncertainty of the measured data, the simplified approach can be also considered acceptable. Further, the simplified approach is preferable due to the lower computational time (three orders of magnitude faster in respect to the detailed one). In conclusion, both approaches reveal to be suitable for fulfill the research goal, the RWB quality state assessment. The approach choice has to be addressed by the accuracy level required for the research that has to be carried out: less uncertainty and higher accuracy are better addressed employing the detailed approach. However, in case of long term simulations computational time may prevent the application of detailed approach and in such case simplified approach can be a good solution to cope with such a problem.

6. CONCLUSIONS

This study considered the comparison between detailed vs. simplified modeling approaches for the assessment of the RWB quality state considering an integrated modeling approach. More specifically, the complete form of the de Saint-Venant equations along with the advection-dispersion equations have been compared with a simplified reservoir model approach. The comparison has been accomplished in terms of uncertainty analysis. This latter has been assessed by means of the GLUE methodology. The results show that both models have a good capability to fit the experimental data giving the impression that all adopted approaches are equivalent both for the quantity and for the quality. The detailed model approach is more robust and presents less uncertainty in terms of uncertainty bands. On the other hand, simplified river water quality model approach shows higher uncertainty and may be unsuitable for RWB quality state assessment. However, the model approach accuracy level is strictly connected with the research goal. Therefore, simplified model approach can be suitable and fulfill the study needs although less accurate in terms of uncertainty than the detailed one. In other words, the choice of the model approach has to be compared with the research goal. When possible, simplified approaches are preferable to detailed ones since they require reduced amount of model parameters and computational times.

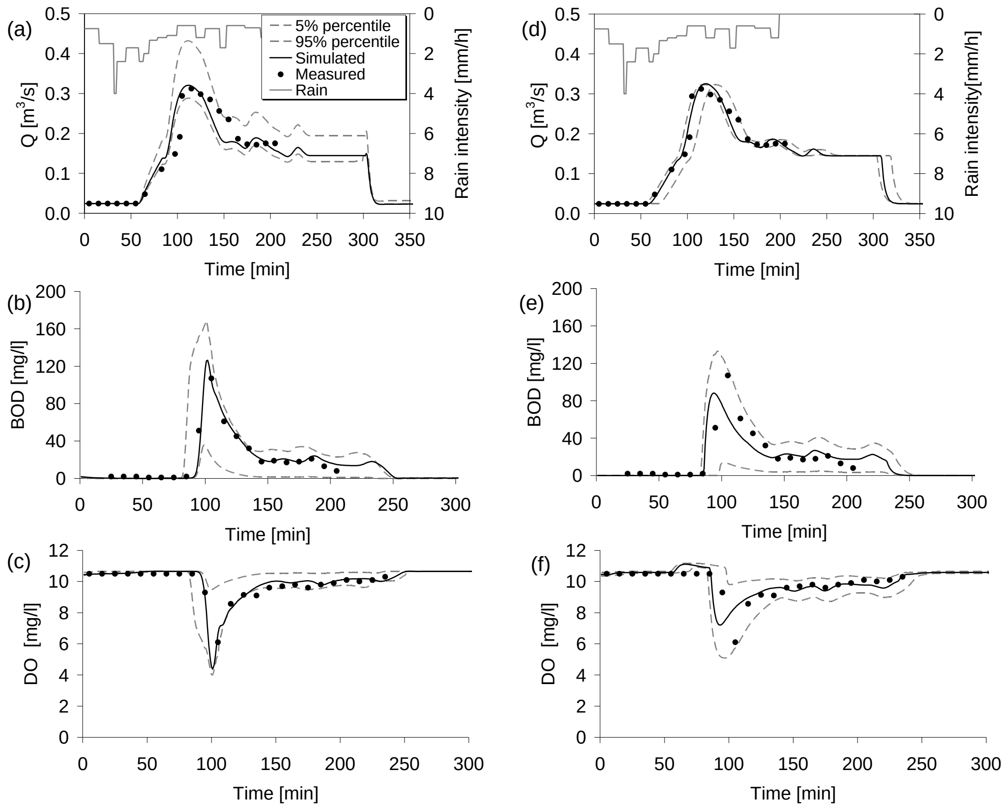

Figure 4 Uncertainty bands in terms of flow rate, BOD, and DO for the detailed model [(a), (b), (c)] and for the simplified model approach [(d), (e), (f)].

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