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Experimental economics and agent-based models

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Abstract:

Validating agent-based models is challenging due to the emergent properties of complex systems. Defining system-level patterns and determining which underlying processes created them is difficult because of the irreducibility of these systems. This paper describes methods to empirically calibrate and validate models, specifically using laboratory experiments with human subjects. Using experimental techniques, well-defined decision situations can be reproduced, and behaviours of human participants revealed. The outcomes of the experiment can be used to calibrate an agent model, offering a way to capture heuristics of decision making and reveal strategies that humans use in dealing with complex situations. This presents an alternative to representing agents as fully informed and rational decision makers, yet offering a defensible alternative that is grounded in empirical data. An example of using this technique is presented for agricultural agents acting in a cap and trade system for water quality management. Human participants are immersed in a running agent model where they control the decisions of one agent. The data revealed by the participants can then be used to calibrate artificial agents. The objective of this paper is to contribute to improved calibration and validation through participatory modelling, and a technique of combining agent-based models with experimental economics is presented as an empirically grounded method of defining agent behaviours.

Keywords: *Validation, Calibration, Laboratory experiments, Participatory modelling*

1. INTRODUCTION

Validating models of complex systems poses unique challenges. The complex nature of agent-based modelling (ABM) and its emergent properties at a system level make validation of models challenging, and some argue inherently impossible due to the irreducibility of emergent properties. This paper overviews issues of empirical calibration and validation of models, and presents a way to address this issue through participatory modelling using experimental economics. Combining experiments and ABM is presented as a way to draw robust behaviours from human subjects, insert them into the agent model, and thereby improve the defensibility of agent representations. The objective of this paper is to contribute to improved calibration and validation through participatory modelling, and a technique of combining agent-based models with experimental economics is presented as an empirically grounded method of defining agent behaviours.

When validating a model, the patterns that arise at the macro-level are comprised of a multitude of underlying agent actions. Identifying the underlying organisation of the system can be difficult, and knowing if the agent behaviours that give rise to systematic patterns are indeed valid representations of real-world behaviour is not a straightforward exercise. As such, an explicit strategy for coping with complexity and uncertainty is not generally used, and model structure is often ad hoc (Grimm 2005, Crooks et al. 2008).

The modeller can represent any number of agent behaviours, control the way agents interact, and define the 'world' in which they function. This flexibility of ABM allows the modeller to use any number of parameters and functions, making it difficult to restrict the ranges of model parameters based on empirical data (Dawid and Fagiolo 2008). Grimm et al. (2005) conclude that based on this, ABMs include too many degrees of freedom, and ABMs may reflect too greatly the perspective of the modellers and users without an understanding of specific interests, beliefs and scales of perception. Coucelis (2001) asks whether the benefits of that flexibility exceed the considerable costs of the added dimensions of complexity, concluding that it likely does not.

Although most ABMs which represent humans in a simulated environment have been inspired by observation of real biological and social systems, many of them have not been rigorously tested using empirical data. In fact, most ABM efforts do not go beyond a "proof of concept" (Janssen and Ostrom 2006). Grimm et al. (2006) reports that no general framework for designing, testing, and analysing bottom-up models has yet been established. Epstein (2006) concludes the field lacks standards for model comparison and replication of results. As a result, a concerted effort has been applied to improve the defensibility of ABM through empirical validation of models, with examples in Marks (2008), Troitzsch (2004), Brown et al. (2005), and notably Robinson et al. (2007), Matthews et al. (2007), Windrum et al. (2007), Fagiolo et al. (2007), Moss (2008), and Janssen and Ostrom (2006). The following section provides an overview of calibration and validation issues in ABM. An example ABM and experiment is then provided to show how building an experimental platform can assist this process.

1.1. Methods for calibration and validation

Techniques to validate ABMs have begun to receive more attention as the field matures. Some techniques are related to simulation modelling in general, and are applicable as much to other modelling techniques. These represent basic forms of simulation best practice, such as sensitivity and uncertainty analysis as outlined in Railsback et al. (2009).

The challenge with ABMs with their feedbacks and multiple interactions is in validating emergent phenomena. A number of studies suggest using patterns or stylized facts of a system. Grimm (2006) suggests the use of patterns to guide model structure and reduce parameter uncertainty. First, alternate theories of agents' decision are formulated and patterns at both individual and higher levels are identified. Theories are tested by how well they reproduce the patterns, rejecting those that fail to do so. Finally, additional patterns with more falsifiable power can be used to design experiments and analyse data. Similarly, Windrum et al. (2007) suggest 'indirect calibration' where stylised facts are identified, and empirical evidence about behaviour and interactions is gathered which supports the selection of model functions. Model outcomes for stylised facts are compared to the evidence, and parameter sets are limited to those which reproduce the stylised facts. In both these examples, the empirical data can be drawn using a number of methods.

Increasingly, researchers are using multiple methods to calibrate and validate models. These include surveys, semi-structured interviews, existing data sources such as GIS and census data, direct participant observation, role playing games, and laboratory experiments. From the data gained through these media, statistical functions can be derived, and / or decision making rules constructed. Heterogeneous 'types' of agents can be identified from empirical data, as outlined in Valbuena et al. (2008). Reviews of validation techniques for ABM are presented in Robinson et al. (2007), Matthews et al. (2007) , Windrum et al. (2007), Fagiolo et al. (2007), Moss (2008), and Janssen and Ostrom (2006).

Surveys can gather information to derive individual or household behavioural models based on microeconomic theory, or to generate statistical descriptions of the attributes of agents. Brown and Robinson (2006) and Heckbert et al. (2009) use econometric estimates from survey data to design agent preference functions. Survey data can identify types of agents based on cluster analysis, and provide information on the distributions of characteristics, beliefs and preferences within a group. Surveys are good for sampling and extrapolating to the population level. Limitations exist in the ability to capture dynamic decision making, given surveys are a snapshot in time and in a certain context. To learn directly *why* people behave the way they do, semi structured interviews can be used in conjunction with surveys. Participant observation from anthropological techniques (e.g. Huigen 2006) can capture in-depth information as well.

Existing data sets at increasingly finer resolutions are becoming available for use in ABM including GIS, census, and other fine-resolution data which can be used to derive agent-agent and agent-environment relationships. Probability functions and transition rules can be derived for spatial cellular automata, and heuristics and decision rules formulated for agent behaviour. Berger and Schreinemachers (2006) present a straightforward way to parameterise ABMs using a common sampling frame to randomly select observation units for both biophysical measurements and socioeconomic surveys. These are then extrapolated over the landscape based on estimated probability functions, and assigned to the model agents and landscape. The resulting landscape and agent population are statistically consistent with empirical data (Berger and Schreinemachers 2006).

Participatory modelling using ABM has been conducted with success using the companion modelling technique (Com Mod). This allows for stakeholders to explore system interactions in role-playing games laden with context. Real-world stakeholders are the participants, and play their roles while information is gathered to be used in developing the associated ABM (Castella et al. 2007; Barrateau and Abrami 2007; Becu et al. 2008, Guyot and Honiden 2006). The information collected on stakeholder behaviour are evaluated by stakeholders, including post-game interviews and cross-checks, then transformed into rule-based agents in the model. This process can offer system-level awareness building and an opportunity to observe agent-agent interactions, but is limited by issues of objective knowledge of stakeholders. Castella

(2008) provides an overview of collaborative research using participatory methods and geovisualisation tools with ABM and role-playing games to analyse decision-making processes by individual farmers based on the resource profiles of their farms, and institutions which regulate resource access and usage. Also of note is Dray et al. (2006) on the use of knowledge engineering techniques to validate heuristics.

1.2. Laboratory Experiments for calibrating models

Laboratory experiments and ABM each have much to gain through combined use given they both are concerned with decision making at the individual level; the former elicits decisions, the latter represents them explicitly in model code. Firstly, agent behaviours can be calibrated from results of experiments to create a population of simulated agents whose behaviours are consistent with the experiment participants. In this way, experiments can be used to bring empirical data into the ABM from data sets of observed behaviour. Second, experiments can help choose between possible sets of decision making algorithms a modeller may be considering (Epstein 2006). The use of combined ABM and experiments is limited, namely being conducted in Evans et al. (2006) and Duffy and Unver (2006). ABMs have been designed with the results of lab experiments in mind, and experiments can reveal behaviours at an individual level whose inclusion in ABM would have novel applications, particularly in macro economics (Duffy 2006), formation of communication networks (Corbae and Duffy 2008), and the formation of cooperation among humans (Kurzban and Houser 2005). ABMs have much to gain by using these types of experiments to better select agent behaviours, calibrate decision making functions based on revealed behaviours, and validate outcomes of ABMs against laboratory findings.

As opposed to Com Mod, which is context rich, laboratory experiments are generally highly abstract and controlled in order to test specific hypotheses of particular decision making situations. Using laboratory experiments alongside other validation techniques can delve into a specific decision making function of agents, while leaving the surveys and interviews to gather more general information. Duffy (2006) discusses using ABM and experiments as a means of isolating the sources of aggregate phenomena. For example, in Evans et. al (2006) examines reforestation and agricultural land use decisions, and present an excellent example of how experiments can be used to test very specific, but highly important assumptions in ABMs. Using a mixed methods approach, first remote sensing data was used to estimate agent preference parameters by regression, then surveys were used to collect social and demographic information. Only a weak relationship between income and reforestation was found using this approach, and other factors such as learning, information, knowledge, risk aversion, and influence of social networks were hypothesised to play a role, but not able to be captured in surveys (Manson and Evans 2007). Lab experiments were designed to further test hypotheses about decision making and resulting behaviours.

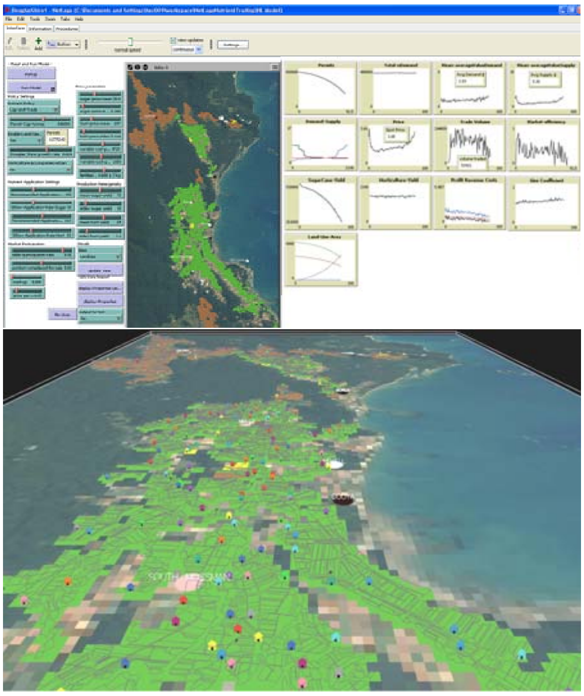

Figure 1: Interactive model interface, map, and graphs tracking simulated model data over time. Properties (depicted as houses) and their paddocks are depicted for the Wet Tropics, Australia.

In the Evans et al. (2006) study, an experiment was devised to assess how people make allocation decisions between agricultural use and reforestation. Subjects were allocated areas to one of two land uses, receiving revenue according to an increasing price for one, and a decreasing price for the other. Experiments found considerable variance in behaviours of allocating land to each of the two uses, and where a ‘rational’ decision maker would have changed land use, the majority of experiment participants took many rounds complete the reallocation, and some persisted in allocating to the disadvantaged option (Manson and Evans (2007). Duffy and Unver (2006) examine a trading market where a modified zero-intelligence agent (Gode and Sunder 1993, McBride 2007) can be used to explore patterns observed in trading laboratory experiments, by generating asset price bubbles and crashes.

2. EXPERIMENTS AND AGENTS: AN APPLICATION IN NATURAL RESOURCES

This section describes an application of using laboratory experiments to calibrate decision making functions in an ABM applied to natural resource management. An ABM was constructed to explore the possibility of managing water quality using a cap and trade system for pollution permits. A full description of the model can be found in Heckbert (2009), but here the focus is rather the calibration technique of using laboratory experiments built to accompany the model outlined in Heckbert (2009). In this study, model results are reported for fully informed rational agents. A method of capturing deviations from this behaviour in experiments with human participants was constructed using an experiment interface which allows participants to login and take control of the decisions of an agent while the model runs. Participants can ‘play’ against a population of artificial rational agents, also with agents who make ‘errors’ in pricing decisions, also versus groups of human participants, or some combination of these. From the data revealed by participants, assumptions regarding agent behaviour can be modified in the original model. By this process, empirical data about how individuals make specific decisions can be brought in to the model, supporting the defensibility of model assumptions. Preliminary results are presented here for trial runs testing the software, and full experimental sessions with students, agronomists, and eventually agriculturalists and policy makers are envisioned.

2.1. Application of Experimental Economics and Agent-based Modelling: Water quality in coastal Australia

Water quality in the Wet Tropics of Queensland Australia has had increasing attention due to potential impacts of agriculture run-off of nutrients and sediments into waters of the Great Barrier Reef World Heritage Area. The Reef Water Quality Protection Plan (AG 2003) sets out goals to manage water quality from land-based pollutants entering coastal waters. Included in a number of possible resource management tools is the use of market-based instruments to regulate agricultural run-off. One possibility is a cap-and-trade system of fertilizer permits, however there is not enough known about how a system like this might operate, specifically how people will participate under different instrument designs. The decision making of individuals is key to making such an instrument function properly.

To explore the performance of a cap-and-trade system, an agent-based model was constructed, and is presented in Heckbert (2009). Agents within the model represent properties as farm households, and are comprised of a number of GIS-based paddocks. The model interface is presented in Figure 1, with interactive map, buttons and sliders to control model functions, and output graphs tracking model data over time. The interface can be constructed alongside policy makers, and used online by stakeholders to facilitate discussions and explore scenarios.

Agents in this model perform a number of land use decision making operations throughout the simulated agricultural production year. Agents face constrained fertilizer application rates through the setting of a cap-and-trade system for fertilizer permits, and can trade fertilizer permits by deciding on price-quantity bundles to enter the fertilizer trading market. A call market structure determines which bids and asks are successful. The outcomes for the fertilizer permit market are estimated for various aggregate fertilizer cap levels within river catchments.

Research questions of interest here pertain to the functioning of the market; how do farm profitability and the distribution of profits within the agent population respond to market configurations? What are modeled fertilizer trading prices, what volume is traded, and how does market efficiency change over time? These outcomes depend on individual decision making, and assumptions about agent rationality, heterogeneity, and pricing strategies all have an impact on how well the market functions.

2.2. Agents versus Humans

Assumptions about agents making rational and fully-informed decisions about price-quantity bundles to trade obviously does not reflect reality, where decisions are influenced by many other factors. Trading fertiliser permit market might be altered through inexperience with trading, lack of knowledge about one's own farm productivity and the effect of fertiliser use on crop yield, or aversions to trading and regulation that might hinder a grower from participating in trading. There are a number of ways to capture further information, as discussed in Section 1 including surveys, interviews, and findings from other studies. However, there are very few examples of water quality trading models internationally, and learnings therein are limited for the conditions of the Wet Tropics. Also, as a 'new' market, growers do not have experience in operating in such a market, so surveys and interviews could not provide sufficient data on this decision. The critical decision process to further explore is the creation of price-quantity bundles for trade. To gather empirical data on how this decision is made, an economic experiment was created that uses the ABM as the experiment platform. Participants login to the model, replacing one agent, and proceed to make the trading decisions for the agent as the model runs (waiting for participant decisions). Figure 2 shows a participant within the model (a participant 'farmer' replacing an artificial agent 'house') and the interface the participants use to make price-quantity decisions. Participants see information about their farm including current bank balance, productivity, current permits and trading price of permits. Participants have a marginal value table (right side of interface) which provides the participant information about marginal values of different size bundles of fertiliser. Lastly, sliders are moved to select the price and quantity of trading fertiliser permits. Data from respondents is collected and outputted to spreadsheets for further analysis.

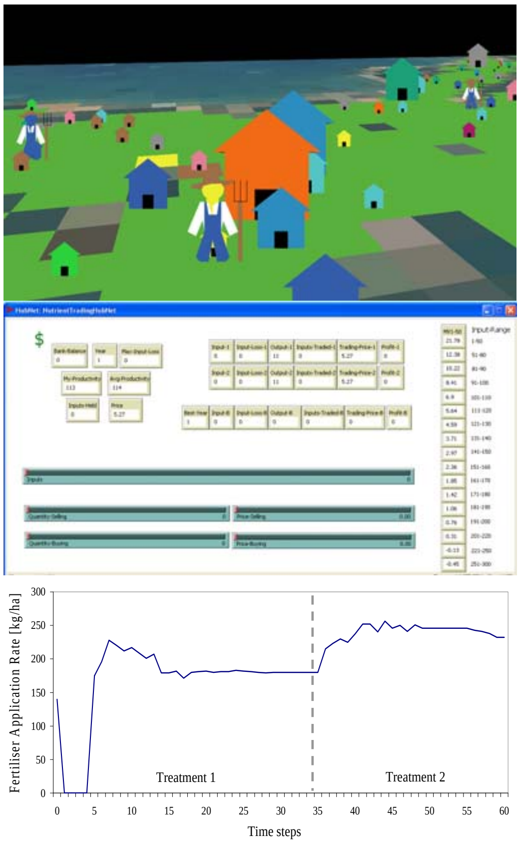

Figure 2: User interface for participants, showing a 'farmer' replacing an artificial agent (houses). The experimental interface provides information about the user's property, productivity, and a history of profitability based on past decisions. Participants move sliders to select price-quantity bundles for trade. The bottom figure shows respondent data on fertiliser application rate increasing in response to a stochastic 'rainfall' event, a strategy observed

Three experimental treatments test the decision making process for the situation where i) participants choose fertiliser application rates and responding to the outcomes for realised profits ii) participants choose fertiliser application rates for the situation where a scholastic variable for 'rainfall' removes an unknown amount of their fertiliser, and iii) agents select price-quantity bundles for exchange on the trading market, informed by their marginal values for different sized bundles via the participant interface. The software has been tested on individual participants, with future uses in a laboratory setting with multiple users. An interesting result of preliminary uses is the finding that between treat 1 and treatment 2, the application rate is increased in response to the unknown rainfall variable, effectively creating an 'insurance' strategy against loss of fertiliser. By over-applying fertiliser, agents opt for more likely higher yield rather than a predicted economically optimal outcome that accounts for their costs of added fertiliser units. This behaviour has been observed in the field by agronomists, and is the topic of education and extension efforts to convince agriculturalists that they can realise a win-win situation with decreased fertiliser use, by saving on fertiliser costs, and improving water quality. Nevertheless, agriculturalists often continue to over-apply fertiliser. The preliminary findings suggest this might be a strategic, and even 'rational' response to uncertainty in production, as revealed through the experimental economics process.

3. CONCLUSIONS

Calibration and validation of agent-based models is challenging due to the emergent properties of complex systems. Laboratory experiments with human subjects can offer a way to reveal behaviours that might not otherwise be included due to the limits of the modeler's understating of the system at hand. Surveys and interviews are often used to populate information on agent characteristics and behaviours, but often these would not be sufficient to explore decision making in dynamic contexts. Companion modelling can be used in context-rich situations where agents' behaviours and relationships are revealed through role-playing games. Experimental economics can explore well-defined decision situations, requiring respondents to reveal their strategies for dealing with dynamic decision making problems. The outcomes of the experiment can be used to calibrate an agent model, offering a way to capture heuristics of decision making and reveal strategies that humans use in dealing with complex situations. This presents an alternative to representing agents as fully informed and rational decision makers, yet offering a defensible alternative that is grounded in empirical data. The application discussed here uses an agent model of fertilizer permit trading in a cap-and-trade system for managing water quality. The experimental platform allows the participant to be immersed in the running model, effectively taking control of the decision of one agent. Revealed behaviour is recorded and can then be used to re-calibrate the artificial agents. The process can be iterative in a participatory fashion using students, agronomists, agriculturalists and policy makers, and can serve to improve the confidence that stakeholders place in modeled behaviours of artificial agents.

The objective of this paper is to contribute to improved calibration and validation through participatory modelling, and a technique of combining agent-based models with experimental economics is presented as an empirically grounded method of revealing agent behaviours. Through this technique, robust decision making strategies can be re-assigned to agents, allowing the experimental medium to reveal strategies which may not be evident to the modeler, and may not be found through other techniques of bringing empirical data into agent-based models.

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