18th World IMACS / MODSIM Congress, Cairns, Australia 13-17 July 2009
http://mssanz.org.au/modsim09

The modelling of business rules for dashboard reporting using mutual information

Gregory Calbert [1]

[1] *Command, Control, Communications and Intelligence Division, Defence Science and Technology Organisation, Edinburgh, South Australia, 5111.*
Email: greg.calbert@dsto.defence.gov.au

Abstract: The role of a business or military dashboard is to form a succinct picture of the key performance indicators that govern an organisation's dynamics or effectiveness. These indicators are based on the judgments of different experts. Variables from one or several databases along with the subjective opinion of the expert are fused or amalgamated to form the so called key performance indicators. A principal example of such a dashboard is the reporting that occurs in the defence preparedness system. Judgments, based on data or opinions within lower level units are formed then fused through the defence hierarchy. The aim of this system is to form a concise picture of the strategically important preparedness issues pertinent to senior leadership.

The issue that we begin to address in this paper is the analysis of just how informative such business dashboards can be. There are three principal reasons as to why the information conveyed by the dashboard to senior boards or defence leadership may be uninformative. The first is that the selection of the key performance variables may not capture the true dynamics of the organisation's state over time. The second is that the models or algorithms used to map fundamental database variables onto key performance indicators may be wrong. Finally, even if the models are correct, the key performance indicator may not convey sufficient information as to the state of the organisation. In this paper we will discuss the last issue-that of the information conveyed through the application of business rules to form the key performance indicator.

A number of examples will suffice to illustrate the loss of information. Important information might be lost when variables are fused or amalgamated due to the requirement for brevity. Take for example overall profit. While the profit for an organisation or division may be good, the successes of some branches may hide a critical loss in others. A simpler example is the requirement for hierarchical reporting. Reports are amalgamated at the branch level and then to the division level with inevitable loss of information.

For this paper we apply the information-theoretic concept of mutual information between the performance indicators -the state- at one level of the organisational hierarchy and the corresponding state, formed by the business rule for fusion at a higher level. The analysis is done for a number of business rules. The business rules analysed are the commonly applied "report by exception" rule or the "report by majority" rule. By calculation of the mutual information, one is able to quantify just how much information is lost through the application of such rules, as reports propagate upward in the hierarchy.

We draw some conclusions as to which business rule is more appropriate in organisations that are either relatively static with few key performance indicators changes, versus organisations which are highly dynamic, where such indicators may change often. A generalised business rule is then defined. This work forms the basis for the measurement of the effectiveness of the reporting system itself. Measures of system bias are formed that suggest the degree of effectiveness of the overall system in the communication of the defence system's overall state to the apex of the hierarchy, that is to senior leadership.

Keywords: *Organisational modelling, information theory, hierarchy, business rule, fusion, threshold.*

1. INTRODUCTION

The defence preparedness system informs senior leadership as to the overall state of the organisation in relation to the readiness of units and the ability to conduct specific operations. Reports regarding the state of a number of independent key performance indicators (KPI) are formed at lower levels of the hierarchy, typically, if using the Army as an example, at the company level. These reports may be in regard to the state of the facilities or stocks at different barracks. Though there may be some minimal dependence on the state of KPI at the different levels of the hierarchy, for pragmatic reasons they are assumed to be independent. They are then fused at the battalion, then the brigade and finally the output group level to form a picture, termed in business a dashboard, to inform senior leadership. Brevity and clarity is paramount and therefore the key performance indicators are drawn from a discrete set. An example of such a set is the traffic light indicator form of green for adequate performance and yellow, red for moderate and inadequate performance respectively. For simplicity of analysis we will only consider the binary valued green and red KPI states of adequate and inadequate performance.

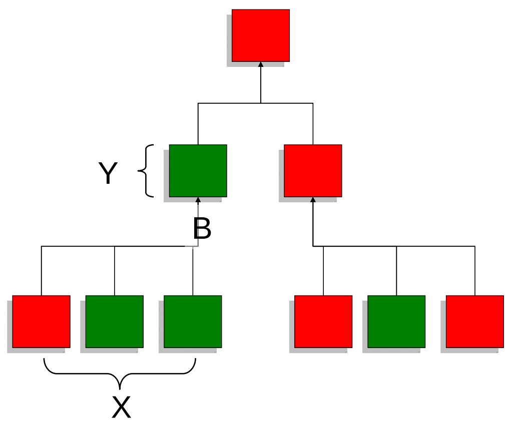

Figure 1. Schematic representation of the defence preparedness system, in which the independent lower level states labelled as X are fused through a business rule B to the state Y, the states being good (green) or bad (red).

As illustrated in Figure 1, independent reports at the lower level of the hierarchy are amalgamated through the application of a business rule to form an overall judgement at the next level, of the hierarchy. This process is repeated to the apex of the hierarchy or to some depth at a higher level.

The aim of this paper is to quantify the performance of the business rules used to form the judgments. We use an information theoretic concept of mutual-information (Shannon, 1948) to quantify such performance. A discussion as to the use of this measure against other measures will be made. It should be emphasised that the application of mutual information in control and to understand networks is not new. Indeed the communications theory subject of rate distortion is devoted to the study of information loss under various compression algorithms. There is an extensive literature on information theory and data fusion (Tay, 2008). Recently, mutual information has been applied to quantifying information propagation in general complex networks (Luque, 1997), neurobiological circuits and networks of gene dependency in biology (Butte, 2000). Our application here is in the specific context of defence preparedness in order to draw conclusions regarding the different business rules applied.

2. MEASURES OF EFFECTIVENESS

Before proceeding, it is worth defining the notation that will be used to model the information propagation in the defence preparedness system. We define the state of a single node in the preparedness hierarchy (that is a unit such as a company lower in the hierarchy or a brigade or output group near the apex) to be $X \in (R, G)$, where R is the state red and G is the state green for inadequate and adequate performance respectively. The levels of the reporting hierarchy are labeled from lowest to highest as $n = 0,1,\cdots,N$. A combined state at level n of the hierarchy (a series of independent red and green reports) $\mathbf{X}_n = \left(X_{n1}, X_{n2}, \cdots, X_{nm}\right)$ is associated with the coordinator or fuser at the next level $n+1$. Here m is the number of performance indicators linked to the fusion state in the next level of the hierarchy, the branching factor. Without loss of generality we label the combined state along with the fusion state as $\mathbf{X}, \mathbf{Y}$ respectively as is shown in Figure 1. The state $\mathbf{X}$ is termed the subordinate state to $\mathbf{Y}$. Business rules are defined to be Boolean functions that map the subordinate level combined state onto the fusion state, $\mathrm{B} : \mathbf{X} \to \mathbf{Y}$. The space of all business rules is termed $\mathbf{B}$. Thus the business rule is defined between the subordinate states to form the fused state of the coordinator.

Now we turn our attention to a discussion of the measures of performance for the preparedness system. One could argue that there are two broad measures. One is to propagate as many reports of inadequate performance through to the next level of the hierarchy as possible. Framed as an optimisation problem, we look for a business rule B that maximises the probability of reporting a red state through to the next level of the hierarchy

$$\max_{\mathrm{B}\in\mathbf{B}} \Pr(\text{red reported in level } \mathbf{Y} \mid \text{red in } \mathbf{X}).$$

It is clear that the business rule termed “report by exception” maximises effectiveness according to this criterion. The report by exception rule sees the coordinator report a red at a node if any one of subordinate nodes is red. This is the commonly used business rule applied in defence preparedness reporting.

Another approach is to find a business rule that reflects, through changes in the fused state $\mathbf{Y}$ the changing states at the subordinate level $\mathbf{X}$. Intuitively, consider two organisations, one where the system is stable and performance is generally good. Here, red reports are rare. Reports to the apex of a multi-level hierarchy will contain a mix of a few inadequately performing divisions, amongst the sound performers. Now consider an organisation in a state of change or urgency. Here, there are many units, branches or divisions with inadequate performance. Applying the report by exception rule will mean that over the levels, the confluence of reports at the apex of the hierarchy will only see inadequate performance across all the divisions, without viewing the changing dynamic of the nodes below. In the limit, there need only be one report of red for the whole system to record inadequate performance as a whole (if all nodes including the apex apply the exception rule), which will not be reflective of the performance of all the nodes below.

To emphasise this point it is worth forming a novel analogy. If all traffic lights within a metropolitan traffic system where red all the time, their effectiveness in the control of traffic would cease as they ultimately would be ignored.

To prevent this problem of reporting in dynamic and changing organisations we propose using the mutual information between levels of the hierarchy as the measure of effectiveness. Suppose that the evolution of the subordinate state over time is formed from a probability distribution $\Pr(\mathbf{X})$. The measure of the information (in bits) of the subordinate state is the information entropy

$$\mathrm{H}(\mathbf{X}) = \sum_{\mathbf{X}} p(\mathbf{X}) \log_2 p(\mathbf{X}).$$

The entropy is a measure of how much “information” the random variable $\mathbf{X}$ conveys over time (Shannon, 1948). Between the subordinate level $\mathbf{X}$ and the fusion state $\mathbf{Y}$ the mutual information is defined as

$$\mathrm{I}(\mathbf{X},\mathbf{Y}) = \mathrm{H}(\mathbf{X}) - \mathrm{H}(\mathbf{X} \mid \mathbf{Y}).$$

This function is symmetric and measures how the dynamic changes of $\mathbf{X}$ over time are reflected in $\mathbf{Y}$. We can observe this noting that if $\mathbf{X}$ and $\mathbf{Y}$ are independent then $\mathrm{H}(\mathbf{X} \mid \mathbf{Y}) = \mathrm{H}(\mathbf{X})$ making the mutual information zero. If $\mathbf{X}$ has the same distribution as $\mathbf{Y}$ then $\mathrm{H}(\mathbf{X} \mid \mathbf{Y}) = \mathrm{H}(\mathbf{X} \mid \mathbf{X}) = 0$ so the mutual information is maximised to $\mathrm{H}(\mathbf{X})$. We thus seek a business rule or series of business rules that maximise the mutual information between levels of the hierarchy, as is seen from the commonly formed capacity equation (Shannon, 1948)

$$\max_{\mathrm{B}\in\mathbf{B}} \mathrm{I}(\mathbf{X},\mathbf{Y}).$$

3. THE MUTUAL INFORMATION OF TWO BUSINESS RULES

We will derive the mutual information of two business rules in this section. The first is the aforementioned “report by exception” rule. We then consider the “report by majority” rule. For this rule, the reporter in the fusion node above observes all subordinate node states. If there are more green states than red then a report of green is made, otherwise red is reported. Both rules are juxtaposed in Figure 2. Of course there will be many other rules that may be applicable. For example there may be special subordinate nodes that have more weight in the reporting than others. We do not consider this, as all nodes are considered equally important. More important nodes may be expanded to multiple nodes with more detailed representation. Generally, we consider threshold business rules (Irving, 1994), where, exceeding some specified fraction of red reports the lowest common ancestor reports red.

Our aim is to examine how these two business rules function in conveying information to the next level of the hierarchy, under different levels of organisational uncertainty. Drawing from the examples of Section 2, organisations that face certain environments have low entropy whilst high uncertainty implies rapid changes over time in the value of the key performance indicators, that is, high entropy.

To calculate the mutual information for the two rules, a model of the key performance indicator values (the value of the nodes) must be formed at level 0 of the hierarchy (the base nodes)-the interface between the organisation and the environment. Our approach is to define a set of distributions parameterised by entropy value h in the following way. Assume there are m subordinate nodes that report to the coordinator or fuser. Suppose $\mathbf{X}_0 = \left(X_{01,} X_{02}, \cdots, X_{0m}\right)$ then in order to tune the distribution over $\mathbf{X}_0$ to have entropy h the first $m-h$ nodes are set or "frozen" to the green state, $X_{01} = X_{02} = \cdots = X_{0(m-h)} = G$, with probability 1. The remaining h nodes are set to have either the green or red state with probability $1/2$. Thus

$$\Pr_h(\mathbf{X}_o) = \Pr(X_{m-h+1})\Pr(X_{m-h+2})\cdots\Pr(X_m),\ \Pr(X_i = G) = 1/2,\ i \in m-h+1, \cdots, m.$$

It is easy to show that the total entropy of the subordinate nodes defined by this distribution has entropy h bits. The base nodes that can switch from red or green states with probability $1/2$ supply the uncertainty into the hierarchy. Thus the entropy $h,$ is both the measure of the uncertainty of the organisation and the number of subordinate nodes that can switch between the red and green states. With this explicit model distribution, we are able to calculate the mutual information precisely for the two business rules. The examination of the effectiveness of each rule can then be ascertained by varying the entropy $h,$ that is changing the number of nodes that can switch between the red and green states.

To calculate the mutual information the conditional entropy between the fusion node states and the subordinate node states

$$\mathrm{H}(\mathbf{X} \mid \mathbf{Y}) = \sum_{\mathbf{Y}=R,G} \sum_{\mathbf{X}} \Pr(\mathbf{Y}) \Pr(\mathbf{X} \mid \mathbf{Y}) \log_2 \Pr(\mathbf{X} \mid \mathbf{Y})$$

is evaluated.

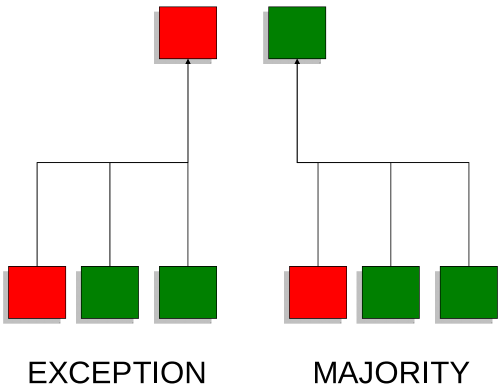

Figure 2. Illustration of the exception and minority reporting rules. On the left hand side a red state is seen so under the exception rules a red is reported for the fusion. On the right hand side there are more green reports than red, so under the majority rule a green is reported.

We omit the details of the derivation albeit to say that the conditional distributions are evaluated through the application of Baye's Theorem, $\Pr(\mathbf{X} \mid \mathbf{Y}) = (\Pr(\mathbf{Y} \mid \mathbf{X}) \Pr(\mathbf{X})) / \Pr(\mathbf{Y})$. Upon viewing the conditional probabilities, the two business rules can now be expressed in general form. For both rules the conditional probability

$$\Pr(\mathbf{Y} = R \mid \mathbf{X}) = \begin{cases} 1 \text{ for } \mathbf{X} \in S_R \\ 0, \text{otherwise}, \end{cases}$$

where for the report by exception rule

$$S_{R,\text{exception}} = \left\{\mathbf{X} : \text{the number of red subordinate reports} \geq 1\right\}$$

and for the report by the majority rule with m subordinates reporting to the coordinator

$$S_{R,\text{majority}} = \left\{\mathbf{X} : \text{the number of red subordinate reports} \geq \lceil m/2 \rceil\right\}.$$

Put in this form, the reporting rule can be generalised to any threshold rule as we will discuss in the following section 4.

The mutual information between the subordinate nodes and the coordinator node for the exception rule, given information entropy level of the subordinate nodes h and the distribution specified above can be shown to be

$$\mathrm{I}_{\text{exception}}(\mathbf{X},\mathbf{Y}) = h - \left(1 - \frac{1}{2^h}\right)\log_2\left(2^h - 1\right),\ h>0.$$

The report by majority rule can be shown to have mutual information

$$\mathrm{I}_{\text{majority}}(\mathbf{X},\mathbf{Y}) = \begin{cases} h - \log_2\left(2^h - \mathrm{K}\right) + \dfrac{\mathrm{K}}{2^h}\log_2\left(\dfrac{2^h}{\mathrm{K}} - 1\right) & \text{for } h > \lceil \mathrm{m}/2 \rceil, \\ 0 \text{ otherwise,} & \end{cases}$$

where the number of combined subordinate states with total entropy h in which the majority report is red (that is in $S_{R,\text{majority}}$)

$$\mathrm{K} = \binom{h}{\lceil \mathrm{m}/2 \rceil} + \binom{h}{\lceil \mathrm{m}/2 \rceil + 1} + \cdots + \binom{h}{h}.$$

The following figure plots the mutual information for the exception and the majority rules, as a function of the information entropy, assuming that there are eleven such subordinate nodes.

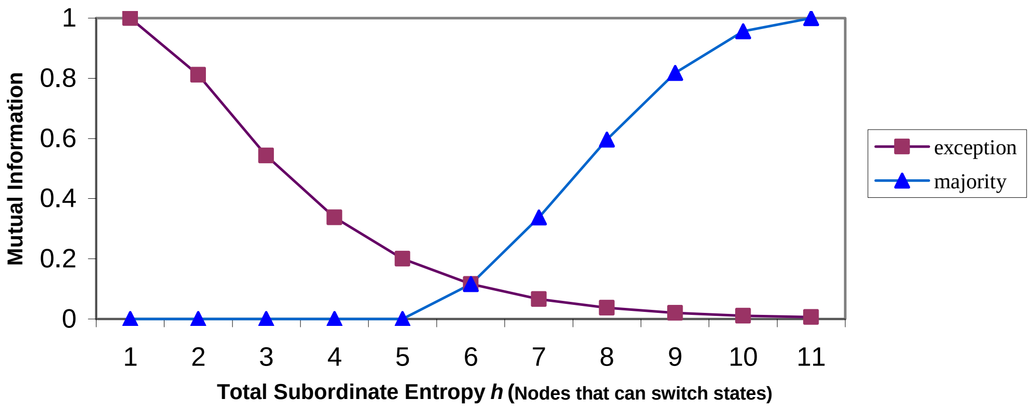

Figure 3: Mutual information between subordinate and fusion nodes of the exception and majority reporting rules as a function of the subordinate nodes total information entropy, which is the number of nodes that can switch between red and green states, given eleven subordinate nodes.

It is clear from inspection of Figure 3 that the exception reporting rule is most effective when the information entropy is low. The interpretation for the preparedness reporting system is that the exception rule applies when most subordinate reports are consistently good (green) over time, reporting few inadequate key performance indicators. However, as the uncertainty increases the majority reporting rule then becomes more informative as to the state of the subordinate nodes below. Analogously, this reflects an organisation where the base key performance indicators are in flux and change over time.

Referring to Figure 3, in the left limit as the information entropy approaches one, any changes in the subordinate states are precisely reflected in the report of the lowest common ancestor, as there is one subordinate state changing. Hence the mutual information is maximised for the exception rule. Conversely, in the right limit, when all subordinate states can change randomly and equiprobably, the majority rule will reflect these changes, though not precisely as there is information loss. The mutual information approaches the maximum of one (this will be the maximum for the coordinator node as it can only be in two states, red and green, one bit). The mutual information reaches this maximum value only because our assumption that all subordinate nodes can change in an equiprobable way. If nodes can change from across red and green states with different probabilities then the mutual information for the majority rule will not reach one. We discuss this issue in the following section.

4. A GENERALISED THRESHOLD BUSINESS RULE

Through the preceding section we have illustrated that the exception business rule may not necessarily be effective in propagating the changing states of the subordinate nodes below. We modeled the majority business rule and found that under the circumstances of dynamic state change of subordinate nodes, this rule better reflects such a change in states. Both heuristics are a version of the general threshold business rule as determined by the value of the conditional probability distribution $\Pr(\mathbf{Y}=R\,|\,\mathbf{X})$ which is one (that is the business rule reports the red state) under our distribution $\Pr_h(\mathbf{X}_0)$ for the set

$$S_{R,a}=\{\mathbf{X}:\text{the number of red subordinate reports}\geq a\}.$$

For the exception rule $a=1$ and for the majority rule $a=\lceil m/2\rceil$ where m is the number of subordinate nodes. It is natural to ask if there is an generalised threshold business rule. Couched in communications theory terms we seek a rule maximises that capacity between the subordinate nodes and the state of the fusion node. Under our distribution $\Pr_h(\mathbf{X}_0)$ defined earlier, we can calculate the mutual information for a general value of a. The resulting equation is identical to that of the mutual information for the majority rule except that the value of K is replaced by a term dependent on a, this being the number of elements in $S_{R,a}$ with entropy h bits

$$\mathrm{K}(a)=\binom{h}{a}+\binom{h}{a+1}+\cdots+\binom{h}{h}.$$

One need only cycle over the values $0\leq a\leq \mathrm{H}$ to find the optimum value of a. These values are shown in the following Figure 4.

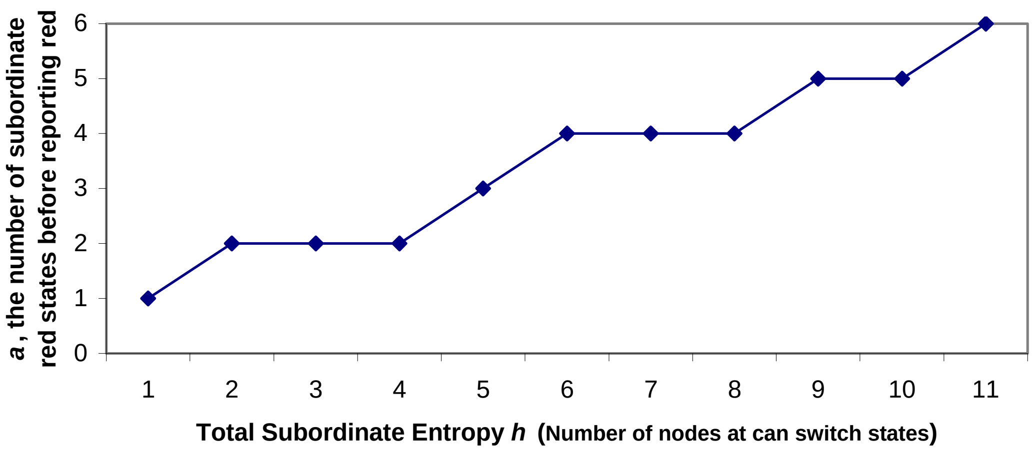

Figure 4: Graph of the optimal number of red states a, given eleven subordinate nodes, before the business rule reports an overall state of red, as a function of the subordinate entropy h.

Referring to Figure 4, as per the observation in Figure 3, as the uncertainty in the organisation increases, the business rule threshold by which the subordinate reports the red state also increases. As an approximation, the threshold number of red reports $a=[h/2]$.

We are able to calculate a generalised business rule under the assumption that all the nodes that can switch between red and green states do so with equal probability $1/2$ over time. If this assumption fails, then inevitably, the difficulty of calculating exact expressions for the mutual information will increase substantially. To overcome this problem, one need only employ Monte Carlo simulation for the given set of state switching probabilities to determine what the threshold will be. For that set of state switching probabilities $\Pr(X_i=G,R), i=1,\cdots m$ the information entropy may be exactly determined. This gives us an upper bound on a as $0 \le a \le h$. It is only the conditional probabilities that need to be estimated from simulation.

5. ESTIMATES OF BIAS IN THE REPORTING SYSTEM

From the discussion above, we are able to establish criteria by which reporting within the preparedness system is biased. This measure of bias will determine the overall system effectiveness. Longitudinal data on the state changes of each node in the current defence system is recorded and therefore the entropy of all subordinate nodes can be estimated. There are two approaches to estimating the internal preparedness system bias, given longitudinal data. One is in reference to the judgments of other reports in the hierarchy and the other is in reference to the deviation from the generalised business rule or the mandated business rule.

To measure the internal bias in reporting, one can estimate, given a particular level of the hierarchy the probability of a fused state $\Pr(\mathbf{Y} \mid \mathbf{X})$. Differences in the fused state probabilities can be calculated and summed to form the bias estimate over the whole hierarchy. Assuming that the subordinate states are the same, that is $\mathbf{X}_{\mathbf{Y}_i} = \mathbf{X}_{\mathbf{Y}_j}$ for two fused states $\mathbf{Y}_i$ and $\mathbf{Y}_j$, a measure of the preparedness system bias is

$$b = \sum_{\text{Hierarchy levels}} \sum_{i,j} \left| \Pr(\mathbf{Y}_i \mid \mathbf{X}_{\mathbf{Y}_i}) - \Pr(\mathbf{Y}_j \mid \mathbf{X}_{\mathbf{Y}_j}) \right|.$$

An alternative is to look at the deviation from the result of the general business rule. For a fusion node, an indicator function may be defined $1_{\mathbf{Y}^*}(\mathbf{Y})$ that is one when the fused state matches that of the state found from the business rule or zero otherwise. Then one measure of the preparedness system bias, given the set of fusion nodes which we label A (that is all nodes except the base nodes of the hierarchy) is

$$b = |A| - \sum_{i \in A} 1_{\mathbf{Y}_i^*}(\mathbf{Y}_i).$$

6. DISCUSSION AND CONCLUSION

We have highlighted that the use of the report by exception rule, whilst appropriate for a preparedness system that has few problems, may not be the appropriate rule when the values of the key performance indicators show dynamic changes over time. The majority business rule or the general threshold rule as discussed in Section 4 is more appropriate. In Defence, decision makers know the problems with the exception rule. At the moment there is considerable subjectivity and variability in fusion decisions made, with limited guidance as to what rules should be applied across different circumstances. Decision makers have the option to fuse decisions based on exception or averaged results from the subordinates or fuse subjectively. Data regarding the longitudinal change in states over time is recorded. As such, the necessary infrastructure is in place at present to apply the best threshold business rules as derived in this paper. This work should improve the guidance on what rules to apply to best relay preparedness information to senior leadership.

REFERENCES

Butte, A. J. and Kohane, I.S. (2000), Mutual Information Relevance Networks: Functional Genomic Clustering Using Pairwise Entropy Estimates. Pacific *Symposium in Biocomputing,* 415-426.

Irving, W. W. and Tsitsiklis, J. N., (1994), Some Properties of Optimal Thresholds in Decentralized Detection. *IEEE Transactions in Automatic Control* 39(4).

Luque, B. and Ferrera, A., (1997), Measuring Mutual Information in Random Boolean Networks. *Complex Systems*, 11.

Shannon, C.E. (1948), A Mathematical Theory of Communication. *The Bell System Technical Journal* 27 379-423 and 623-656.

Tay, W. P., Tsitsiklis, J. N., and Win, M. Z., (2008), Data Fusion Trees for Detection: Does Architecture Matter? *IEEE Transactions in Information Theory* 54(9), 4155-4168.