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Modeling Market Memory As Potential Indicators of Market Informational Efficiency

Dani A. C. Foo

Department of Finance, Waikato Management School, University of Waikato, New Zealand

Email: dfoo@waikato.ac.nz

Abstract: The purpose of this paper is to find a way of objectively benchmarking a selection of regional market against a relatively advanced market in the Asia Pacific in terms of their relative efficiency. Theoretically the Hurst Index ("*H*-Index") seems to be an appropriate *absolute* measure of the efficiency of a market as the Index purportedly reveals long run dependencies ("persistence") embedded in a candidate dataset. However, there have been some controversies about the various methodologies used in calculating the *H*-Index rendering the reliance on the *H*-Index as an *absolute* measure somewhat complicated and problematical.

A large number of long range dependence estimators have been proposed from various research fields including physics, biology, engineering, and economics. Greene and Fielitz (1977) employed the classical Rescaled Range (R/S) analysis, first proposed by Hurst (1951). They analysed the daily returns of 200 common stocks listed in the New York Stock Exchange, and found that long-term dependence is a common characteristic amongst the sample stocks.

Methods of determining long run dependence can be generally classified into two different domains: time domain based method and frequency domain based method. The time domain-based methods include Detrended Fluctuation Analysis (DFA) proposed by Peng et al (1994) and the Fractal Dimension method (Higuchi, 1988). The main advantages of the time domain based method are intuitive appeal, user friendliness and graphical visibility. On the other hand, the lack of a confidence interval and the necessity of Gaussian assumptions are some of the drawbacks. The absence of an analytic confidence interval makes it difficult to discriminate between small but statistically significant LRD from complete absence of LRD. Frequency domain-based methods include the Periodogram regression of Geweke and Porter-Hudak (1983), Wavelet estimator (Veitch and Abry, 1998), and Local Whittle estimator (Kunsh, 1987). Frequency domain methods are normally difficult to apply and to interpret, but they do provide an asymptotic distribution, hence provide a basis for conducting formal hypothesis test.

There are merits in using *H-Index* as a relative measure of efficiency. We have chosen classical Hurst Rescaled Range Analysis (R/S Method) to derive the H-Index from the three methodologies including, Detrended Fluctuation Analysis (DFA) and semi-parametric GPH method. One of the reasons for the choice is the relative ease of implementation and its well understood algorithm. Further, since we are attempting to obtain a *relative* measure, most of the weaknesses that might emanate from using the R/S Method should, generally and mutually, be negated, when the different markets are compared.

It is found that the use of the *H*-Index discriminates the lesser developed markets from the more developed ones. This is a first step in constructing an objective measure of relative efficiency of markets. If ongoing work proves this instrument to be robust, it can be adopted by international investors, fund managers and other interested parties to augment their respective decision making capabilities in assessing the relative efficiency, transparency, governance, and potential risks of their target markets.

***Keywords*:** *Relative Market Efficiency, Hurst Index, Detrended Fluctuation Analysis, benchmarking markets*

1. Introduction

The purpose of this paper is to investigate the relative efficiency of selected Asian-Pacific stock markets and objectively benchmarking this selection against an arguably more advanced stock market in the region, such as the Sydney Stock Exchange. "Efficiency" is defined in terms of the Efficient Market Hypothesis (EMH) developed by Fama (1969), a cardinal but much criticised financial hypothesis. The EMH states that financial markets are efficient, inter alia, when prices on traded assets, e.g., stocks, bonds, or property, already reflect all known information – "information efficiency". It follows that it is impossible to consistently outperform the market by using any information that the market already knows, except through luck. Information or *news* in the EMH is defined as anything that may affect prices that is unknowable in the present and thus appears randomly in the future. Hence, markets that exhibit these characteristics are said to be "efficient." It follows that efficient markets generally provide a "level playing field" for all participants and prices fluctuate only in response to the random news events that affect the market concerned. Presumably, these news events are disseminated more widely and quickly in a more efficient market than one which is less efficient.

Conventionally, stock prices have been assumed to follow a Brownian process where prices change randomly and each price change is independent of all past prices. However, many studies have shown that real economic and financial data do not perform the "random walk" (e.g. Mandelbrot (1967), Baillie and Bollerslev (1994)) and price changes do not follow a normal distribution, as assumed in EMH. Many studies (e.g. Mandelbrot (1971) and Greene and Fielitz (1977)) report that there are manifestations of long range dependence (LRD) in financial time series. LRD here implies the presence of embedded "long range memory" and "persistence" The extent to which LRD manifests in a price series generated by a market may be broadly construed as the extent to which the said market may have deviated from the EMH "efficiency." The presence of LRD in a series of price data implies the presence of embedded "information" that could probably be exploited by savvy market traders. Numerous methods have been devised to detect and measure the presence of LRD see for instance Taqqu, Teverovsky and Willinger (1995). Most of these were criticised for deficiency in statistical power or being weak in discriminating between the genuine presence long memory processes and other confounding factors such as "shifting means" (e.g. Rea, W., L. Oxley, M. Reale, and J.Brown (2008)).

Theoretically, a stationary process X_t is long-range dependent, if there exists a real number $a \in (0,1)$ and a constant $c_1 > 0$ such that $\lim_{k \to \infty} \rho(k)/[c_1 k^{-a}] = 1$ where the sample correlation function and k is number of lags. According to this definition the autocorrelation function of long memory processes, decay to zero with rate approximately k^{-a}. The parameter that characterizes long-range dependence is the Hurst exponent (H), where $H = 1 - a/2$. Long-memory occurs when $½ < H < 1$ (persistence) and $0 < H < ½$ (anti-persistent). Note that while it is not easy to calculate the Hurst exponent in a direct and definitive way, it can be estimated. Long memory process can generate non-periodical cyclical patterns as observed by Hurst (1951) for the Nile River, where long periods of drought are followed by long periods of plenty. Mandelbrot and Wallis (1968) called this phenomenon as the "Joseph" or "Hurst" effect. The *H* Index is an acceptable indicator for measuring the presence of LRD. The strength of the presence of LRD in a market generated series, implies the degree of weakness of the market's efficiency. We employ the *H* Index for the benchmarking the relative "efficiency" of the selected markets. The *H* Index is named after John Hurst and may be derived from a version of Hurst Rescaled Range Analysis (R/S Analysis) and from other methods. Most of the controversies stem from the methods used to calculate the *H* Index and detection of LRD..

In this paper, three widely used methods of detecting and measuring the presence of LRD are discussed, namely, the classical Hurst R/S method, Detrended Fluctuation Analysis and semi-parametric GPH method. Each of these methods has their own strength and weaknesses. Classical Hurst R/S method is chosen because the others are found to be cumbersome computationally, results are difficult to interpret and they offer very little marginal benefits to enhance the objective of this paper.

2. Long range dependence – A brief survey

Greene and Fielitz (1977) employed the classical Rescaled Range (R/S) analysis, first proposed by Hurst (1951). They analysed the daily returns of 200 common stocks listed in the New York Stock Exchange, and found that long-term dependence is a common characteristic amongst the sample stocks. Their findings of LRD in stock returns were criticized, as classical R/S analysis might be biased by the presence of short term dependence. Wallis and Matalas (1970) and Aydogan and Booth (1988) have evidence that the Greene and

Fielitz's results have been confounded by inherent "short term dependence" biasness of classical R/S analysis along with non-stationarity of their data. Peters (1994) attempted to correct the above-mentioned bias, by "pre-whiting" the data. He applied a first-order autoregressive (AR(1)) techniques on the data, then used classical R/S analysis on the estimated residuals. He compared the values of the R/S statistics obtained for different lag lengths with the expected value of the R/S statistics computed by Anis and Lloyd (1976) for white noise process. The correction term used by Peters (1994) was derived by the simulation. Whether his AR(1) filter was successful in erasing all short term serial dependence in his data series was debatable.

Lo (1991) refined to the classical R/S methods, allowing formal statistical testing that was robust to serial correlation and some forms of non-stationarity. His modified R/S tested on daily and monthly stock return indices concluded that Greene and Fielitz (1977) overstated the existence of the long memory in stock returns. Subsequently, the Lo's modified R/S method became standard model in this field and was applied by several researchers to other financial data sets. E.g., Cheung and Lai (1993) applied it to gold market returns, Crato (1994) to international stock markets, Goetzmann (1993) to historical stock return series, and Mills (1993) to UK stock series. The evidence produced by these paper were largely consistent with Lo's (1991) result, i.e. with the transformed R/S statistic showing little evidence of long memory in the returns of those respective financial assets. Recent studies using Lo's modified R/S method include Jacobsen (1996), Hiemstra and Jones (1997). Jacobsen (1996) showed that none of the return series of indices of five European countries, the United States and Japan exhibit long term dependence. He also reported in the classical rescaled range statistic, after adjusting the series for short term dependence, no long-term dependence was evident. Hiemstra and Jones (1997) similarly applied Lo's method to the return series of 1952 common stocks. Their results indicated that long memory was not a significant characteristic of most these stock returns and persistent long memory in the returns was found in a small proportion of stocks. These findings support our assumption that developed markets have little or no evidence of LRD, which imply that they are "efficient" to a large degree.

Teverovsky, Taqqu and Willinger (1999) found vexing shortcomings in Lo's modified R/S, which was its propensity for Type II error bias, not rejecting the null hypothesis regardless of whether long range dependence was present in the data or not. As a result, Teverovsk, et al.(1999) cautioned against its use as the sole technique for testing for LRD in any given data set and suggested instead the use of a diverse number of graphical and statistical methods (see Beran (1994)). Subsequent studies use a range of concomitant methods for testing LRD. Studies of stock price returns included Skjeltorp (2000), Grau-Carles (2001), Weron (2002), Henry (2002), and Liu et al. (1999) who investigated the long memory properties of stock volatility. Crato and Ray (2000) studied the future price, and Lillo and Farmer (2004) examined signs of order in London Stock Exchange. Henry (2002) employed GPH and ARFIMA estimators to investigate the long range dependence in nine international stock indexes. Matos et al (2004) analyzed the time series structure of the Portuguese stock market index from 1993 to 2001 and looked for evidence of market maturation and the appropriateness of the standard finance model. Carbone et al (2004) calculated the Hurst exponent of several time series by a dynamical implementation of the Detrended Moving Average (DMA). Lillo and Farmer (2004) studied the long memory properties for the signs of order in London Stock Exchange using the Hurst exponent. They used four popular Hurst exponent estimators, namely, the Periodogram method, the classical R/S method, Detrended Fluctuation Analysis and the fit of the autocorrelation function.

All these are essentially attempts to trawl for a best method of operationalizing the concept of "efficiency" via the detection of LRD through the *H* Index or other surrogate measures such as a biased-corrected version of the Hurst statistic, a nonparametric spectral test, and a spectral-regression. Results based on these three methods provided no evidence of persistent behavior studies of futures' returns. However, they provided overwhelming evidence of long-memory behavior for the volatility of future's returns, which is beyond the scope of this paper.

In a less developed market, Costa and Vasconcelos (2003) investigated the Ibovespa index of the São Paulo Stock Exchange and appeared to have detected the existence of long-range dependence. They used a rescaled variant of the usual Detrended Fluctuation Analysis which enabled them to obtain the Hurst exponent through a one-parameter fitting. The authors also computed a time-dependent Hurst exponent H(t) using 3-year moving time windows. The results suggested that before the launch of the Collor Plan, a series of inflation-stabilization and economic reforms effected by Brazilian President de Mello in 1990, the curve of H(t) remained well above 0.5, while after the reforms, H stayed close to 0.5. They argued that the structural reform set off by the Collor Plan had led to a more efficient stock market in Brazil. In a lesser developed sector, Jin and Frechette (2004) used a "corrected t-test" to enable the R/S test to measure statistical significance in a set of daily and weekly agricultural cash price returns. They found evidence of long memory in more than half of the agricultural commodities analysed. However, the values of estimated *H* Index statistics were less than 0.6, indicating relatively weak memory.

Given the ongoing debate about the efficacy, robustness and efficiency of the various methods discussed, a way to circumvent some of these limitations is to use a *relative* measure where these limitations can minimized, negated or circumvented. Conceptually, when we adopt a specific *H* Index, we are in fact relying on the *absolute* efficacy, robustness and efficiency of the methodology by which the said *H* Index was derived. By adopting a *relative* approach, we should be able to avoid most of the limitations encountered in the *absolute* approaches. As long as comparable samples with the same characteristics are collected, i.e. of the same time period, and using the same procedures to calculate the measure, *H*, any differences between the respective *H*'s should be attributable to the inherent differences between the markets under consideration. Further, since we use the same procedures to obtain *H* for each of the markets, measurement artefacts that may confound *absolute* measurements should not be a major issue. The candidate markets should experience the same biases resulting from the effects of the measurement artefacts. Hence any perceivable differences in these markets should be primarily attributable to inherent differences of the information generation processes of these respective markets. Thus, the method used to estimate *H*, for gauging the markets' relative performance should not affect our conclusions materially.

3.1 The Classical R/S Methodology

The Classical R/S method was first proposed by Mandelbrot and Wallis (1969). It was based on Hurst's (1951) work in hydrology. One way f calculating H is, first, to divide a selected time series *Z* (say, of returns) of length *L* into *d* subseries of length *n*, so $n = L/d$. Next, for each subseries *m* where $m = 1, \dots, d$, find the subseries mean (E_m) and standard deviation (S_m). Then the data is normalized by subtracting the subseries mean, (E_m), from the each of the elements from the data subseries *m*, thus $X_{i,m} = Z_{i,m} - E_m$ for $i = 1,\dots, n$. Now, we create a cumulative time series $Y_{j,m} = \sum_{j=1}^{m} X_{j,m}$ for j = 1, …m and calculate the range Rm = $\max\{Y_{1,m},\dots Y_{n,m}\} - \min\{Y_{1,m},\dots Y_{n,m}\}$ and rescale the range as R_m/S_m. Finally, the mean value of the rescale range for all subseries of length n is calculated as:

$$(R/S)_n = \frac{1}{d}\sum_{m=1}^{d} R_m / S_m$$

It can be shown that the R/S statistics asymptotically follows the relation: $(R/S)_n \sim cn^H$. Thus, the value of *H* can be obtained by running a simple linear regression over a sample of increasing time horizons $\log(R/S)_n = \log c + H \log n$ Equivalently, one can plot the $\log_{10}(R/S)$ statistics against $\log_{10} n$ in a scatter plot. If the time series is white noise then the plot is roughly a straight line with slope of 0.5. If the underlying process is persistent, the slope is larger than 0.5; if it is an anti-persistent then the slope is smaller than 0.5. Weron (2002) argued that a major weakness of the R/S analysis for *absolute* performance evaluation was that no theoretical asymptotic distribution could be derived for a Hurst parameter that was obtained from linear regression. Many previous studies using the R/S method as an *absolute*, stand alone measure, that did not address this problem, left their respective findings opened to question.

4. Data analysis

4.1 Data and distribution properties

The data used in this study were aggregate stock market indices from thirteen countries as follow: ASX All Ordinaries Price Index (Australia); NZSX All Price Index(New Zealand); Hang Seng Price Index (Hong Kong); MSCI Singapore; MSCI Taiwan; MSCI Malaysia; MSCI Indonesia; SHANGHAI SE Composite Index; NIKKEI 500 Price Index; MSCI Thailand; INDIA BSE National Price Index; MSCI Philippines; MSCI Korea. These were daily data extracted from DataStream with all market prices recorded from 1/1/1990 to 24/12/2004. This period was selected because it straddles the 1997/1998 "Asian Financial Crisis," which may be considered as a "major global/exogenous shock", placing the "shock" somewhere in the middle of the range. The market returns ("returns") are calculated as $r_t = \ln(p_t / p_{t-1})$.

Preliminary statistical properties of the data are summarized in Table 1, which shows that all the indices have asymmetric distribution, the skewness coefficients being different from zero. The returns series of Australia, New Zealand, India and Hong Kong are negative skewed while others are positive skewed. The kurtosis of four series are larger than 4 indicating that the tails of their distribution are fatter than normal distribution. Amongst these markets, the Nikkei 500 index has the thinnest tail compared to other markets and China has the fattest tail, which suggests there are more extreme returns in this index. Similarly, the standard deviation suggests that China is also the most volatile of these countries. The Jarque-Bera tests suggest that the normality hypothesis can be rejected in all markets as all p-values are smaller than 5%. The rejection of normality will be examined further by using a more robust test. It is observable that Korea, Thailand and Indonesia are systematically affected by the1998 Asian Financial Crisis, which is reflected in larger variation in the post-crisis period.

Conventional finance theory posits that stock returns have a normal distribution. The Jarque-Bera test as shown in Table 1 strongly suggests this is not the case. A further examination the Quantile-Quantile(QQ) plots of the various returns indicate that all have leptokurtic distributions, suggesting thicker tails than Normal distribution. The QQ plots of Indonesia, Malaysia, Philippines and Singapore appear more deviant than the other four markets, suggesting substantial fat tails in their returns distributions.

Table 1. Summary Statistics of Daily Returns of Market Indices.

	Mean	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera*	P-value
Australia	0.0002	0.061	-0.074	0.008	-0.405	8.434	4922	0.00
New Zealand	0.0001	0.092	-0.129	0.009	-0.672	19.745	46025	0.00
Singapore	0.0001	0.150	-0.092	0.014	0.295	10.400	8986	0.00
Hong Kong	0.0004	0.172	-0.147	0.016	-0.032	13.000	16310	0.00
Indonesia	0.0002	0.168	-0.191	0.019	0.087	16.438	29457	0.00
Malaysia	0.0001	0.233	-0.242	0.016	0.748	40.652	231560	0.00
Taiwan	-0.0001	0.126	-0.103	0.020	0.051	5.904	1377	0.00
China	0.0004	0.707	-0.388	0.029	4.220	121.499	2147554	0.00
India	0.0003	0.162	-0.140	0.019	-0.087	9.042	5958	0.00
Japan	-0.0002	0.103	-0.086	0.015	0.065	5.721	1210	0.00
Korea	-0.0001	0.272	-0.226	0.025	0.373	13.815	19165	0.00
Thailand	-0.0002	0.184	-0.145	0.022	0.658	11.153	11123	0.00
Philippine	-0.0002	0.213	-0.110	0.018	0.664	12.578	15247	0.00

4.2 LRD in stock indices return

When we calculated the *H* index of these selected markets as surrogated by stock market indices, all exhibit varying degree of LRD.

Table 2. Hurst exponents using R/S method

	Hurst coefficient	Constant	R-square	sum of squares
Australia	0.552	-0.019	0.993	1.398
New Zealand	0.573*	-0.056	0.995	1.510
Hong Kong	0.553	-0.017	0.996	1.407
Singapore	0.567	-0.030	0.996	1.478
Taiwan	0.586**	-0.092	0.996	1.579
Malaysia	0.583**	-0.043	0.996	1.560
Indonesia	0.608**	-0.053	0.994	1.698
Japan	0.585**	-0.068	0.990	0.767
Thailand	0.562	-0.029	0.997	1.449
India	0.588**	-0.070	0.996	1.591
Philippine	0.609**	-0.109	0.998	1.704
Korea	0.565	-0.077	0.998	1.469
China	0.585**	-0.068	0.990	0.767
90% confidence interval	0.427	0.569		
95% confidence interval	0.413	0.582		

The empirical confidence interval is based on hypothesis of no long range dependence.

* and ** denote significance at 90% and 95% level respectively.

Table 2 shows the results of linear regression on the empirical $\log_{10}$(R/S) values versus box size $\log_{10}(N)$.The regression results show that all R-squares are close to one indicating a remarkably good fit. All index series present persistent LRD having Hurst exponents greater than 0.5. Empirical confidence intervals provided by Weron (2002) enable us to examine the significance of the estimation results. Seven out of thirteen countries significantly rejected the null hypothesis of "no long range dependence" at 95% confidence level, including Taiwan, Malaysia, Indonesia, Japan, India, Philippines, and China; together with New Zealand we can reject the null hypothesis at the 90% confidence level.

The Hurst coefficients for the more developed countries are consistently lower than those for developing countries. The average Hurst exponent for Australia, New Zealand, Hong Kong, Japan, South Korea and Singapore is 0.566, which is significantly lower than 0.589 of Thailand, India, Philippines, China, Taiwan, Malaysia and Indonesia. It is expected that developed markets are observed to be more efficient than the lesser developed markets and they tend to have less persistence in market price. We now may have an empirical measure of this relative efficiency based on well researched grounds.

Alternatively, using Detrended Fluctuation Analysis (DFA) on the selected markets, we have the following:

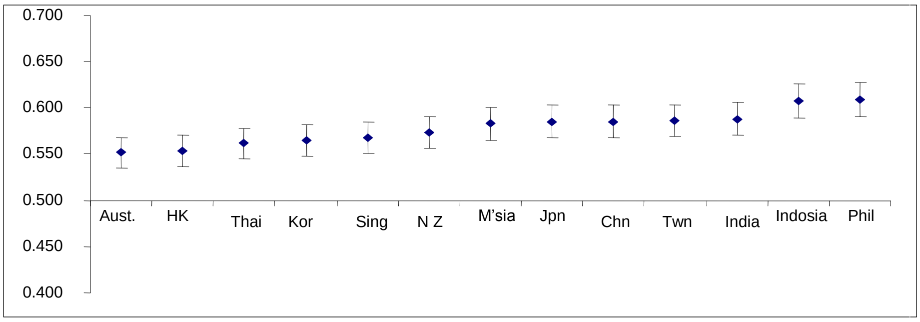
Table 3. Estimated Hurst exponents using Detrended Fluctuation Analysis (DFA)

	Hurst coefficient	Constant	R-square	Sum of Squares
Australia	0.481	-2.681	0.989	1.063
New Zealand	0.542	-2.733	0.993	1.349
Hong Kong	0.516	-2.432	0.991	1.226
Singapore	0.547	-2.524	0.990	1.373
Taiwan	0.562*	-2.421	0.991	1.452
Malaysia	0.568*	-2.539	0.985	1.484
Indonesia	0.578**	-2.432	0.990	1.537
Japan	0.622**	-2.437	0.988	1.632
Thailand	0.537	-2.322	0.992	1.323
India	0.555*	-2.430	0.996	1.416
Philippines	0.571*	-2.452	0.991	1.499
Korea	0.564*	-2.398	0.993	1.460
China	0.595**	-2.379	0.981	0.792
90% confidence interval	0.440	0.554		
95% confidence interval	0.423	0.572		

* The empirical confidence interval is based on hypothesis of no long range dependence.
* * and ** denote significance at 90% and 95% level respectively.

DFA (Table 3) test results are consistent with our previous R/S test. Here, all series seem to exhibit the long memory property (i.e. H>0.5). We can significantly reject the null hypothesis of "no long range dependence" in the following markets: Taiwan, Malaysia, Indonesia, Japan, India, Philippines, Korea and China. More significantly, the DFA method confirms that developed stock markets have smaller Hurst exponents (hence more efficient) than their developing counterparts.

Figure 1. Hurst exponent from R/S method



Results from the two methods show that developed countries have smaller Hurst exponents than the developing markets. It is expected that developed stock markets have more efficient information dissemination processes hence exhibit less persistence in market price than stock markets of developing countries. Other interpretations are that the Hurst exponent may reflect the degree of market maturation. Figure 1 presents the H Index from R/S method for all markets arranged from left to right in ascending order of the H Index. It can be observed that developed markets tend to have low H Index and developing markets have high H Index. The apparent exceptions are, first, Thailand, a perceived developing market having low H Index from both the R/S and DFA method and Japan with a relatively high H-Index. An explanation for the apparent anomaly is that Thailand's low H can be due to its very diverse types of market participants and open market while Japan's participants are similarly concentrated and the Japanese market being relatively insular.

5. Concluding Remarks.

In conclusion, strong evidence for a H persistence is found in the returns of at least six stock markets, with most of which were developing markets. Malaysia, Taiwan, India, and Philippines consistently show evidence of persistence, which may suggest that the price movements in these markets are more seriously influenced by realizations from both the recent and remote past than the more advanced Australian market. New Zealand shows some signs of long range dependence which may be an artifact of its relatively small size as compared with the markets of the rest of the region. Significant persistence is not evident in the Australian and Hong Kong's market indices which make these good candidate anchors for determining relative "efficiency" of regional markets.

References:

Anis, A.A. and E.H. Lloyd (1976), The Expected Value of the Adjusted Rescaled Hurst Range of Independent Normal Summands. *Biometrika*, 63, 1, 111-116.

Aydogan, K. and G.G. Booth (1988), Are There Long Cycles in Common Stock Returns, *Southern Economic Journal*, 55(July), 141-149.

Bachelier, L (1900), Theorie de la speculation, *Annales de L'Ecole Normale Superieure* 17, 21-86

Baillie, R.T and T. Bollerslev (1994), The long memory of the forward premium, *Journal of International Money and Finance*, 13, 565-71

Beran , J. (1994), Statistics for long memory process. Chapman & Hall.

Cheung, Y. and K.S. Lai (1993), Do Gold Market Returns Have Long Memory?,*Financial Review*, 28, 2, 181-202.

Costa, R. L. and G. L. Vasconcelos (2003), Long-range correlations and nonstationarity in the Brazilian stock market. *Physica A: Statistical Mechanics and its Applications* 329(1-2): 231-248.

Crato, N. (1994), Some international evidence regarding the stochastic behavior of stock returns, *Applied Financial Economics* 4, 33–39.

Crato, N. and B. K. Ray (2000), Memory in Returns and Volatilities of Futures' Contracts, *The Journal of Futures Markets* Vol. 20(No. 6): 525–543.

Fama, E.F. (1965), The Behavior of Stock Market Prices. *Journal of Business*, 38, 34-105.

Fama, E.F. (1969), Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance*, Volume 25, Issue 2.

Geweke, J. and S. Porter-Hudak (1983), The estimation and application of long memory time series models, *Journal of Time Series Analysis* 4, 221–238.

Goetzmann, W.N.(1993), Patterns in Three Centuries of Stock Market Prices,*Journal of Business*, 66, 2, 249-270.

Grau-Carles, P. (2001), Long-range power-law correlations in stock returns. *Physica A: Statistical Mechanics and its Applications* 299(3-4): 521-527.

Green, M. and B. Fielitz (1977), Long term dependence in common stock returns. *Journal of Financial Economics*, 4, 339-349

Hiemstra, C. and J. D. Jones (1997), Another look at long memory in common stock returns. *Journal of Empirical Finance* 4(4): 373-401.

Highchi, T (1988), Approach to irregular time series on basis of fractal theory, *Physica D* 31, 277-283

Hurst H (1951), Long term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers* 116: 770-799

Jacobsen, B. (1996), Long term dependence in stock returns. *Journal of Empirical Finance* 3(4): 393-417.

Jin, H. J. and D. L. Frechette (2004), A new t-test for the R/S analysis and long memory in agricultural commodity prices. *Applied Economics Letters* 11: 661–667.

Kunsh. H. R. (1987), Statistical aspects of self-similar processes. 1, 67–74, Utrecht, VNU Science Press.

Liu, Y., P. Gopikrishnan, et al. (1999), Statistical properties of the Volatility of Price fluctuations. *Physical Review* 60(2).

Lo, A.W.(1991), Long-Term Memory in Stock Market Prices, *Econometrica*, 59, 5,1279-1313.

Mandelbrot, B (1967), the variation of some other speculative prices, *Journal of Business*, 40, 393-413

Mandelbrot, B. (1971), When can a price be arbitraged efficiently? A limit to the validity of the random walk and martingale models, *Review of Economics and Statistics*, 53, 225-236.

Mandelbrot, B.B. and J. Wallis, (1968), Noah, Joseph and operational hydrology, *Water Resources Research* 4, 909-*918*

Matos, J. A. O., S. M. A. Gama, et al. (2004), An econophysics approach to the Portuguese Stock Index--PSI-20. *Physica A: Statistical Mechanics and its Applications* 342(3-4): 665-676.

Mills, T.C. (1993), Is there long memory in UK stock return? *Applied Financial Economics*, 3, 303-306

Peng C-K, Buldyrev SV, Havlin S, Simons M, Stanley HE, Goldberger AL. (1994), Mosaic organization of DNA nucleotides. *Physical Reviews E* 1;49:1685-1689.

Peters, E.E. (1991), Chaos and Order in the Capital Markets, John Wiley & Sons, Inc. and Peters, E.E. (1994) Fractal Market Analysis, Wiley, New York.

Rea, W., L. Oxley, M. Reale, and J.Brown (2008), The Empirical Properties of Some Popular Estimators of Long Memory Processes, *Working Paper No.13/2008, Department of Economics, College of Business and Economics, University of Canterbury, Christchurch, New Zealand*

Taqqu, M. S and Teverovsky, V. (1996), Semi-parametric graphical estimation of long memory data. In P.M. Robinsion & M. Rosenblatt (Eds.) *Athens conference on applied probability and time series. Vol 2: Time series analysis in memory of E.J. Hannan.* Springer.

Sadique, S. and P. Silvapulle (2001), "Long-term memory in stock market returns: international evidence." *International Journal of Finance and Economics* **6**: 59-67.

Skjeltorp, J. A. (2000), Scaling in the Norwegian stock market. *Physica A: Statistical Mechanics and its Applications* 283(3-4): 486-528.

Veitch, D. and P. Abry (1998), A Wavelet Based Joint Estimator of the Parameters of Long-Range Dependence, *IEEE Transactions on Information Theory*, 45

Wallis, J.R. and N.C. Matalas(1970), Small Sample Properties of H and K Estimators of the Hurst Coefficient H, *Water Resources Research*, 6, 6, 1583-1594.

Weron, R. (2002), Estimating long-range dependence: finite sample properties and confidence intervals. *Physica A: Statistical Mechanics and its Applications* 312(1-2): 285-299.

Willinger, W., M. S. Taqqu, and V. Teverovsky (1999), Stock market prices and long range dependence, *Finance and Stochastics* 3,1-13.