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Numerical Study on Evolution and Characteristics of Mixed-Sized Windblown Sand Flux

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Abstract: In this paper, on the basis of the splash function of stochastic collisions with the sand grain size considered, the numerical method is employed to study the evolution of mixed-sized windblown sand flux. The evolution of the grain number flux and the streamwise sand transport, the profiles of the mass flux and the wind velocity, the influences of the friction velocity and of the size distribution of sand grains on the streamwise sand transport are all discussed in detail. It is shown that the time needed for the mixed-sized windblown sand flux to reach the steady state(duration time for short) is much shorter than that for the single-sized sand flux; as the friction velocity increases, the duration time and the time needed for the initial stage of the wind effect (the period when the grain number flux per unit area and the streamwise sand transport increase very little with the passing of time, initial entrainment time for short) decrease, while the streamwise sand transport increases; as the diameter of volume average of the bed sand grains increases, both the streamwise sand transport and the duration time decrease.

Keywords: *Mixed-sized sand grains, windblown sand flux, evolution, numerical simulation*

1. INTRODUCTION

Desertification is causing a series of environment and economy problem. Around the globe, 25% of the land is more or less desertified and 415,000 km^2 of the farm land becomes partially or completely unproductive (Dooley, 2002). Desertification not only disturbs the ecological balance, leads to soil erosion, but also threatens the human existence and social and economic development. It expresses itself mainly in the form of sand movement in the wind field, which, in turn, is the major cause of the sand storm, therefore, the study on the dynamic mechanism of windblown sand flux is always a major concern of blown sand physics and the sand control project. Although the studies of blown sand physics based on the field observation and experiment were first conducted as early as 1930s-1950s and a large number of scholars devoted themselves to mathematic modeling and quantitative simulation in the following decades, the association between the micro- and macro-studies of the windblown sand flux is not yet satisfactorily investigated. Just as Anderson *et al.* suggested, the present models were not adequately sophisticated to predict such macro quantities of the windblown sand flux as the mass flux and the wind velocity profile (1991).

The windblown sand saltation is the primary form of the windblown sand movement. The research of its mechanism, especially its evolution, is crucial to bridge the gap between the micro- and macro-studies. Many scholars have conducted field observation, experimental measurement and numerical simulation on the evolution of the saltation. For example, Anderson and Haff discovered from their numerical simulation of the evolution of saltation that the duration time was about 3.0s (1991); Shao and Raupach, drew conclusion from their observation of sand saltation that the duration time was approximately 2.2s (1992); Zheng *et al.*, taking into consideration the influence of the electric field on the saltation, suggested the duration time as 2.6s (2006). However, all these numerical simulations are on the saltation of single-sized sand grains, while the sizes of sand grains in the realistic windblown sand movement are definitely different from each other. According to numerous experiments of scholars, most of the grain distributions of deserts in nature are approximately unimodal (Bagnold, 1941, Zhou et al., 2002 and Zheng et al., 2003), though it is a fact that it differs slightly in deserts of different regions and that, even in the same desert, statistics collected by one researcher might be a bit different from those by another (Bagnold, 1941, Zhou et al., 2002 and Zheng et al., 2003). The size distribution analysis of the sample sand grains by *Zhou et al.* shows that the size distribution of sand grains is a logarithmic normal distribution, as shown in Figure 1, where the volume mean diameter is 0.228 mm (Zhou et al., 2002). A great deal of effort has been devoted to understanding the mechanism of mixed-sized wind-blown sand flux (Bagnold, 1941, Zhou et al., 2002, Rice et al., 1995, Zhou et al., 2006 and Li, Zhou, 2007). Therefore, authors of the paper consult the studies of Li and Zhou on the particle-bed collision of mixed-sized sand grains, and employ the numerical method to simulate the whole process of the mixed-sized windblown sand flux in the saltation from the initial phase to the steady state.

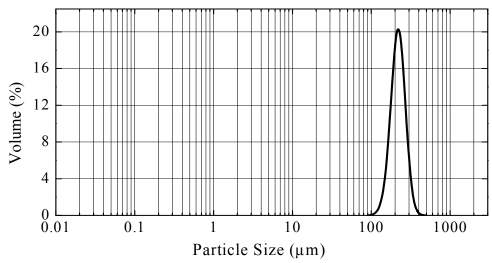

Figure 1. Particle size distribution of sand sample

2. FUNDAMENTAL

The self-equilibrium of the saltation includes the following major processes (Anderson, *et al.*, 1991, Bagnold, 1941): first, the sand grains driven by wind are entrained into saltation; second, the grains, affected by the force of wind and gravity, saltate above the bed; third, saltating grains extract energy from the wind field and exert influences through the coupling action between the wind and sand grains, which affects the wind velocity; fourth, the saltating grains descend to the bed, convey their energy to the stationary grains on the bed and make the stationary ones eject and saltate.

2.1. Initial Entrainment of Sand Grains by Wind

The grain bed studied in this paper is composed of grains in different sizes. Although the grains are different among their diameters, it is assumed that they are all spherical with identical density ρ_g. It is known that, if the shear stress of wind velocity reaches and exceeds a critical value, then, the grains will leave the bed and saltate. Then, it is advisable to suppose that N^a, the number of entrained grains per unit area of bed per unit time is in direct proportion to the excess shear stress (Anderson and Haff, 1991, Anderson and Haff, 1988)

$$N^{\mathrm{a}}(t) = C(\tau_a - \tau_c)\,, \tag{1}$$

where, τ_a is the short term mean shear stress at the bed, $\tau_c = \rho_a u_{c*}^2$ is the critical fluid shear stress for entrainment, $u_{c*} = A\sqrt{(\rho_g - \rho_a)/\rho_a g \mu_D}$ is the critical friction velocity (Bagnold, 1941, Greeley *et. al.*, 1982, Greeley and Iversen, 1985), C and A are constants, g is the gravity acceleration, ρ_a is the air density, and here μ_D is the diameter of volume average of the sand grains.

2.2. Saltation of Sand Grains in the Air

The forces acting on the grain of diameter D_i are gravity $\mathbf{F}_g$, aerodynamic drag $\mathbf{F}_d$ and aerodynamic lift $\mathbf{F}_l$. Consider a coordinate system in which positive x is in the downwind direction and positive z is upward. Then the fundamental dynamic function for a single saltating grain can be described as

$$\frac{1}{6}\pi\rho_g D_i^3 \ddot{x} = -\mathbf{F}_{dx} \tag{2a}$$

$$\frac{1}{6}\pi\rho_g D_i^3 \ddot{z} = -\mathbf{F}_g - \mathbf{F}_{dz} + \mathbf{F}_l \tag{2b}$$

where the aerodynamic drag is expressed as

$$\mathbf{F}_d = \frac{1}{8}\pi D_i^2 \rho_a C_d V_r \mathbf{V}_r \tag{3}$$

in which $Vr = \left[(\dot{x} - u)^2 + \dot{z}^2\right]^{1/2}$ is the velocity of the grain relative to the wind, C_d is the drag coefficient taken as an empirical formula (White, 1974)

$$C_d = 24.0/R_e + 6.0/(1.0 + \sqrt{R_e}) + 0.4 \tag{4}$$

where the Reynold's number R_e is defined as $R_e = V_r D_i/\nu$, ν is the kinematic viscosity of air. The magnitude of the lift F_l is expressed as (Anderson and Hallet, 1986)

$$\mathbf{F}_l = \frac{1}{8}\pi D_i^2 \rho_a C_l (u_{top}^2 - u_{bot}^2) \tag{5}$$

in which u_{top} and u_{bot} are respectively the wind velocities respectively at the top and the bottom of the grain. The lift coefficient C_l is taken as $0.85C_d$ (Anderson and Hallet, 1986).

2.3. Coupling Action between the Wind and the Sand Grains

The saltating sand grains can change the wind velocity profile with the counterforce of the aerodynamic drag (Anderson, Haff, 1991 and Bagnold, 1941) and the Navier-Stokes equation of the wind flow can be expressed as (Anderson, Haff, 1991 and McEwan et al., 1999)

$$\rho_a \frac{\partial \mathbf{u}}{\partial t} + \rho_a \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \rho_a + \nabla \cdot \boldsymbol{\tau} + \rho_a \mathbf{g} + \mathbf{F}_x \tag{6}$$

In a two-dimensional wind field, the differential equation of the wind velocity in the x-direction is expressed as (Anderson, Haff, 1991 and Zheng et al., 2006)

$$\rho_a \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left[\rho_a k^2 z^2 \frac{\partial \mathbf{u}}{\partial z}\left|\frac{\partial u}{\partial t}\right|\right] + \mathbf{F}_x(z) \tag{7}$$

in which κ is the von Karman constant, $\boldsymbol{\tau} = \rho_a k^2 z^2 \frac{\partial \mathbf{u}}{\partial z}\left|\frac{\partial u}{\partial t}\right|$ is the turbulent (Reynolds) shear stress, $\mathbf{F}_x$(z) is the force on the wind per unit volume in x-direction by the sand grains.

By Newton's third law, the force exerted by the wind field on the saltating grains and the force exerted by the grains on the wind field are equal in magnitude and opposite in direction. If sand grains lift off from the bed at various velocities and the ejecting velocity distribution is denoted as $f(v)$, then the drag acting on the wind per unit volume is expressed as

$$\mathbf{F}_x(z) = \frac{1}{6}\pi\rho_g \int_{D_{\min}}^{D_{\max}} f(D)D^3 \int N(t,D) f(v) \left[\frac{\mathbf{a}_{x\uparrow}(z,v)}{\dot{\mathbf{z}}_{\uparrow}(z,v)} - \frac{\mathbf{a}_{x\downarrow}(z,v)}{\dot{\mathbf{z}}_{\downarrow}(z,v)} \right] dv dD \quad (8)$$

where $f(D)$ is the size distribution of sand grains, $N(t, D)$ is the number of grains with the diameter D ejecting from the bed per unit area per unit time at moment t, $a_{x\uparrow}$ and $a_{x\downarrow}$ are the respective accelerations of the saltating sand grain at height Z in ascending and descending.

The corresponding boundary and initial conditions are

$$z = z_0: \quad u = 0; \quad z \to \infty: \quad u_* = \lim_{z\to\infty} (zk\frac{du}{dz}); \quad t = 0: \quad u(z) = \frac{u_*}{k}\ln(\frac{z}{z_0}) \quad (9)$$

where the aerodynamic roughness z_0 is $\mu_D/30$, u_* is the friction velocity.

2.4. Impacting of Sand Grains on the Bed

By the means of the particle dynamics method, Li and Zhou (2007)studied the realistic sand bed consisted of mixed-sized grains and analyzed the stochastic particle-bed collisions. They discovered: the number of ejecting grains, N^{ej}, is related to the velocity and size of the incident grain; the size distributions of ejecting grains caused by incident grains in different sizes are basically the same; the size distribution of ejecting grains is relevant to that of grains on the bed and the diameter of volume average of the ejecting grains is a bit smaller than that of grains on the bed; N^{reb}, the velocity of the rebounding grain, is hardly influenced by the size of the incident grain and is about 60% of the velocity of the incident grain; if the size of the incident grain increases, the rebounding angle θ^{rb} decreases exponentially. The larger the size of an incident grain is, the more likely it is to be captured by the bed. If the size of the incident grains increases, the non-rebounding rate of the incident grains η^{dr} increases exponentially, and then

$$N_{ej}(M^{in}) = 17.96 + 18.05\ln(M^{in} + 0.37) \quad (10)$$

$$\theta^{rb}(D^{in}) = 161.46e^{-D^{in}/2.5} + 0.15 \quad (11)$$

$$\eta^{dr}(D^{in}) = 0.064e^{D^{in}/349.06} - 0.036 \quad (12)$$

where M^{in} is the momentum of the incident grain and D^{in} is its diameter.

Since the size distribution of the ejecting grains is of no great difference from that of grains on the bed (Li and Zhou, 2007), it is quite advisable to suppose that the two are in the same pattern. In this paper, the grain size distribution on the bed follows the principle of logarithmic normal distribution given by *Zhou et al.* (2002), that is

$$f(D) = \frac{1}{\sqrt{2\pi}D\sigma_D}\exp(-\frac{(\ln D - \mu_D)2}{2\sigma_D^2}) \quad (13)$$

in which σ_D is the standard deviation. To compare the features of the windblown sand flux made up of grains with different diameters of volume average, the three size distribution patterns in Figure 2 are adopted in the paper.

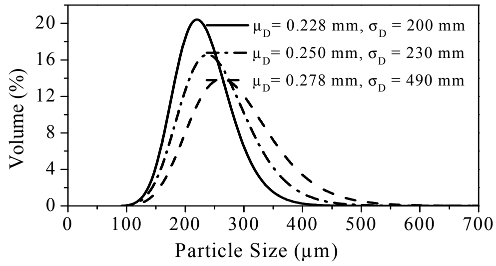

Figure 2. Particle size distribution of mixed-sized grain bed with the principle of logarithmic normal distribution.

Supposing the ejecting velocity of grains in the identical size conforms to exponential distribution (Anderson, Hallet, 1983), then

$$f(v) = \frac{1}{0.63u_*}\exp(-\frac{v}{0.63u_*}) \quad (14)$$

The ejecting velocity of a grain is related to both the friction velocity of the wind and the size of the grain itself (Rice et al., 1995). Anderson and Hallet suggested that Ev_i, the mean ejecting velocity of a grain of a given size D_i, is in inverse proportion to the square root of its mass (1983). To a sand grain with a diameter D_{ref} of 0.35mm, the amount of its mean ejecting velocity Ev_{ref} is 63% of that of the friction velocity u_*, then

$$Ev_i = Ev_{ref}[(\frac{4}{3}\pi D_{ref}^3\rho_g)/\frac{4}{3}\pi D_i^3\rho_g]^{1/2}. \quad (15)$$

Then the ejecting velocity distribution of grains in each size can be worked out.

According to the above description of the fundamental function of saltation, the vertical profile of the mass flux and the streamwise sand transport are respectively

$$Q(z) = \frac{1}{6}\pi\rho_g \int_{D_{\min}}^{D_{\max}} f(D)D^3 \int_0^{\infty}\int N(t,D)f(v)\left[\frac{\dot{x}_{\uparrow}(z,v)}{\dot{z}_{\uparrow}(z,v)} - \frac{\dot{x}_{\downarrow}(z,v)}{\dot{z}_{\downarrow}(z,v)}\right]dvdD, \quad (16)$$

$$Q_m = \int_0^{\infty} Q(z)dz. \quad (17)$$

3. NUMERICAL SIMULATION AND RESULTS ANALYSES

In numerical computing, the grain density □g is 2650 kgm-3，C is 1×105，the short term shear stress acting on the bed τ_a is $\rho_a u_*^2$. The validity of the model needs to be testified. Anderson and Haff (1991) and Zheng *et al.*(2006) have conducted numerical studies on the evolution of the windblown sand movement of single-sized grains in size of 0.25mm with the friction velocity of 0.5ms^{-1}. For the sake of an easy comparison between the results of the two previous studies and results of this paper, the numerical method is employed to study the evolution of the number of the ejecting grains and the streamwise sand transport from the initial phase to the steady state, with the diameter of the sing-sized grains as D=0.25mm and the diameter of volume average of the mixed-sized grains as μ_D=0.25mm and the friction velocity as u_*=0.5ms^{-1} (see Figures 3 and 4).

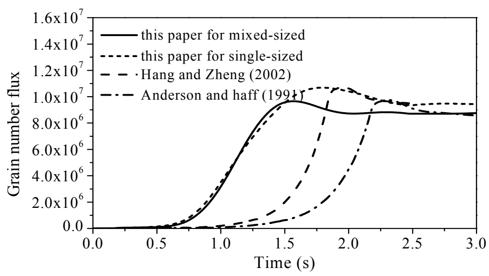

Figure 3. Evolution of the grain number flux.

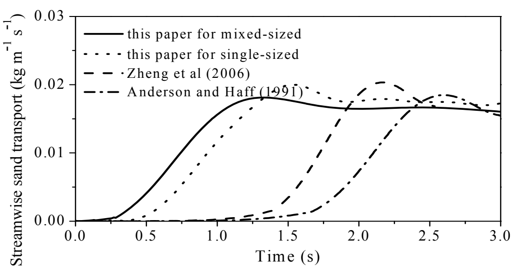

Figure 4. Evolution of the streamwise sand transport.

As shown in the above two figures, the evolution trend of the single-sized grains simulated by the model of this paper fundamentally agrees with those simulated by Anderson and Haff (1991) and by Zheng et al. (2006) and the duration time of the former is slightly smaller than the latter. Therefore, the results of the model are in substantial conformity with those obtained by the former scholars and thus, to some extent, the conformity testifies the validity of the model.

Meanwhile, the two figures show that the evolution of the grain number flux and the streamwise sand transport on a mixed-sized grain bed is qualitatively in the same pattern as that on a single-sized grain bed. In both circumstances, there is an initial entrainment time. Afterwards, the two increase drastically as time advances and finally approach the steady state. The duration time is about 1.75s, which is greatly shorter than that of 3s given by Anderson and Haff (1991) and 2.6s by Zheng *et al.* (2006). In addition, the grain number flux and the streamwise sand transport on a mixed-sized grain bed are also slightly smaller than those on a single-sized grain bed given by Anderson and Haff(1991) and by Zheng *et al.* (2006).

Figure 5 and Figure 6 show the profiles of the mass flux and wind velocity at steady state over a mixed-sized grain bed in comparison with those over a single-sized bed given by Anderson and Haff (1991) and Zheng *et al.* (2006).

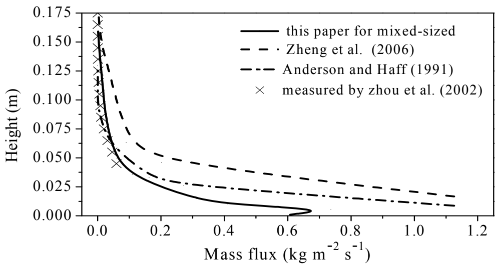

Figure 5. The mass flux profiles at steady state

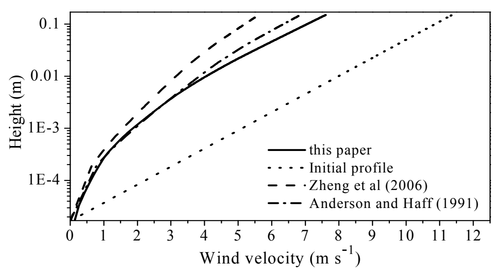

Figure 6. The wind profiles at steady state

It is shown that, in equilibrium, the mass flux of a mixed-sized windblown sand flux is smaller than that of a single-sized windblown sand flux given by Zheng *et al*. (2006); at height below 0.0625 m, it is smaller than that of a single-sized windblown sand flux given by Anderson and Haff (1991); at height above 0.0625m, it is larger than that of a single-sized windblown sand flux given by Anderson and Haff (this might result from the fact that small grains are more likely to be blown high into air) and agrees with the experimental result of Zhou *et al*. (2002) to a great degree. In addition, the mixed-sized mass flux intensity in this paper displays the stratification feature of increasing, saturating and decreasing, which is in accordance with the experimental results of *Yin* (1989) and the model of Zheng *et al*.(2004). Moreover, the magnitude of the wind velocity over a mixed-sized grain bed is larger than that at the corresponding height given by Zheng *et al* and slightly larger than that given by Anderson and Haff. The above comparison between the result produced by the model in this paper and results of previous experiments and models shows that this model is valid.

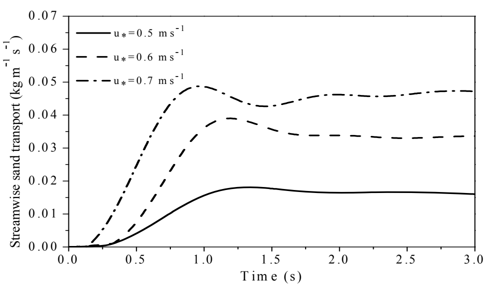

Figure 7. Evolution of the streamwise sand transport at different friction velocity.

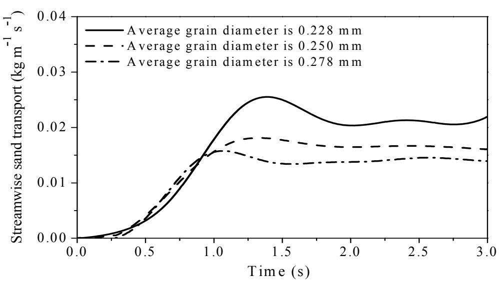

Figure 8. Evolution of the streamwise sand transport on beds of different size distributions

Figure 7 shows the evolution of the streamwise sand transport on a mixed-sized grain bed with a diameter of volume average of 0.228 mm at different friction velocities. It is shown in Figure 7, as the friction velocity increases, both the initial entrainment time and the duration time decrease, while the streamwise sand transport increases.

Figure 8 shows the evolution of the streamwise sand transport on beds of different size distributions of sand grains at the friction velocity of 0.5ms^{-1}. It is clear that, as the diameter of volume average of the mixed-sized grain bed increases, the streamwise sand transport decreases and the duration time decreases.

Figure 9 shows the streamwise sand transport on a mixed-sized grain bed with a diameter of volume average of 0.228mm at different friction velocities in comparison with extant results. It is shown that, on the whole, the streamwise sand transport of this paper is smaller than those calculated by other scholars. When the friction velocity $u_* \leq 0.34 m/s$, the results of this paper are close to those of Zhou et al(2002). and when $u_* \geq 0.34 m/s$, the results of this paper are close to those of Bagnold(1941). The validity of the model is thus further testified.

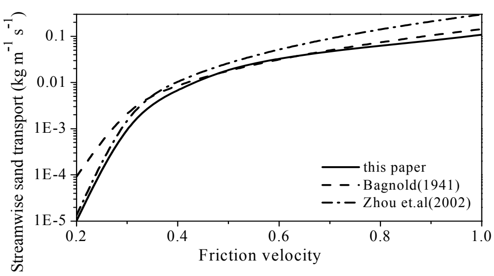

Figure 9. The streamwise sand transport at friction velocity

4. CONCLUSIONS

On the basis of above analyses, it is clear that, the introduction of the size distribution features of the sand grains into the studies is helpful to construct a more precise model of the stratification structure of the windblown sand flux; the duration time of the mixed-sized windblown sand flux is greatly shorter than that for the single-sized one; as the friction velocity increases, both the initial entrainment time and the duration time decrease and the streamwise sand transport increases; as the diameter of volume average of the mixed-sized grain bed increases, the streamwise sand transport and the duration time decrease. The research reveals the basic principles of the windblown sand movement and its structure and paves the way for a more precise prediction of such macro physical quantities as the streamwise sand transport and the wind velocity profile. It also demonstrates that studies of the windblown sand flux evolution and its structure are not supposed to be conducted on a single-sized windblown sand flux instead of a mixed-sized one, otherwise, the results are doomed to stray away from reality. Since this paper is merely a preliminary study into the evolution of the mixed-sized windblown sand flux, only the saltation of sand grains is considered but the suspension of the small grains and the creep of the large grains are not mentioned, which demand more attention in the further studies.

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