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Accurate costing in mixed integer utilisation mining models

C. Burt[1] and Y. Chan[1]

[1]ARC Centre of Excellence for Mathematics and Statistics of Complex Systems,
Department of Mathematics and Statistics, University of Melbourne, Australia
E-Mail: c.burt@ms.unimelb.edu.au, y.chan@ms.unimelb.edu.au

Abstract: In this paper, we study the problem of truck and loader equipment selection for surface mines, where we wish to select the best truck and loader fleet for a multi-period mining schedule. In particular, we look at the costing aspect of this problem. The operating cost is known to dominate the overall cost of materials handling over time.

When performing equipment selection, we can best account for the operating cost by considering the number of hours that the equipment has been utilised. In a surface mine, equipment is often not utilised to full capacity and not accounting for this difference may lead to inferior solutions. Generally, the cost of operating equipment depends on the age of that equipment, while the decision to use a piece of equipment or not is based on the cost. This co-dependency of the age and utilisation of the equipment has so far provided a barrier to tractable equipment selection models.

In the mining industry, it is a common practice to discretise both the age of the equipment and the current time into discrete blocks. However, since the running cost of a piece of equipment depends on its age, an undesirable side-effect of this discretisation is that the cost of operating a piece of equipment over a given time period must be determined by its age at the start of that period. It would be more accurate to account for changes in the age of the equipment as time passes within the period.

In this paper, we present a way in which we can capture the effect of these changes, using linear constraints and adjustments to the objective function. These constraints and adjustments are intended to be added to a mixed-integer linear program for equipment selection, which accounts for equipment utilisation, pre-existing equipment and heterogeneous fleets. However, with these additions this mixed-integer programming model requires further work to achieve tractability in large-scale case studies.

Keywords: *Integer programming, surface mining, equipment selection, utilisation models*

1. INTRODUCTION

In this paper, we consider the problem of equipment selection. Equipment selection is an important part of any materials handling operation, where the final choice in purchase and salvage policy has the potential to affect both the profitability of the operation as well as the ability to meet production requirements. In surface mining, a common equipment selection problem is the selection of haul trucks and excavating loaders – the loaders will be used to load the mined material into the haul trucks; the trucks will in turn transport the material to a dumpsite or mill. We wish to consider a set of trucks and loaders that are suitable for the mining method and mine conditions, and to determine a purchase and salvage policy over the given mining schedule that will optimise the cost of materials handling.

In equipment selection, it is known that operating expense dominates the cost of materials handling over time (Haidar, Naoum, Howes & Tah 1999). The operating expense reflects the cost of operating and maintaining the equipment. It takes into account varying maintenance expenses, availability and productivity levels, all of which are known to vary with the age of the equipment. We can best account for the operating cost by considering the number of hours that the equipment operates; equipment is often not utilised to full capacity, and not accounting for this difference may lead to inferior solutions. However, including cost as a function of utilisation and equipment age adds great complexity to the problem, and has the potential to introduce nonlinearities (Hassan, Hogg & Smith 1985).

In the mining industry, the nonlinear operating cost is commonly simplified by creating a piecewise linear function that is divided into age brackets, so that the cost is constant while the equipment is in any given age bracket (Figure 1). While this discretisation allows us to easily apply linear techniques (such as integer or mixed-integer programming) to model problems involving the costs, it has disadvantages when it comes to actually calculating this cost. Traditionally, it is assumed that a unit stays in the same age bracket for an entire period of time, and so the cost is calculated based on the age of the equipment at the start of that period.

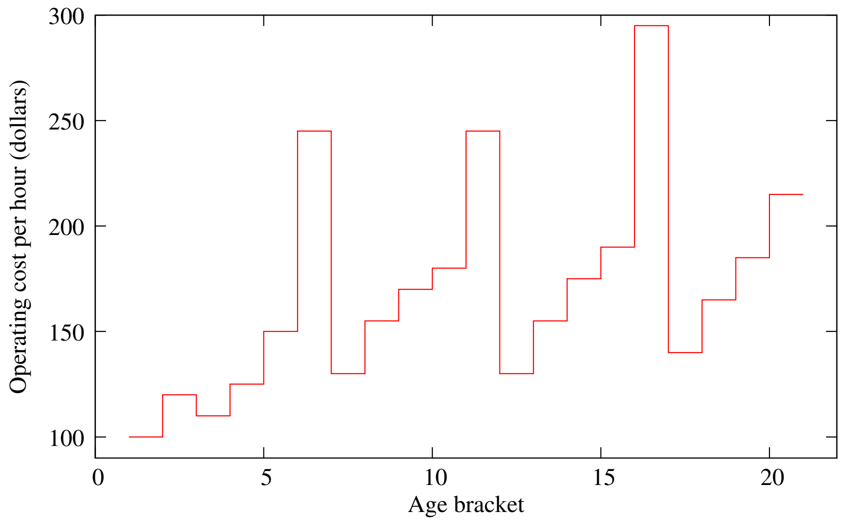

Figure 1. The variability of equipment operating cost against age bracket. The rise in operating cost is due to increased maintenance expense over time. Large drops in the operating expense occur when an overhaul has taken place.

Unfortunately, this is an unrealistic assumption. In many cases, a piece of equipment will sit in the starting age bracket for only a fraction of the period, and spend the remainder of the period in the next bracket, resulting in a different cost to the one calculated. Clearly this will introduce error into the objective function. On the other hand, since time periods are generally shorter than age brackets, it is also possible that the equipment may stay in the same age bracket for the entire period. These two possibilities are illustrated in Figure 2. To accurately account for cost in case (b), we have to determine the amount of time that the unit spends in the next age bracket.

In a departure from cost optimisation, some solution methods in the literature look at optimising productivity (Smith, Wood & Gould 2000), where the cost is constrained with some upper bound (Cebesoy, Gzen & Yahşi 1995). The problem with this approach in a mining application is that the schedule has already been chosen – we need to select equipment to meet the schedule. Selecting equipment that will outperform the schedule could be costly. Other models use life-cycle costing techniques to obtain an equivalent unit cost for

Figure 2. Possible scenarios that occur within a time period. In case (a), the unit stays in the same age bracket for the entire period. In case (b), the unit moves into the next age bracket within the period.

the equipment [(Markeset & Kumar 2000),(A.Bozorgebrahimi, Hall & Morin 2005)]. These costs estimate the average lifetime cost per hour of use or tonne of mined resource. However, this is not practical if we are considering salvaging equipment when it is no longer useful or has reached the end of its optimal replacement cycle.

Burt previously presented models for mining equipment selection, one of which (Chapter 6) accounted for utilisation of equipment (Burt 2008). In this paper, we extend these results by presenting a method for determining the actual cost when equipment moves into the next age bracket during a period. Section 2 shows how accurate costing can be achieved in a non-utilisation model. In Section 3, we observe how we can account for utilisation and age brackets in an equipment selection model. In Section 4, we show how accurate costing can also be achieved for a utilisation model. Finally, we note in Section 5 how we can use this framework to improve the accuracy of other parts of the model.

2. ACCURATE COSTING IN A NON-UTILISATION MODEL

Firstly, we show how we can produce accurate costing in a non-utilisation model. In such a model, we assume that the number of hours that a piece of equipment works in any time period (given its equipment type, age, and the operating hours of the mine) is pre-determined. This means that the working age of the equipment is determined by the length of time since it was bought, and this makes it quite simple to figure out when a piece of equipment crosses into a new age bracket.

Our basic usage variable is $\mathbf{x}_j^{k,l}$, which we define to be 1 if equipment j is owned in time period k, having been bought l periods before that, and 0 otherwise. We assume that age brackets are of constant size, denoted by B_0, and the operating hours of the mine are H^k for period k. In our model, we had $B_0 = 5000$ hours and $H^k = 3500$ hours (for all k). We use a_j^l to denote the availability of equipment j when it is l periods old. The availability is the proportion of the operating hours that the equipment works: since this is a non-utilistion model, all equipment is worked to the fullest in each period.

Assuming that $\mathbf{x}_j^{k,l} = 1$, the age bracket that equipment j lies in at the start of period k is

$$b_j^k(l) = \left\lfloor \frac{\sum_{h=k-l}^{k-1} a_j^{l+h-k} H^h}{B_0} \right\rfloor .$$

We now define $B_j^{k,l}$ to be the number of hours in period k that the equipment lies in age bracket $b_j^k(l) + 1$ (Figure 3).

By assumption, equipment cannot jump age brackets twice in one period, so $a_j^l H^k - B_j^{k,l}$ is the number of

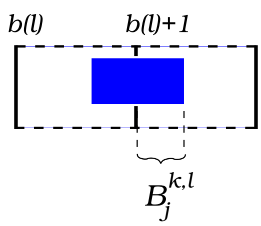

Figure 3. Parameter B represents the amount of time the equipment spends in the next age bracket within the period. We write $b(l)$ as a simplified form of $b_j^k(l)$.

hours spent in bracket $b_j^k(l)$. The equipment stays in bracket $b_j^k(l)$ throughout period k if and only if

$$\sum_{h=k-l}^{k} a_j^{l+h-k} H^h < (b_j^k(l)+1)B_0,$$

and in this case, $B_j^{k,l} = 0$. Otherwise,

$$B_j^{k,l} = \sum_{h=k-l}^{k} a_j^{l+h-k} H^h - (b_j^k(l)+1)B_0.$$

To calculate our cost, we let V_j^l be the variable cost (i.e. cost per operated hour) for equipment j that is in age bracket $b_j^k(l)$, and let D^k be the net present value factor associated with time period k. Then the cost of operating equipment j in time period k is

$$\sum_l D^k V_j^{b_j^k(l)}(a_j^l H^k - B_j^{k,l})\mathbf{x}_j^{k,l} + \sum_l D^k V_j^{b_j^k(l)+1} B_j^{k,l}\mathbf{x}_j^{k,l}. \tag{1}$$

By adding this term to the objective function for every j and k, we produce an accurate assessment of the operating costs of the mine.

3. UTILISATION AND COST BRACKETS IN A LINEAR MODEL

In order to accurately assess the cost of operating a piece of equipment, we need to know the age of the equipment. In a utilisation model, the length of time since the equipment has been bought does not necessarily reflect the age of the equipment in terms of usage (although it does provide an upper bound).

We have a different usage variable for this model: we set the variable $\mathbf{x}_j^{k,l}$ to be 1 if equipment j is owned in time period k and starts this period in age bracket l, and 0 otherwise. This is different to the non-utilisation model because l now counts age brackets, rather than time periods. We also include a continuous utilisation variable, $\mathbf{f}_j^{k,l}$, which is the proportion of the operating hours that equipment j works in time period k, having started it in age bracket l.

We note that the age bracket that equipment j lies in at the start of period k is given by

$$\left\lfloor \sum_{h<k,l} \frac{H^h}{B_0}\mathbf{f}_j^{h,l} \right\rfloor.$$

Therefore, to ensure that only the correct $\mathbf{x}$ is nonzero, we include the constraints

$$l \le \sum_{h<k,l} \frac{H^h}{B_0}\mathbf{f}_j^{h,l} \le l+1-\epsilon \tag{2}$$

when $\mathbf{x}_j^{k,l} = 1$, where ϵ is an arbitrarily small positive number. To apply these constraints only when $\mathbf{x}_j^{k,l} = 1$, we add or subtract a large multiple of $\mathbf{x}_j^{k,l}$ to the appropriate side, so that the constraints vanish when $\mathbf{x}$ is 0. This is shown in the following lemma.

Lemma 3.1. *To ensure that only the* $\mathbf{x}$ *with the correct age bracket is nonzero, we include the constraints*

$$\sum_{h<k,l} \frac{H^h}{B_0}\mathbf{f}_j^{h,l} \ge M(\mathbf{x}_j^{k,l}-1)+l \qquad \forall \quad k,l,j \tag{3}$$

and

$$\sum_{h<k,l} \frac{H^h}{B_0} \mathbf{f}_j^{h,l} \leq l+1-\epsilon+M(1-\mathbf{x}_j^{k,l}) \qquad \forall \quad k,l,j \tag{4}$$

where M is an arbitrarily large integer.

We note that, in practice, M does not need to be arbitrarily large, but merely at least as big as the maximum number of age brackets.

The above results do not apply to pre-existing equipment, because such equipment already has a nonzero age at time 0. We can use similar constraints, but with a small modification to account for this. If we define $P(j)$ to be the age in hours of equipment j when the mine schedule starts, the age bracket that it lies in at the beginning of period k is now

$$\left\lfloor \frac{P(j)+\sum_{h<k,l} H^h \mathbf{f}_j^{h,l}}{B_0} \right\rfloor .$$

Constraints (3) and (4) can then be repeated by replacing the left-hand side with this expression.

4. ACCURATE COSTING IN A UTILISATION MODEL

In a utilisation model, we do not assume that all equipment is worked to its fullest capacity. Instead, we allow ourselves to choose which pieces of equipment work, and for how long. This means that it is not so easy to calculate the true age of a piece of equipment.

As for the non-utilisation model, we need to know whether a given piece of equipment crosses into a new age bracket in the middle of a time period, and if so, for how long. To find this, it is not enough to know the age bracket of a piece of equipment – we must know its age in hours as well. With this in mind, we let $\boldsymbol{\beta}_j^{k,l}$ be a variable which is the amount of time, in hours, that equipment j operates in age bracket $l+1$ in period k, given that it started that period in age bracket l (Figure 4). We set a salvage variable, $\mathbf{s}_j^{k,l}$, to be 1 if equipment j is salvaged in time period k and starts that period in age bracket l, and 0 otherwise.

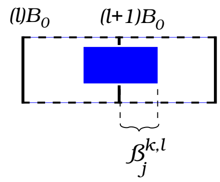

Figure 4. Variable $\boldsymbol{\beta}$ represents the amount of time the equipment spends in the next age bracket within the period.

The first constraint on $\boldsymbol{\beta}$ is that it is non-negative:

$$\boldsymbol{\beta}_j^{k,l} \geq 0, \qquad \forall\, j,k,l. \tag{5}$$

Next, we restrict $\boldsymbol{\beta}$ to be 0 if the corresponding $\mathbf{x}$ is zero, indicating that the equipment was either not owned or in a different age bracket in the period:

$$\boldsymbol{\beta}_j^{k,l} \leq M\mathbf{x}_j^{k,l}, \qquad \forall\, j,k,l. \tag{6}$$

Now, the age in hours of equipment j at the end of time period k is given by

$$\sum_{h\leq k,l} H^h \mathbf{f}_j^{h,l}.$$

Since the start of the $(l+1)$th age bracket is $(l+1)B_0$ hours, we need $\boldsymbol{\beta}_j^{k,l}$ to be the difference between the age and $(l+1)B_0$, if this difference is positive. This gives a lower bound on $\boldsymbol{\beta}$, with the caveat that the corresponding $\mathbf{x}$ must be nonzero:

$$\boldsymbol{\beta}_j^{k,l} \geq \sum_{h \leq k,l} H^h \mathbf{f}_j^{h,l} - (l+1)B_0 - M(1 - \mathbf{x}_j^{k,l}) \qquad \forall\, j,k,l. \tag{7}$$

Together with the restriction that $\boldsymbol{\beta}$ is non-negative, this provides a tight lower bound for $\boldsymbol{\beta}$. However, the upper bound is slightly trickier. We cannot simply set $\boldsymbol{\beta}$ to be less than or equal to this difference, because it may be negative. We get around this by noting that we can tell when the equipment crosses over into the next age bracket, by observing the value of $\mathbf{x}_j^{k+1,l} + \mathbf{s}_j^{k+1,l}$. This is 1 if and only if the equipment starts period $k+1$ in age bracket l, and therefore has not shifted age brackets. This gives us the final constraint on $\boldsymbol{\beta}$:

$$\boldsymbol{\beta}_j^{k,l} \leq \sum_{h \leq k,l} H^h \mathbf{f}_j^{h,l} - (l+1)B_0 + M(1 - \mathbf{x}_j^{k,l} + \mathbf{x}_j^{k+1,l} + \mathbf{s}_j^{k+1,l}) \qquad \forall\, j,k,l. \tag{8}$$

For pre-existing equipment, the age in hours of the equipment is increased by $P(j)$. This propogates through to constraints 7 and 8 in the obvious way.

Now that we have set the value of $\boldsymbol{\beta}$, we can include it in the objective function. We define V_j^l and D^k as for the non-utilisation model. Accounting for the possibility of an age bracket shift gives us the operating cost

$$\sum_{j,k,l} D^k V_j^l H^k \mathbf{f}_j^{k,l} + \sum_{j,k,l} D^k (V_j^{l+1} - V_j^l) \boldsymbol{\beta}_j^{k,l}.$$

If we assumed that the cost is determined by the age bracket of the equipment at the start of the period, then the operating cost would have been just the first of these terms.

The addition in complexity to the model is not great, as there are the same number of $\boldsymbol{\beta}$ variables as $\mathbf{x}$ or $\mathbf{f}$ variables. However, we have not yet managed to implement this model to achieve a satisfactory solution time. This problem may be alleviated by technical integer programming cleverness, such as cuts and heuristics, that tighten the solution space and thus shorten the time required to find the optimal solution.

5. ACCURATE UTILISATION IN A UTILISATION MODEL

Calculating the number of hours spent in the next age bracket also allows us to bound the utilisation variable $\mathbf{f}$ in a better way. Previous models merely assumed that the utilisation was bounded by the availability of the equipment, as determined at the start of the time period. In other words, we redefined a_j^l to be the availability of equipment j when it is in age bracket l, and applied the constraints $\mathbf{f}_j^{k,l} \leq a_j^l$.

However, this suffers from the same problem that we encountered with costing: the equipment may cross into a new age bracket with significantly different availability during the time period. To solve this problem, we note that equipment j operates for $\boldsymbol{\beta}_j^{k,l}$ hours in age bracket $l+1$ during period k. The total number of hours spent in age bracket $l+1$, including non-operating time, must therefore be at least $\frac{1}{a_j^{l+1}} \boldsymbol{\beta}_j^{k,l}$. This gives us an upper bound for the total number of hours spent in age bracket l, which in turn gives an upper bound on the operating hours spent in age bracket l:

$$\mathbf{f}_j^{k,l} H^k - \boldsymbol{\beta}_j^{k,l} \leq a_j^l \left(H^k - \frac{1}{a_j^{l+1}} \boldsymbol{\beta}_j^{k,l} \right).$$

Rearranging gives the modified bound on $\mathbf{f}$:

$$\mathbf{f}_j^{k,l} + \frac{1}{H^k}\left(\frac{a_j^l}{a_j^{l+1}} - 1\right)\boldsymbol{\beta}_j^{k,l} \leq a_j^l \qquad\qquad \forall\, j, k, l. \tag{9}$$

If $\boldsymbol{\beta}_j^{k,l} = 0$, this reduces to the original constraint.

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