Water Balance Modelling Over Variable Time Scales

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EXTENDED ABSTRACT

Partitioning of rainfall into evapotranspiration and runoff is controlled by climate and catchment characteristics. The degree of control exerted by these factors varies with the spatial and temporal scales of processes being modelled. Understanding individual hydrological processes and their relationships with climatic and catchment characteristics is an important step in predicting catchment water balance. However, this does not mean that one should try to incorporate every known process into a model. The interaction and co-evolution of these processes may manifest themselves in such a way that the overall behaviour of the catchment can be described by simple relationships.

A top-down approach was used on a whole catchment-scale water balance to determine the minimum level of model complexity required for predicting runoff over a range of time scales. Monthly values of precipitation, potential evapotranspiration, and streamflow from 265 catchments in Australia were used in this study. A model based on the "limits" concept and an index of dryness was adopted and proved to be adequate for the selected catchments. On mean annual basis, the index of dryness defined as the ratio of potential evapotranspiration to precipitation was found to be a dominant factor in determining the water balance.



Figure 1. Ratio of mean annual evapotranspiration to rainfall (E/P) as a function of the index of dryness (E_0/P) for different values of parameter.

At annual time scale, interannual variability in streamflow can be reasonably well estimated from the index of dryness without additional inputs. Results however suggested increased model complexity is required on finer time scale such as monthly. In response to this, the mean annual water balance model was modified to include additional factors and this resulted in a parsimonious lumped conceptual model on monthly time scale. The model was calibrated against the measured runoff and showed encouraging results (Figure 2). The model developed from this study can be applied to ungauged catchments and also be used to investigate the impact of land use and climate change on catchment- scale water balance.



Figure 2. Comparison of observed (line with dot) and predicted (solid line) monthly streamflow for selected catchments. The results are shown for best (bottom) and worst (top) cases as indicated.

INTRODUCTION

The movement of water through the continuum of the soil, vegetation, and atmosphere is an important process. Understanding the water balance in relation to climate and catchment characteristics provides insight into the complex processes operating over a range of spatial and temporal scales. At the catchment scales, quantifying the effect of land use change on water balance and prediction of streamflow in ungauged catchments are the major scientific challenges for hydrologists (Zhang, et al., 2004, PUB initiative (http://iahs.info), Sivapalan, et al., 2003). While there have been major advances in our knowledge of the physical and biophysical processes controlling the water balance, it remains a difficult task to develop models that can be used to make hydrological predictions at the catchment scales. This is partly due to the fact that for such a model to be of practical use it must meet the requirements of parsimony in terms of data inputs and model parameters. This means inevitably that the model has to be simple enough and parameters can be estimated from known climate and catchment characteristics. The uncertainties in determining spatial and temporal distributions of the climatic variables, especially rainfall, constitute a major obstacle to the understanding of hydrological behaviour at the catchment scales (Milly et al., 2002).

In an attempt to understand relationships between water balance and climate, Budyko (1958) postulated that long-term average annual evapotranspiration from a catchment is determined by rainfall and available energy. Based on this assumption, Budyko (1958) derived a simple water balance model known as Budyko's curve that showed good agreement with the long-term water balance data for a number of catchments in the former USSR. The work of Budyko (1958) has led to more theoretical studies trying to understand how climatic and catchments characteristics affect equilibrium or long-term average water balance 1994; Koster and Suarez, 1999; (Milly, Choudhury, 1999; Zhang et al., 2001, 2004; Farmer et al., 2003; Raupach et al., 2001). The importance of these studies is that they showed a hierarchy of controlling factors exerted by climate and catchment characteristics over various temporal scales. At mean annual scale, climatic factors such as precipitation and potential evapotranspiration are the main controls on water balance and inclusion of these two factors may be sufficient for many purposes (Budyko, 1958). When finer time scale is of interest, one has to include additional factors such as rainfall seasonality and catchment water storage capacity

into the water balance model. These studies also demonstrated clearly the scientific merits and practical benefits of the method developed by Budyko (1958), which is an example of a topdown approach. The main feature of the top-down approach is the empirical nature, seeking an understanding of overall catchment behaviour and the function based on observed data. In the topdown approach, one starts from exploring firstorder controls in catchment water balance, and model complexity is increased only when deficiencies are identified.

The purpose of this study is to (1) investigate degrees of controls exerted by climate and catchment storage on water balance over time scales ranging from mean annual to monthly; (2) determine the first order factors affecting water balance and minimum model complexity required for Australian catchments.

1. STUDY CATCHMENTS AND DATA

The catchments included in this study have at least 5 complete years of unimpaired streamflow data and a catchment area between 50 and 2000 km². Unimpaired streamflow is defined as streamflow that is not subject to regulation or diversion. The streamflow data was assembled by Peel et al. (2000). In total, 265 gauging stations were selected. Of these, 125 catchments with at least 10 complete years of unimpaired streamflow data were used for the calibration dataset, and the remainder of 11 catchments with 5 to 20 complete years of unimpaired streamflow was used as evaluation dataset.

Monthly rainfall was estimated from 5 km by 5 km gridded daily rainfall (Peel et al., 2000) based on interpolation of over 6000 rainfall stations in Australia. The interpolation uses monthly rainfall data, ordinary Kriging with zero nugget and a variable range. Monthly rainfall for each point is converted to daily rainfall using daily rainfall distribution from the station closest to that point. Catchment average rainfall was estimated from the rainfall. Mean monthly dailv potential evapotranspiration was calculated based on the Priestley-Taylor equation (Priestley and Taylor, 1972). Details of the calculation can be found in Raupach et al. (2001).

2. METHOD

2.1. Equilibrium water balance model

The dynamic water balance of a catchment can be written as

$$\frac{d}{dt}w(t) = P(t) - E(w,t) - Q(w,t)$$
(1)

where w(t) is the soil water store in the catchment, and P(t), E(w, t), and Q(w, t) are precipitation, evapotranspiration and runoff.

When Equation (1) is integrated over a time interval T, one obtains

$$\frac{w(T) - w(0)}{T} = P - E - Q$$
(2)

where P, E and Q are time-averaged water fluxes given by

$$P = \frac{1}{T} \int_{0}^{T} P(t) dt, \quad E = \frac{1}{T} \int_{0}^{T} E(w, t) dt, \quad Q = \frac{1}{T} \int_{0}^{T} Q(w, t) dt,$$
⁽³⁾

The left hand side of equation (2) accounts for the effect of water storage changes in the timeaveraged water balance and decreases relative to the right hand side as T increases. If T is long enough, *i.e* decades, the storage term can be neglected relative to the fluxes and Equation (2) becomes

$$0 = P - E - Q \tag{4}$$

which can be thought as the "equilibrium" or "steady-state" water balance.

The equilibrium water balance model is based on methods proposed by Budyko (1958) and Fu (1981) and further developed by Milly (1994) and Zhang et al. (2001, 2004). It is a holistic approach that assumes the equilibrium water balance is controlled by water availability and atmospheric demand. The water availability can be approximated by precipitation, the atmospheric demand represents the maximum possible evapotranspiration and is often considered as evapotranspiration. potential Based on phenomenological considerations, Fu (1981) developed the following relationships for estimating mean annual evapotranspiration:

$$\frac{E}{P} = 1 + \frac{E_0}{P} - \left[1 + \left(\frac{E_0}{P}\right)^{\alpha}\right]^{1/\alpha}$$
(5)

where E_0 is potential evaporation and α is a model parameter. These relationships are shown in Figure 3 and details of the solutions are given in Zhang et al., (2004).

The method of Fu (1981) is similar to Budyko (1958) and assumes that the equilibrium water balance is controlled by water availability and atmospheric demand. On mean annual time scale, the water availability can be approximated by precipitation, while the atmospheric demand is represented by potential evapotranspiration.

By combining equations (4) and (5), one obtains the following expression for mean annual runoff:

$$Q = \left(P^{\alpha} + E_0^{\alpha}\right)^{1/\alpha} - E_0 \tag{6}$$



Figure 3. Ratio of mean annual evapotranspiration to rainfall (*E*/*P*) as a function of the index of dryness (E_0/P) for different values of parameter α .

2.2. Dynamic water balance models

Interannual variability in water balance

In moving from mean annual to shorter time scale, *i.e.* decreasing T, one generally has to account for the effect of catchment water storage change on the water balance. Unlike the equilibrium water balance modelling, little effort has been made to address the issue of interannual variability in water balance. Koster and Suarez (1999) assumed that interannual changes in catchment water storage are much smaller than the annual precipitation, evaporation, and runoff. As result, they suggested the following expression for evaporation ratio in a given year i:

$$\left(\frac{E}{P}\right)_{i} = F\left(\left(\frac{E_{0}}{P}\right)_{i}, \alpha\right)$$
(7)

where F is Equation (5). Based on the above relationship and by assuming negligible interannual variations in potential evapotranspiration, Koster and Suarez (1999) further showed that the ratio of the standard deviation of annual evaporation to that of annual precipitation can be expressed as a function of index of dryness:

$$\frac{\sigma_E}{\sigma_P} = F(\Phi) - \Phi F'(\Phi) \tag{8}$$

where Φ is the dryness index. Equation (8) is called the evaporation deviation ratio. Large values of this ratio indicate that most of precipitation variability becomes evapotranspiration variability, whereas small values mean that evapotranspiration variability is largely insensitive to variability in rainfall. From this relationship, Koster and Suarez (1999) showed that the ratio of the standard deviation of annual runoff to the standard deviation of annual precipitation can also be expressed as:

$$\frac{\sigma_{\varrho}}{\sigma_{p}} = 1 - \frac{\sigma_{E}}{\sigma_{p}} \tag{9}$$

Equation (9) can be used to describe interannual variability of streamflow (Sankarasubramanian and Vogel, 2002). One can also estimate annual runoff from equation (10) in analogy to equation (6) by ignoring interannual water storage change:

$$Q_{i} = \left(P_{i}^{\alpha} + E_{0i}^{\alpha}\right)^{1/\alpha} - E_{0i}$$
(10)

where subscript *i* represents year.

Water balance model at monthly time scale

As we move from mean annual to monthly time scale, the effect of catchment water storage on water balance becomes significant. As a result, variations in rainfall, potential evapotranspiration, and water storage need to be considered. Following a similar framework as the equilibrium water balance model or the mean annual water balance model, we need to appropriately define new state variables and fluxes.

The idea here is to develop a dynamic water balance model for monthly time scale using a method similar to Budyko's concept of water availability and atmospheric demand or the concept of "limits and controls" (Calder, 1998). In so doing, a few generalisations have to be made. It is assumed that rainfall in time step t will be partitioned into direct runoff $Q_d(t)$ and evapotranspiration plus increase in catchment water storage. In order to estimate direct runoff, the amount of rainfall available for storage and evapotranspiration is denoted as X(t). Following a similar argument to Budyko (1958), we can postulate that when the sum of available storage capacity and potential evapotranspiration is very large compared to P(t), X(t) will approach P(t) as little direct runoff will occur under this condition, while when the sum of storage capacity and potential evapotranspiration is very small, X(t) will approach the storage and evapotranspiration limit. The partitioning of rainfall can be expressed as:

$$X(t) = \begin{cases} P(t)f(\frac{S_{\max} - S(t-1) + E_0(t)}{P(t)}, \alpha_1), & P(t) \neq 0 \\ 0, & P(t) = 0 \end{cases}$$
(11)

where *f* is a function similar to equation (5) and α_I is a model parameter.

The monthly direct runoff $Q_d(t)$ is calculated as:

$$Q_d(t) = P(t) - X(t) \tag{12}$$

For the time scales considered here, soil water storage at the beginning of the time step S(t-1) also needs to be considered. Hence, water availability W(t) can be defined as :

$$W(t) = X(t) + S(t-1)$$
(13)

Evapotranspiration at the time step t can be determined from the water availability and atmospheric demand or potential evapotranspiration in a similar way to that of X(t):

$$E(t) = \begin{cases} W(t) f(\frac{E_0(t)}{W(t)}, \alpha_2), & W(t) \neq 0\\ 0, & W(t) = 0 \end{cases}$$
(14)

where α_2 is a model parameter. It should be noted that equation (14) is similar to Budyko's curve except that rainfall has been replaced with the available water W(t), which takes into consideration the effect of catchment water storage.

An intermediate step is necessary for the estimation of water storage S(t) and groundwater recharge R(t). Firstly, the following state variable is defined:

$$Y(t) = E(t) + S(t) \tag{15}$$

Potential evapotranspiration $E_0(t)$ can be considered as an upper limit for actual evapotranspiration E(t) in terms of energy availability, while W(t) represents upper limit in terms of water availability. Similarly, water storage capacity S_{max} can be considered as the upper limit for water storage S(t). It is obvious that the upper limit for Y(t) can be estimated as the sum of potential evapotranspiration and soil water storage capacity, i.e. E_0+S_{max} . It should be noted that Y(t) is also called *evapotranspiration* opportunity after Sankarasubramanian and Vogel (2002). Secondly, Y(t) can be estimated from the following relationship:

$$Y(t) = \begin{cases} W(t) f(\frac{E_0(t) + S_{\max}}{W(t)}, \alpha_2), & W(t) \neq 0\\ 0, & W(t) = 0 \end{cases}$$
(16)

Given the estimates of Y(t) and E(t) from equations (14) and (16), soil water storage S(t) is calculated from equation (15) and groundwater recharge R(t) is estimated as:

$$R(t) = W(t) - Y(t)$$
(17)

Finally, groundwater storage is treated as linear reservoir, so that the groundwater balance and baseflow can be calculated as:

$$Q_b(t) = dG(t-1) \tag{18}$$

$$G(t) = (1 - d)G(t - 1) + R(t)$$
(19)

where G is groundwater storage and d is a constant. Equation (18) represents a linear storage-discharge relationship and constant d resembles the reciprocal of the retention constant (Wittenberg, 1999).

The evaluation of the water balance equation requires extra information about catchment physical characteristics, climatic variables, and further relationships. In the case of the dynamic water balance, the extra information may include equations for estimating evapotranspiration and deep drainage. Such models are generally complicated with a large number of parameters (Walker and Zhang, 2002). The models for the equilibrium water balance are much simpler, but can still provide useful insight into the key processes responsible and they are very robust.

RESULTS AND DISCUSSION 4.1 Equilibrium water balance modelling

Equation (5) can be used to calculate actual evapotranspiration when mean annual values of rainfall and potential evapotranspiration are known. A comparison of observed and calculated evapotranspiration is shown in Figure 4. In the calculation, a single value of 2.63 was used for the α parameter and it resulted in a mean annual error (MAE) of 60 mm or 8%. The correlation coefficient is 0.87 and the best-fit slope through the origin is 0.99. Comparable results were obtained for mean annual runoff.



Figure 4. Comparison of predicted and measured mean annual evapotranspiration

3.2. Interannual variability in water balance

Figure 5 compares the predicted and observed values of interannual variability of streamflow for all the catchments. The correlation coefficient is 0.83 and the slope of regression line through the origin is 0.89, indicating underestimation of streamflow variability by the model. These results are consistent with the findings of Sankarasubramanian and Vogel (2002)for catchments in the U.S. and Arora (2002) for GCM The advantage of the method simulations.

developed by Koster and Suarez (1999) is that the interannual variability in streamflow can be calculated from a simple relationship using the dryness index. The scatter in the results may be due to rainfall seasonality and the omission of interannual storage variations. The dynamic water balance model with annual time scale works well for the majority of the catchments (see an example in Figure 6) and this indicates that the storage effect can be neglected in these catchments.



Figure 5. Comparison of interannual variability of streamflow predicted using the relationship in equation (9) with observed values



Figure 6. Comparisons of predicted and observed annul runoff for the Pipers River catchment.

4.3. Monthly water balance modelling

The monthly water balance model was calibrated against observed monthly streamflow to obtain best fit parameter sets. Then relationships between the parameter sets and catchment characteristics were developed using regression analysis to allow monthly streamflow predictions of ungauged catchments. In the model validation, the monthly water balance model was tested using data for 11 catchments from various regions in Australia.

For model validation, it is important to select appropriate statistics to avoid misleading conclusions (Garrick et al., 1978, Legates and McCabe, 1999). A number of performance statistics have been advocated for determining the applicability or accuracy of hydrologic models (Legates and McCabe, 1999; Refsgaard and Knudsen, 1996). In this study, we decided to use five statistics following recommendations by Legates and McCabe (1999) and they are: the modified coefficient of efficiency (E1), the modified index of agreement (d1), mean absolute error (MAE), the mean (Q), and the standard deviation (σ). Table 1 lists the statistics calculated for the test catchments. The difference between the predicted and observed mean monthly streamflow over the whole period of record ranged from -0.2 to 22.2 mm. The maximum mean absolute error was 34.9 mm. The modified index of agreement is high for all the catchments. The performance statistics listed in Table 1 are comparable with the ones reported by Legates and McCabe (1999) and indicate that the model is adequate in reproducing the observed monthly streamflow.

Table 1. Statistical results comparing predictedversusobservedstreamflowforvalidationcatchments

ID	E1	d1	MAE	\overline{Q}_{obs}	$\overline{Q}_{_{pred}}$	$\sigma_{\scriptscriptstyle obs}$	$\sigma_{\scriptscriptstyle pred}$
237205	0.25	0.68	4.6	7.5	6.0	8.8	11.7
319204	0.59	0.76	11.0	27.0	27.5	36.5	26.8
405228	0.66	0.83	5.4	15.9	15.3	21.5	23.7
611111	0.48	0.72	6.8	10.9	11.2	18.9	18.4
111007	0.68	0.84	34.9	122.9	145.1	180.9	197.7
143110	0.66	0.83	6.0	12.8	10.5	36.4	39.4
208019	0.70	0.84	14.9	42.5	35.3	81.1	66.0
410105	0.75	0.87	3.8	12.2	12.0	32.7	31.5
215005	0.37	0.66	24.2	36.7	12.9	64.5	30.6
318900	0.58	0.80	11.1	24.4	24.7	43.5	45.4
421106	0.52	0.76	3.5	6.0	6.4	13.2	12.7

The observed and predicted monthly streamflow time series were plotted for two selected catchments (Figure 7). To show the full extent of the model performance, we selected the best and worst cases. In the best case, not only was the model able to capture high flows but also low flows in terms of magnitude and timing. In the worst case, the model showed some discrepancies compared with the observed streamflow.



Figure 7. Comparison of observed (line with dot) and predicted (solid line) monthly streamflow for

selected catchments. The results are shown for best (bottom) and worst (top) cases as indicated.

4. CONCLUSIONS

We used a top-down approach to explore the effects of climate and catchment characteristics on water balance over variable time scales. At mean annual scale, the water balance is dominated by climatic factors such as average precipitation and potential evapotranspiration. Based on the "limits" concept, Budyko (1958) developed a simple water balance model and it includes only the first-order factors. The key to the success of Budyko-like models is to capture the dominant controls on water balance at the expenses of minor processes. At annual time scale, results from this study showed that the interannual variability of water balance in many Australian catchments can be reasonably well estimated from the dryness index using the method of Koster and Suarez (1999). Predicted annual runoff using this method showed good agreement with the observed runoff for some selected catchments, indicating minimum effect of storage change and rainfall seasonality in these catchments. At monthly time scale, one has to take into consideration the effects of soil moisture dynamics on water balance and hence increased model complexity is required. Following the topdown approach, we modified the mean annual water balance model of Fu (1981), a Budyko-like model, to include additional processes and factors. The resulting model is a parsimonious conceptual monthly water balance model, which showed encouraging results when compared with measured monthly streamflow.

Understanding individual hydrological processes and their relationships with climate and catchment characteristics is an important step in predicting catchment water balance. However, this does not mean that one should try to incorporate every known process into a model. The interaction and co-evolution of these processes may manifest themselves in such a way that the overall behaviour of the catchment can be described by simple relationships. The strength of purpose derived relationships, such as Budyko (1958) and Fu (1981), is to bypass the need to describe each of these processes in detail. Rather they try to encapsulate the combined effect of these competing processes within a range of the parameter space that describes catchments. The development of the monthly water balance model followed the same path and proved to be beneficial.

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