# Modeling And Simulating Hierarchies Using An Agent-Based Approach

<sup>1</sup><u>Müller, J.-P.</u>, <sup>1,2</sup>Ratzé, C., <sup>3</sup>Gillet, F. and <sup>2</sup>Stoffel, K.

<sup>1</sup>CIRAD-TERA-GREEN, France, <sup>2</sup>Neuchâtel University, Switzerland, <sup>3</sup>EPFL-ENAC-ISTE-ECOS, Switzerland E-Mail: <u>jean-pierre.muller,ratze}@cirad.fr</u>

Keywords: Hierarchies, multi-agent systems, complex systems, AGR.

#### EXTENDED ABSTRACT

The notion of complex system covers many different meanings. Socio-economical and ecological systems are typically kind of 'organized complex systems', defined as a middle number of heterogeneous components interacting in an intricate manner and producing emergent properties and dynamics.

The only possibility to understand and to model such complex systems is to exhibit some organizational principles behind the apparent inextricability. The hierarchy theory in ecology allows a decomposition of this kind of systems that could improve our understanding of the underlying dynamic processes. Formally, it is a view of ecological systems, which takes the scales of observation explicitly into account and which tries to conceptualize the phenomena at their proper scale. This conceptualization mostly exhibits nested hierarchical systems in the sense that a system at a given scale contains its component subsystems at the underlying scale. Socio-economical systems additionally exhibit nonnested hierarchies in which representatives of social groups (and not the groups themselves) form higher level groups or organizations.

The aim of this paper is to propose a set of concepts to formalize both nested and non-nested hierarchical systems. For the first kind of systems, we will review the hierarchy theory in ecology to extract the main concepts, mainly the concept of holon. Independently, we will review the concepts of agents, groups and roles (AGR) to represent ecosociological systems as proposed in Ferber et Gutknecht (1998). Finally, we will propose a modeling and simulation formalism able to represent these concepts in a unified framework and discuss some of its advantages and disadvantages.

## 1. INTRODUCTION

The notion of complex system covers many different meanings. Some complex systems can be found in so-called 'self-organized systems' or 'complex adaptive systems' in which the dynamics and the emergent properties are consequences of interactions between heterogeneous components at different spatio-temporal scales (Cowan et al., 1994; Wu and Marceau, 2002). Socio-economical and ecological systems are typically a kind of 'organized complex systems', defined as a middle number of heterogeneous components interacting in an intricate manner (O'Neill et al., 1986). This kind of complex systems must be distinguished from 'disorganized complex systems' whose interactions among a large number of homogeneous components can be captured by an average behavior (like in statistical dynamics).

The only possibility to understand and to model such organized complex systems is to exhibit some organizational principles behind the apparent inextricability. The hierarchy theory in ecology (O'Neill et al., 1986) allows a decomposition of this kind of systems that could improve our understanding of the underlying dynamical processes. Conceived by its authors as an epistemology, this theory has the important property to emphasize on both a top-down and bottom-up perspective (Wu and David, 2002). Formally, it is a view of ecological systems, which takes the scales of observation explicitly into account and which tries to conceptualize each phenomenon at its proper scale. This conceptualization mostly exhibits nested hierarchical systems in the sense that a system at a given scale contains its component subsystems at the underlying scale. Socio-economical systems additionally exhibit non-nested hierarchies in which representatives of social groups (and not the groups themselves) form higher level groups or organizations: a company contains its departments but the manager does not contain its middlemanagers, however we are facing two hierarchies.

The aim of this paper is to propose a set of concepts to formalize both nested and non-nested hierarchical systems. For nested hierarchies, we will review the hierarchy theory in ecology to extract the main concepts, and in particular the concept of holon. For non-nested hierarchies, we will review the concepts of agents, groups and roles (AGR) to represent eco-sociological systems as proposed in (Ferber and Gutknecht, 1998). Finally, we will propose a modeling and simulation platform called MIMOSA able to represent these concepts in a unified framework and discuss some of its advantages and disadvantages.

#### 2. SOME CONCEPTS OF THE HIERARCHY THEORY

The central idea of the hierarchy theory is to derive the hierarchical organization from differences in temporal and spatial scales between the phenomena of interest.

In an observational approach, empirical studies reveal the existence of thresholds in the continuum possible scales of observations, which of correspond to distinct levels of organization (Marceau, 1999). Two complementary concepts are important in the understanding of the link between scales and levels of organization: i) scale domain is a region of the scale spectrum over which the structure and the functional relationships between variables describing a particular object of interest (process, entity, phenomenon) do not change or change monotonically (in an easily predictable way) with change in scale; ii) such domains are separated by thresholds that are relatively sharp transitions or critical points along the scale continuum where a shift in the relative importance of variables influencing a process occurs (Marceau, 1999).

At the beginning (O'Neill et al., 1986), hierarchy theory in ecology has focused on temporal and functional aspects of ecosystems: the hierarchical levels were defined by different characteristics of the processes (e.g. behavioral frequencies, relaxation time, cycle time or response time). In the spatial context of landscape ecology, a more structural approach has emerged that integrates the spatial aspects, in the so-called 'hierarchical patch dynamics' (HPD) paradigm (Wu and Levin 1997). The idea is that we can relate functional processes with structural spatial properties across the scales with the help of patches (Reynolds and Wu, 1999).

The next problem is to find appropriate scaling laws in order to appropriately relate information across a wide range of scales (Wiens, 1995 in Blaschke and Petch, 1999). Extension across the scale thresholds may be difficult or impossible because of the instability in the dynamics of the transition zone between two domains of scale (Marceau, 1999). If entities and relationships between variables emerge at specific scales, there must be a way to define and relate them across discrete levels of organization (Marceau, 1999).

In the following, we shall review the main concepts of the hierarchy theory in order to precise and to justify the meaning we want to consider.

#### 2.1. The concept of scale

In a recent review, Dungan et al. (2002) identified three dimensions of the scale concepts in a spatial context: phenomenon, sampling and analysis; we will outline: (i) an observational meaning, (ii) an ontological meaning and (iii) a representational meaning: (i) in the observational meaning, scale conceptually represents a filter or a window of perception through which the world is quantified (Hay et al., 2002). This observation scale is not a property of the world, but is generated by the sampling of an external observer (Allen and Hoekstra, 1992); (ii) the ontological meaning of scale refers to the notion of characteristic (or inherent or intrinsic) scale of an object (entity, process or phenomenon), i.e. to the effective size or measure of the object and/or its properties and attributes (Marceau, 1999). The intrinsic scale of existence of an entity determines its proper window of interaction within its environment; (iii) in order to build new scales, two scaling operations are possible: scaling up and scaling down. Scaling refers here to the transfer of information or data from one scale to another (Blaschke and Petch, 1999).

For a constructive simulation of hierarchical systems, we shall consider the two last definitions, i.e. we have to account for the intrinsic scale of the entities we are modeling and to be able to express scaling-up and scaling-down operations.

# 2.2. The concept of holon

As defined in the introduction, a level of organization is composed of interacting components. When facing "organized complexity", some of the components will interact weakly and others strongly, creating boundaries around strongly interacting components regarding their surrounding components. This is a functional, spatial and temporal way to delimit the subsystems and organize them into hierarchies.

Introduced by Koestler (1968), the notion of holon is defined as being both an atomic whole and a compound. As such, this notion naturally implies for the hierarchy theory to articulate a subsystem as a set of strongly interacting components at a given level of organization and its representation as an atomic whole in the next upper level of organization (Figure 1). It can also naturally scale up and down the information between the two levels of organization it relates, taking into account the representational meaning of scale. For uniformity, we will consider that any component at any level of description is a holon, being an atomic whole potentially or effectively decomposable into a subsystem at the lower level of organization. Therefore, the term "holon" will be used uniformly thereafter making the distinction between atomic or "component holons" when talking about the components and "compound holons" when talking about the subsystems.





# 2.3. Hierarchies as points of view

Additionally, a given hierarchy derives from a given perspective or point of view on the ecosystem. Furthermore, the hierarchy theory considers the dynamics resulting from the interplay of different hierarchically structured perspectives (O'Neill et al., 1986). Various thematic points of view (e.g., population, matter and energy flows) can lead to different organizations that are interacting at different levels of organization. Therefore, the hierarchy theory will encompass all the thematic points of view interacting in the ecological system.

## 3. THE AGENT-GROUP-ROLE PARADIGM

In a constructive approach, that is a modeling and simulation perspective, we want to express a hierarchically structured model in a modeling framework effectively functioning with various explicit levels of organization based on multiple scale domains of description and interaction, which can be used to assess and perhaps better understand the consequences of the organizational principles of embedded hierarchies. We follow here the ideas of Bragg et al. (2004). In this direction, a lot of work remains to be done. Most of the simulations exhibiting multiple scales or even a hierarchical structure were built on a caseby-case basis (e.g., Wu and Levin, 1997). So far, no specific modeling method, such as differential equations, cellular automata or multi-agent systems, permits to properly and specifically express a model based on the main ideas of the hierarchy theory in a constructive way.

The hierarchy theory in its spatial form calls for a modeling and simulation formalism which is mainly individual- or agent-based<sup>1</sup>. Effectively, the

<sup>&</sup>lt;sup>1</sup> Usually the term 'individual-based modeling' (IBM) is used when the entities are homogeneous and the term

very notion of holon (either component or compound) can more easily be modeled by interacting entities. The mapping of patches at various scales into agents seems direct. However, if we consider the epistemological perspective, we are not facing the description of an entity per se but only the entity from a given point of view, i.e. only an aspect of the entity. Moreover, the combinations of different perspectives can only be made if we consider combining aspects of various entities rather than the entities themselves. For example, a tree can be considered from the perspective of the carbon cycle as a producer or consumer of carbon dioxide, from the perspective of the water cycle as a producer or consumer of water and from the perspective of forestry as a producer of wood, etc.. From each perspective, we are not describing a tree but only some aspects of it. The tree becomes the place of coordination of these various aspects or, better expressed, these functional roles.

In the same way, the multi-agent domain recently moved from 'agent-centered multi-agent systems' or ACMAS to 'organization-centered multi-agent systems' or OCMAS in which the roles are described first and put together within organizations. In this new paradigm, the agents become the place of coordination of various roles to be played in various groups. This approach has first been used by Benoit Durand (Durand, 1996) for modeling the epidemiology of the foot-andmouth disease as a combination of various perspectives, among which the sanitary, the epidemiological production, the and the economical perspectives. This approach has been formalized by Ferber and Gutknecht under the term AGR for Agent-Group-Role (Ferber and Gutknecht, 1998), implemented into the MadKit platform (http://www.madkit.org/, Gutknecht and Ferber, 2000). The related definitions are the following:

- A *role* is a functional abstraction of the expected behavior of an agent within a group;
- An *agent* is an autonomous entity (Ferber, 1999) able to play different roles in different groups;
- A group is a set of agents where each agent has to play a dedicated role.

For example, the figure 2 describes the functioning of a program committee made of various groups: i) the submission group allows the agents playing the role of authors to submit the papers to the submission receiver; ii) the program committee group distributes the paper to the committee members and makes the decision on selected papers; iii) the evaluation groups organize the reviewing of the submitted papers through the roles of reviewing manager and reviewers.

An agent (represented as a skittle) can simultaneously be the submission receiver, a reviewer, an author and a program chair (with adequate rules of equity regarding the reviewing process) as suggested by the skittles vertically crossing several groups.

This paradigm has two advantages regarding our needs: i) It is easy to model non-nested hierarchies by having agents of one group playing the role of representative within another group; ii) The representation of multiple epistemological perspectives is easy through the notion of organization seen as a set of interacting roles (i.e. aspects of entities) regardless of the entities playing these roles. Moreover, the notion of agent is used to coordinate the various perspectives in an elegant way.

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Figure 2. The program committee example.

However, this formalism is not suited for modeling nested hierarchies because AGR does not allow groups to be agents. Therefore, the notion of holon which is simultaneously a set of interacting components (i.e. a group of agents) and a whole (i.e. an agent) cannot be represented.

MIMOSA is designed to overcome this limitation by allowing an additional coordination of perspectives: the perspective where entities are seen as wholes and the perspective from which entities are seen as compounds. In the following we will review some modeling platforms before introducing our proposal.

<sup>&#</sup>x27;agent-based modeling' (ABM) or 'multi-agent system' (MAS) when the entities are heterogeneous.

#### 4. THE MIMOSA PLATFORM

#### 4.1. State of the art

It is not possible to describe all the modeling and simulation platforms but we will concentrate on the main available tools for multi-agent simulation used by an important user community and dealing in a way or another with hierarchies.

Swarm (Daniels, 1999) is suited for embedded hierarchies by the very concept of swarms as sets of interacting agents or swarms (therefore the definition is naturally recursive). DEVS (Zeigler et al., 2000), and SME (Voinov et al., 2002) are also suited for recursive definition of models but. unlike Swarm, in a purely static way (the connectivity between entities are fixed but some extensions). None of these platforms allows for multiple perspectives. As we have seen, MadKit is suited for multiple perspectives and non-nested hierarchies but does not allow for recursive definition of agent-hood. Finally, none of these platforms are able to dynamically form groups (and therefore new entities) from the observation of emerging organizations.

MIMOSA is designed to overcome these limitations and additionally allow for a variety of formalisms in addition to multiple levels and scales. The general structure of MIMOSA is the following: i) the level 0 is a middleware allowing multiple automata (in the sense of DEVS, i.e. black boxes with incoming and outgoing events) to communicate through well defined interfaces, ensuring time coherence of the simulation; ii) the level 1 defines three kinds of automata: the elements (or aspects, or roles) called the components, the sets of elements called the compounds and the relations within or between compounds. Any formalism is assumed to be a combination of specific components (its building blocks) through specific compounds and relations (its grammar); iii) the level 2 defines a set of formalisms using the structures provided at level 1 for describing automata, dynamic systems, multiagent systems, spatial and temporal structures, etc., from which specific models can be build and put together in various ways.

We will not go into the details of level 0. We shall directly recall some notion of level 1 before proposing a way to represent AGR and the holons within this framework. Further details on MIMOSA itself can be found in Müller (2004).

#### 4.2. The Level 1

The level 1 of MIMOSA is a toolbox to describe the structure of the formalism, its dynamics being provided by the lower level. As Reynolds & Wu (1999) for ecological systems, a correspondence is assumed between functional and structural decompositions. We shall present these two aspects successively.

#### The structure

Basically, any formalism is made of parts and wholes. For example, a space is made of places, a state-chart is made of states, a cellular automata is made of cells, etc. Consequently, we have introduced the notion of *component* for describing the parts seen as indecomposable (atomic) and the notion of *compound* for describing the wholes seen as sets of components. The structure is not recursive as compounds cannot be themselves components. Additionally the compounds introduce the way to name their components. For example, a space names its components (places) either by names (Lisbon, France, etc.) or by coordinates.

In addition, we introduce three notions of *relation* for various purposes:

**intra-compound relations**: are relations between the components of a compound. They can be used to describe the transitions between the states, the relationships between places (adjacency, containment, etc.) or the neighborhood relation among the cells of a cellular automaton;

**inter-compound relations:** are relations between two compounds (i.e. relations between two sets). For example, they can be used to describe the mapping of one space into another;

**component-compound relations:** are relations between a component and a compound. They are used to describe that an atomic component from one point of view can be seen as compound from another point of view. They are the basis for describing some form of holons as we shall see later on.

#### The dynamics

For introducing the dynamics, any object of the system (components, compounds and relations) is provided with an internal *state* which is encapsulated (in the sense that it is not accessible from the outside of the object) and can evolve over time. Additionally, any object of the system can change state in reaction to *events*. An event usually

results in a state change and the production of new events. We are using discrete event simulation mechanisms for the event management. However, fixed time step simulation can be described by using clocks explicitly as generators of stamped events. Moreover, the use of state and events as continuous functions of time is implemented for articulating both continuous and discrete time. Time coherence and the simulation mechanism is provided by the level 0.

## **4.3.** Implementing non-nested hierarchies

AGR being suitable for non-nested hierarchies, we shall successively describe the agents, the groups and the roles.

# The agents

An agent can be seen as a component of a population of agents (the compound). Therefore we introduce both the notion of population and agent. Regarding the dynamics, the population can grow and shrink in response to related events. Additionally, each agent is itself provided with a state and in charge of coordinating its multiple roles.

In order for an agent to be properly situated in an environment, we define an intra-compound relation between the agents of the population. This relation can change over time as the acquaintances between the agents change.

# The groups and roles

In the AGR model, an agent can only communicate within a group by playing a role and can play several roles in several groups at the same time (said otherwise, two groups can only communicate through a shared agent: the notion of representative). Therefore when an agent wants to communicate, it must create a new group or enter an existing group. The formulation in Mimosa is straightforward. The components are the roles and the compounds are the groups. Each role (as each group) has a name like in MadKit. A participation relation maps the set of agents to the set of roles specifying which agent plays which role. An instance of this relation is created for each group and allows the agent to send messages (a particular kind of event). The role receives the message and sends it to the related role which posts it into its related agent mailbox. In the current implementation, the roles only specify the destination of the outgoing messages. Another possibility is to embed into the roles the full interaction protocols, allowing to describe them separately from the agents and therefore making

them reusable. This approach has been proposed by Hilaire et al. (2000) and fully implemented by Amiguet (Amiguet et al., 2002). We intend to port this latter implementation into MIMOSA.

# 4.4. Implementing nested hierarchies

In the previous paragraph, we have only used the intra-compound relations for describing an abstract topology on the agents' population, and the intercompound relations to relate the agents to the roles (more precisely, the set of agents to the set of roles). The nested hierarchies can be implemented in various ways depending on the kind of holons we want to represent: i) if the hierarchy is fixed, a recursive definition of compounds as components can be used even if not directly implementable in MIMOSA. In this case, a component-compound relation must be used to link a component seen as a whole to a compound which is itself a set of agents. In this way, it is easy to describe fixed hierarchies; ii) if the hierarchy is emerging, we have to relate a set A of interacting components (or agents) to another set B of interacting components (also agents) where each component of B is related to a subset of components of A. In a first approximation, such a relation is composed of:

- a set of hyper-edges, each of these relating a component to a set of components;
- a criterion for determining in a computational way when a subset of the components of A forms a coherent entity to be represented as a component in B. Following the hierarchy theory in its observational version, this criterion could be based on change in exchange rates between the components as well as their topology.

# 4.5. An example

The figure 3 shows a screen of the fireman example. In this example, a cellular automaton has been added for simulating fire propagation within a forest. A *position* relation relates each fireman to it's the cell it is onto, to the cellular automaton. The population of firemen is fighting the fire in the following way:

- If a fireman is near a fire cell, it puts water onto the fire cell and it becomes the manager of a coordinating group.
- If a fireman is not near a fire cell, it becomes coordinated by all the existing coordinating groups, if not any, it moves randomly. If coordinated, it moves towards the position of the closest manager.

This basic behavior uses the usual AGR paradigm. In addition, all the groups are represented as agents in a population of fireman teams (where each component or agent is a team). This is a straightforward implementation of the concept of holon. The role of this particular kind of agent is to ensure that the groups remain as scattered as possible. Therefore, they implement the notion of constraint as described in the hierarchy theory in the sense that the group as a whole constrains the dynamics of its components (the firemen). Of course the dynamics between the firemen on one hand and the firemen teams on the other hand are taking place on different scale or rate: the dynamics within teams are faster than between the teams.

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Figure 3. The fireman example.

#### 5. CONCLUSIONS

After having introduced the notions of nested hierarchies as described in the hierarchy theory in ecology and the non-embedded hierarchies as described in the AGR paradigm, we have reviewed most of the existing modeling and simulation platforms suitable for our purpose. None of them being adequate to express both embedded and nonembedded hierarchies as well as multiple epistemological perspectives one can draw on a complex system, we proposed a meta-platform called MIMOSA designed to overcome these limitations. After having described the basic concepts of the platform, we have described how the AGR concepts solving the multiple perspective and the non-embedded hierarchies on one hand and the concept of holon solving the embedded hierarchies on the other hand can be described within the MIMOSA framework. A toy example is provided to illustrate our proposal.

A lot remains to be done. In particular, the concept of holon, both static and emerging, has to be further described and formalized as a way to specify the relations used for their implementations. Furthermore, the observation of one level of simulation to dynamically build a higher level of simulation from emerging organization has to be explored and experimented. A more complex example has to be built to really validate the approach. We intend to apply this framework for modeling and simulating a sylvopastoral ecosystem submitted to human activities.

# 6. ACKNOWLEDGMENTS

This work has been partly financed by the Swiss National Science Foundation (grant FNRS 2153-63958.00 and NCCR Plant Survival) and CIRAD.

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