Bayesian Spatial Panel Probit Model with an Application to Business Cycle in Japan[1]

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EXTENDED ABSTRACT

This paper considers the panel probit model with spatial dependency from a Bayesian point of view. We consider Markov chain Monte Carlo methods to estimate the parameters of the model. Our approach is illustrated with simulated data set. Furthermore, we explore the spatial interaction of business cycle across 47 prefectures from the period 1991 to 2000 in Japan. Spatial dependency can be found in business cycle in Japan.

The probit model has been widely used in qualitative data regression, especially in microeconometrics or business cycle analysis. It is also extended to panel probit model. Maddala (1977) gives excellent surveys of panel probit model. Although the probit model is useful for qualitative data analysis, it is difficult to estimate by maximum likelihood, when we use the panel data as is pointed out by Bertschek and Lechner (1998). Therefore, alternative estimation methods like GMM methods are proposed.

Since the seminal work by Anselin (1988), the spatial dependency becomes the concern of economic activity. Therefore, the problem of spatial dependency in probit model has been examined by some researchers, both analytically and empirically. As it is difficult to estimate by maximum likelihood method, all of the models are estimated by Markov chain Monte Carlo (MCMC) following LeSage (2000). Holloway *et al.* (2002) applies the LeSage's (2000) spatial probit model to HYV rice adoption. Smith and LeSage (2004) also proposes the spatial dependency with individuals in each location and apply to the 1996 presidential election results for 3110 US counties.

In business cycle analysis, it is very important to capture the turning point. Therefore, several models are proposed: probit and logit model (Maddala (1992)); sequential probability recursion model (Neftci (1982)); Markov switching model (Hamilton (1989)); dynamic Markov-switching factor model (Watanabe (2003)). If we consider this fact that the spatial dependency is the concern of economic activity, we also have to consider the spatial interaction in examining business cycle. But there do not exist such panel models with spatial dependency.

From a Bayesian point of view, Kakamu and Wago (2005) examine a spatial panel model. This paper considers the properties of panel probit model with spatial dependency extending Kakamu and Wago (2005) following LeSage (2000). This paper also applies to business cycle in Japan from the period 1991 to 2000 across 47 prefectures. From our empirical results, we can find (1) if the unemployment rate rise, the probability of business rising fall, (2) there exists spatial dependency, (3) the potential probability is different among each prefectures and we can capture the changes in probability of business cycle in each prefectures.

From the results for the simulated data, we found serious spatial correlation biases and the importance of spatial correlation. As for the real data example, we considered the business cycle in Japan. From the results, we can find (1) if electricity demand, sum of annual sales of industrial goods and balance at the bank rise, the probability of business rises, (2) there exists spatial dependency, (3) the potential probability is not different among each prefecture and (4) we can capture the changes in probability of business cycle in each region.

Finally we will discuss our remaining issues. In this paper, we proposed spatial panel probit model. But if the period becomes long, it takes very long time to estimate the model analytically. Therefore, we can only capture the changes in probability and cannot examine the turning point of business cycle empirically. It is also important to consider the efficient method to estimate the long run panel data. But we think this is the first step to examine the spatial interaction in econometrics using panel data.

This paper is organized as follows. In the next section, we summarize the spatial panel probit model and discuss computational strategy of the MCMC methods. In Section 3 our approach is illustrated with simulated data set. Section 4 presents the empirical results based on the business index records across 47 prefectures from 1991 to 2000 in Japan. Section 5 summarizes the results with concluding remarks.

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1. INTRODUCTION

The probit model has been widely used in qualitative data regression, especially in microeconometrics or business cycle analysis. It is also extended to panel probit model. Maddala (1977) gives excellent surveys of panel probit model. Although the probit model is useful for qualitative data analysis, it is difficult to estimate by maximum likelihood, when we use the panel data as is pointed out by Bertschek and Lechner (1998). Therefore, alternative estimation methods like GMM methods are proposed (Bertschek and Lechner (1998); Greene (2004) and so on).

Since the seminal work by Anselin (1988), the spatial dependency becomes the concern of economic activity. Therefore, the problem of spatial dependency in probit model has been examined by some researchers, both analytically and empirically. As it is difficult to estimate by maximum likelihood method, all of the models are estimated by Markov chain Monte Carlo (MCMC) following LeSage (2000). Holloway *et al.* (2002) applies the LeSage's (2000) spatial probit model to HYV rice adoption. Smith and LeSage (2004) also proposes the spatial dependency with individuals in each location and apply to the 1996 presidential election results for 3110 US counties.

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This paper is organized as follows. In the next section, we summarize the spatial panel probit model and discuss computational strategy of the MCMC methods. In Section 3 our approach is illustrated with simulated data set. Section 4 presents the empirical results based on the business index records across 47 prefectures from 1991 to 2000 in Japan. Section 5 summarizes the results with concluding remarks.

2. THE SPATIAL PANEL PROBIT MODEL

2.1. Joint Posterior Distribution

First of all, to derive the joint posterior distribution to implement Bayesian analysis, we will consider the likelihood function of spatial panel probit model. Let y_{it} denote a binary 0/1 response on *i*th unit and *t*th period, let $Y = (y_{11}, y_{21} \cdots, y_{it}, \cdots, y_{NT})'$ denote the collection of responses of all *i*th unit and *t*th period and let θ_{it} denote the spatial interaction effect of *i*th unit and *t*th period. According to the panel probit model (see e.g. Maddala (1977)), the probability that $y_{it} = 1$ conditioned on parameters α_i, β, a set of covariates x_{it} and spatial interaction θ_{it}, is given by $\Pr(y_{it} = 1) = \Phi(\alpha_i + x_{it}\beta + \theta_{it})$ where $\Phi(\cdot)$ denotes the cumulative distribution function of normal distribution. Therefore, the likelihood function is as follows;

$$p(Y|\alpha,\beta,X,\theta) = \prod_{i=1}^{N}\prod_{t=1}^{T} \Phi(\alpha_i + x_{it}\beta + \theta_{it})^{y_{it}} \{1-\Phi(\alpha_i + x_{it}\beta + \theta_{it})\}^{1-y_{it}},$$

where $\alpha = (\alpha_1, \cdots, \alpha_i, \cdots, \alpha_N)'$, $X = (x'_{11}, x'_{21}, \cdots, x'_{it}, \cdots, x'_{NT})'$ and $\theta = (\theta_{11}, \theta_{21}, \cdots, \theta_{it}, \cdots, \theta_{NT})'$. But it is difficult to estimate by maximum likelihood since we cannot identify the spatial interaction dependency by itself.

Then, we consider the spatial autoregressive process following LeSage (2000). To specify the spatial dependency, θ_{it}, we introduce the latent variables[1] $Z = (z_{11}, z_{21}, \cdots, z_{it}, \cdots, z_{NT})'$ such that;

$$z_{it} = \begin{cases} \geq 0, & \text{if } y_{it} = 1 \\ < 0, & \text{if } y_{it} = 0. \end{cases}$$

Suppose also that the $N \times N$ weight matrix W and spatial interaction effect ρ are constant in each period. Then, the spatial dependency can be expressed as $\theta_{it} = \sum_{j=1}^{N} \rho w_{ij} z_{jt}$ and the spatial panel probit model will be written as;

$$z_{it} = \alpha_i + x'_{it}\beta + \sum_{j=1}^{N} \rho w_{ij} z_{jt} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0,1) \qquad (1)$$

where w_{ij} means the *ij*th element of the weight matrix. Therefore, the cumulative distribution

[1] See Taner and Wong (1987).

function becomes as follows;

$$\Phi(\alpha_i + x_{it}\beta + \sum_{j=1}^{N} \rho w_{ij} z_{jt}) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[- \frac{(z_{it} - \alpha_i - x_{it}\beta - \sum_{j=1}^{N} \rho w_{ij} z_{jt})^2}{2} \right] \times \mathbf{1}_{[0,\infty)}(z_{it}) dz_{it},$$

$$1 - \Phi(\alpha_i + x_{it}\beta + \sum_{j=1}^{N} \rho w_{ij} z_{jt}) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[- \frac{(z_{it} - \alpha_i - x_{it}\beta - \sum_{j=1}^{N} \rho w_{ij} z_{jt})^2}{2} \right] \times \mathbf{1}_{(-\infty,0)}(z_{it}) dz_{it}.$$

where $\mathbf{1}_{[0,\infty)}(z_{it})$ and $\mathbf{1}_{(-\infty,0)}(z_{it})$ denote the indicator functions which takes 1 if $z_{it} \in [0,\infty)$ and $z_{it} \in (-\infty, 0)$, respectively.

Since we adopt a Bayesian approach, we complete the model by specifying the prior distribution over the parameters. In addition, as Kakamu and Wago (2005) suggests the hierarchical Bayes model to estimate the spatial panel model, we will also consider the following hierarchical priors;

$$p(\beta, \rho, \alpha, \alpha_0, \xi^2) = p(\beta)p(\rho)p(\alpha, \alpha_0, \xi^2)$$
$$p(\alpha, \alpha_0) = \left\{ \prod_{i=1}^{N} p(\alpha_i|\alpha_0) \right\} p(\alpha_0|\xi^2)p(\xi^2)$$

where α_0 and ξ^2 means the average of α and the variance. And prior distributions are as follows;

$$\begin{aligned} p(\beta) &\sim N(\beta_*, A_*^{-1}), \\ p(\alpha_0|\xi^2) &\sim N(\mu_*, \xi^2/N_*), \\ p(\alpha_i|\alpha_0, \xi^2) &\sim N(\alpha_0, \xi^2), \\ p(\xi^2) &\sim G^{-1}(\nu_*/2, \lambda_*/2), \\ p(\rho) &\sim U(\lambda_{\min}^{-1}, \lambda_{\max}^{-1}), \end{aligned}$$

where $G^{-1}(a, b)$ denotes an inverse gamma distribution with parameters a and b. $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues of W. As is shown in Sun *et al*. (1999), it is well known that $\lambda_{\min}^{-1} < 0$ and $\lambda_{\max}^{-1} > 0$ and ρ must lie in the interval. Therefore, we restrict the prior space as $\rho \in (\lambda_{\min}^{-1}, \lambda_{\max}^{-1})$.

Given a prior density $p(\beta, \rho, \alpha, \alpha_0, \xi^2)$, the joint posterior distribution can be expressed as

$$\begin{aligned} p(\beta,\rho, \alpha, \alpha_0, \xi^2, Z|Y, X, W) &\\ \propto p(\beta, \rho, \alpha, \alpha_0, \xi^2) & \qquad (2)\\ \times \prod_{i=1}^{N}\prod_{t=1}^{T} f(y_{it}|\beta, \rho, \alpha_i, \alpha_0, \xi^2, Z, y_{-it}, x_{it}, W) & \end{aligned}$$

where $y_{-it} = (y_{1t}, \cdots, y_{i-1t}, y_{i+1t}, \cdots, y_{Nt})$ and where

$$f(y_{it}|\beta, \rho, \alpha_i, \alpha_0, \xi^2, Z, y_{-it}, x_{it}, W) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[- \frac{(z_{it} - \alpha_i - x_{it}\beta - \sum_{j=1}^{N} \rho w_{ij} z_{jt})^2}{2} \right] \times \{y_{it}\mathbf{1}_{[0,\infty)}(z_{it}) + (1 - y_{it})\mathbf{1}_{(-\infty,0)}(z_{it}) dz_{it}\}.$$

2.2. Posterior Simulation

Since the joint posterior distribution given by (2) is much simplified, we can now use MCMC methods. The Markov chain sampling scheme can be constructed from the full conditional distributions of β, ρ, α_i $(i = 1, \cdots, N)$, α_0, ξ^2, z_{it} $(i = 1, \cdots, N,\ t = 1, \cdots, T)$.

2.2.1. Sampling ρ

From (2), the full conditional distribution of ρ is written as

$$\rho|\alpha, \beta, Z, Y, X, W \propto |I_N - \rho W|^T \exp\left(- \frac{e'e}{2} \right)$$

where

$$e = (I_T \otimes (I_N - \rho W)Z - \Delta\alpha - X\beta),$$

and I_N is $N \times N$ unit matrix and $\Delta = (\overbrace{I_N, \cdots, I_N}^{T})'$, which we cannot be sampled by standard methods. Therefore, we adopt the Metropolis algorithm (see e.g., Tierny (1994)).

The following Metropolis step is used: Sample ρ from

$$\rho = \rho^* + c\phi,\ \phi \sim N(0, 1),$$

where c is called tuning parameter. Next, we evaluate the acceptance probability

$$\alpha(\rho^*, \rho) = \min\left(\frac{p(\rho)}{p(\rho^*)}, 1 \right),$$

and finally set $\rho = \rho$ with probability $\alpha(\rho^*, \rho)$, otherwise $\rho = \rho^*$. It should be mentioned that the proposal value of ρ is not truncated to the interval $(\lambda_{\min}^{-1}, \lambda_{\max}^{-1})$ since the constraint is part of the target density. Thus, if the proposed value of ρ is not within the interval, we will adopt the rejection sampling (see LeSage (2000) and Smith and LeSage (2004)).

2.2.2. Sampling the Other Parameters

If α and ρ be given, then as $Z^* = AZ - \Delta\alpha$ become constant, where $A = I_{N \cdot T} - I_T \otimes \rho W$, it is reduced to linear regression model. Therefore, for β, it can be easily obtained that

$$\beta|\alpha,\rho,Z,Y,X,W \sim N(\tilde{\beta},\tilde{\Sigma})$$

where

$$\begin{aligned}\tilde{\beta} &= (X'X+A_*)^{-1}(X'X\hat{\beta}^*+A_*\beta_*),\\ \hat{\beta}^* &= (X'X)^{-1}X'Z^*,\\ \tilde{\Sigma} &= (X'X+A_*)^{-1}.\end{aligned}$$

Given β, ρ and Z, (1) becomes

$$\underbrace{z_{it}-(x_{it}\beta+\sum_{j=1}^{N}\rho w_{ij}z_{jt})}_{e_{it}} = \alpha_i+\epsilon_{it}, \qquad (3)$$

$$\epsilon_{it} \sim N(0,1).$$

We can regard α_i as a data generated from $N(\alpha_i,\xi^2)$. Therefore, full conditional distribution of α_i follows;

$$\alpha_i|\alpha_0,\xi^2,\beta,\rho,Y,Z,X,W \sim N(\hat{\alpha_i},\hat{\xi}^2),$$

where

$$\begin{aligned}\hat{\alpha_i} &= \frac{\xi^{-2}\alpha_0+T\bar{e_i}}{\xi^{-2}+T},\\ \bar{e_i} &= T^{-1}\sum_{t=1}^{T}e_{it},\\ \hat{\xi}^2 &= (\xi^{-2}+T)^{-1}.\end{aligned}$$

Full conditional distribution of α_0 and ξ^2 can be regarded as the mean and variance of normal distribution of the posterior distribution using $\{\alpha_i\}_{i=1}^N$ as data. Therefore they follow;

$$\begin{aligned}\alpha_0|\alpha,\xi^2 &\sim N\left(\tilde{\mu},\frac{\xi^2}{\hat{N}}\right),\\ \xi^2|\alpha &\sim G^{-1}\left(\frac{\hat{\nu}}{2},\frac{\hat{\lambda}}{2}\right),\end{aligned}$$

where

$$\begin{aligned}\tilde{\mu} &= \frac{N_*\mu_*+N\bar{\alpha}}{\hat{N}},\\ \bar{\alpha} &= N^{-1}\sum_{i=1}^{N}\alpha_i,\quad \hat{\nu}=\nu_*+N,\\ \hat{\lambda} &= \lambda_*+\nu_*s_\alpha^2+N_*N(\mu_*-\bar{\alpha})^2/\hat{N},\\ \hat{N} &= N_*+N,\quad s_\alpha^2=\nu_*^{-1}\sum_{i=1}^{N}(\alpha_i-\bar{\alpha})^2.\end{aligned}$$

Finally, full conditional distribution of Z follows;

$$Z|\beta,\rho,\alpha,Y,X,W \sim MTN(\mu,V) \qquad (4)$$

where

$$V=(A'A)^{-1},\quad \mu=VA'(\Delta\alpha+X\beta).$$

$MTN(\mu,V)$ denotes a multivariate truncated normal distribution with mean μ and scale matrix V. Here, we use the property that $(A'A)^{-1} = I_T\otimes\{(I_N-\rho W)'(I_N-\rho W)\}^{-1}$, we can save the calculation time a little.

3. NUMERICAL EXAMPLE BY SIMULATED DATA

To illustrate the Bayesian approach discussed in the previous section, z_{it} $(i=1,\cdots,50\ t=1,\cdots,T)$ and α_i $(i=1,\cdots,50)$ were generated from the distribution

$$\begin{aligned}&z_{it}=\alpha_i+1.0x_{1it}+1.0x_{2it}+\sum_{j=1}^{50}\rho w_{ij}y_{jt}+u_{it}\\ &u_{it}\sim N(0,1),\end{aligned}$$

where

$$y_{it}=\begin{cases}1, & \text{if } z_{it}\geq 0\\ 0, & \text{if } y_{it}<0\end{cases},$$

and from the distribution $\alpha_i\sim N(2,2)$, respectively, where x_{1it}, x_{2it} were normal variables with mean -1. To see the effect of values of ρ, we consider the three cases of ρ values 0.3, 0.6, 0.9. Although we also considered the cases of negative value of ρ, the results obtained were similar to those given here. Therefore we report only the case of positive values of ρ to save the space. To see the effect of length of time periods, we also consider the three cases of the length of period $T=5$, $T=10$ and $T=15$. For the prior distributions, the hyper-parameters are set as follows;

$$\begin{aligned}&\beta_*=0,\ A_*=100^{-1}\cdot I_k,\ \mu_*=0,\\ &\nu_*=2,\ \lambda_*=0.01,\ N_*=N-1\end{aligned}$$

Since it is interesting to see the effect of ignoring spatial interaction, we estimated the model with the restriction $\rho=0$ as well as the model without the restriction. With the simulated data, we ran the MCMC algorithm, using 20000 iterations and discarding the first 5000 iterations. The chain was considered to have practically converged after 5000 iterations based on a diagnostic proposed by Geweke (1992). All results reported here were generated using Ox version 3.4 (See Doornik (2001)).

Table 1 shows the posterior estimates of the parameters[2]. From Table 1, we can find that as the value ρ increases, distinct differences appear in posterior means, especially in the estimates of β. For the restricted model, we can see serious spatial correlation biases, that is, the posterior mean of β decrease. The unrestricted model exhibits

[2] In this case we show that the case of $T=10$. Because as is shown in below, we can find that our estimator hardly depend on the length of period.

better estimates of the parameters compared with the restricted model. Thus it is important to take the spatial correlation into account.

Table 1. Simulated Data

Spatial panel probit model with the restriction $\rho = 0$				
	true value	$\rho = 0.3$	$\rho = 0.6$	$\rho = 0.9$
α_0	2	1.2120	0.7735	0.6868
		(0.3021)	(0.0426)	(0.0394)
β_1	1	0.7118	0.6409	0.3044
		(0.0752)	(0.0724)	(0.0626)
β_2	1	0.7006	0.6737	0.4609
		(0.0743)	(0.0713)	(0.0642)
ξ^2	2	0.7611	1.1832	1.7338
		(0.0440)	(0.2946)	(0.4728)
Spatial panel probit model without the restriction				
	true value	$\rho = 0.3$	$\rho = 0.6$	$\rho = 0.9$
α_0	2	1.0882	0.9831	1.1755
		(0.2742)	(0.2258)	(0.3094)
β_1	1	1.1245	0.9270	1.0021
		(0.1470)	(0.1157)	(0.1808)
β_2	1	1.0506	0.9452	1.2644
		(0.1313)	(0.1118)	(0.1951)
ξ^2	2	5.6786	3.9357	6.5967
		(1.7638)	(1.1213)	(2.2897)
ρ		0.1738	0.5783	0.9147
		(0.1473)	(0.0719)	(0.0120)

Posterior means and standard deviations (in parentheses) are shown.

We can also find that the posterior mean of ρ has bias and it becomes larger when ρ becomes small. The approximate posterior distributions of ρ are shown in Figure 1. From the figure, it can be seen that the posterior distributions exhibit the skewness for large values of ρ. Furthermore, we can observe that the posterior modes are closer to the true values than posterior mean.

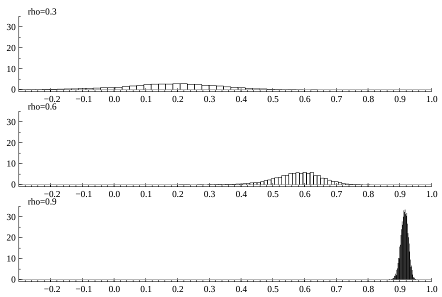

Figure 1. The approximate posterior dist. of ρ

Finally, as is shown in Maddala (1977), the biases in constant terms are serious problem in estimating panel probit models. From Table 1, we can also find the under bias of α_0. But surprisingly the mean of α_i without restriction is around the true value. Then, Figure 2 shows the relationship between true values and estimated coefficients in each length of periods with $\rho = 0.6$. From the figure, we can observe that the posterior modes improve in many units as the length of period becomes longer. But if we see Table 2, which shows the correlation of constant terms, the correlation improve from $T = 5$ to $T = 10$ and we can find similar correlation between $T = 10$ and $T = 15$ with true value. This result implies that 10 period is enough to estimate the parameter by our method.

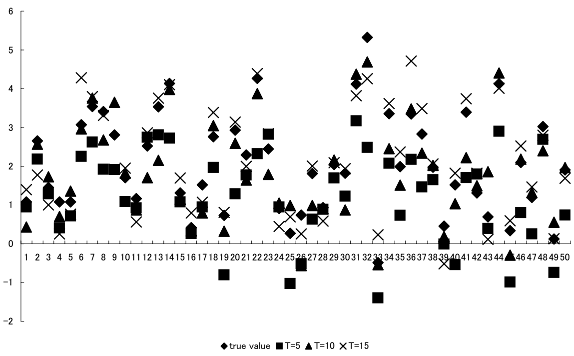

Figure 2. The Constant Terms in each unit

Table 2. Correlation of Constant Terms

	true value	T=5	T=10	T=15
true value	1.000			
T=5	0.861	1.000		
T=10	0.941	0.801	1.000	
T=15	0.944	0.824	0.868	1.000

4. EMPIRICAL EXAMPLE

As an example of real data set, we will consider the business cycle across 47 prefectures ($N = 47$) in the period from 1991 to 2000 ($T = 10$) in Japan. As dependent variable, we use the data of the increase and decrease of real gross regional product (GRP) from Vital Statistics prepared by the Cabinet Office of Japan, where 1 if GRP increases and otherwise 0. As explanatory variables, we use , electricity demand from electricity project handbook prepared by Ministry of Economy, Trade and Industry, sum of annual sales of industrial goods from survey of industrial statistics by Ministry of Economy, Trade and Industry and balance at the bank from Monthly Report of Financial Statistics by Bank of Japan. All the data are changed into per capita data and we take the rate of increase. Finally, as a weight matrix, we use the matrix proposed by Kakamu *et al.* (2005), which considers the connection of economic activities[3].

[3]All except one (Okinawa) Japanese prefectures are situated on the four major islands, Hokkaido, Honshu, Shikoku and Kyushu. But these four islands are connected by train and roads, despite

Using the same hyper-parameters as in the previous section, we ran the MCMC algorithm using 20000 iterations and discarding the first 5000 iterations. As in the previous simulated data example, the convergence of the MCMC simulation was checked by the Geweke's diagnostic.

Table 3 shows the estimation result. The estimation result gives us three major implications. First, electricity demand, sum of annual sales of industrial goods and balance at the bank affect to the rise and fall of business index. We can find if the variables rise, the probability of business index increases. This result suggests that it is effective to think of these policy for business policy.

Table 3. Empirical Result

	Estimates	St. Dev.
α_0	-0.023	(0.055)
Electricity Demand	10.151*	(2.230)
sum of annual sales of industrial goods	8.062*	(1.453)
Balance at the bank	8.797*	(2.022)
ρ	0.368*	(0.060)
ξ^2	0.008	(0.010)

a. Posterior means and st. dev. (in parentheses) are shown.

b. * means the 95% credible interval does not include zero.

Second, we can find spatial interaction in business cycle in Japan, since the parameter ρ is estimated 0.368 positive and significantly. This result suggest that if we work out a regional business policy, we cannot ignore the neighbor regions business.

Third, we can find that the potential probability in each prefecture is different. Figure 3 shows the distribution of constant terms in each prefectures. From the figure, we can find that it shows positive value only in Shimane. Conversely, Shiga, Kumamoto and Tochigi shows high negative values. But we cannot find the relationship between the order of constant terms and other economic situation like GRP. Therefore, it may mislead to wrong economic policy that we use the GRP as a policy index. In addition, if we take into account that the constant terms are not significant by the test of credible interval, there is no potential probability difference among prefectures.

Finally, we can see the difference of changes in probability in each prefectures, especially among 7

the fact that islands are separate geographical entities. But for example, the most northern island Hokkaido is connected by the Seikan railway tunnel to Honshu. And Honshu is connected by the Awaji and Seto Bridge to Shikoku, and the southern island of Kyushu is also connected by the Kanmon Tunnel and Bridge to Honshu. Therefore, Okinawa is the only prefecture which is independent of all other prefectures.

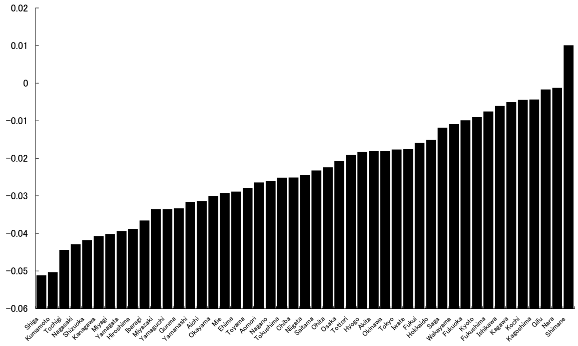

Figure 3. The Constant Terms in each prefecture

regions. Figure 3 shows the changes in probability in probability. From the figure, we can find the similar tendency in each region except for Kyushu region. For example, we can see the fall of probability in 1998 and rise in 1999. It may be the reason that the spatial dependency affects to the similarity of the business cycle in each region. But we can also find the difference among regions. Although the stable with probability period continued from 1994 to 1997, the probability in Kansai region is not so high compared with other regions. Especially, the probability in Osaka is around 0.5.

5. CONCLUSIONS

This paper has examined the panel probit model with spatial dependencies from a Bayesian point of view. We expressed the joint posterior distribution, and proposed MCMC methods to estimate the parameters of the model. We have illustrated our approach using simulated and real data.

From the results for the simulated data, we found serious spatial correlation biases and the importance of spatial correlation. As for the real data example, we considered the business cycle in Japan. From the results, we can find (1) if electricity demand, sum of annual sales of industrial goods and balance at the bank rise, the probability of business rises, (2) there exists spatial dependency, (3) the potential probability is not different among each prefecture and (4) we can capture the changes in probability of business cycle in each region.

Finally we will discuss our remaining issues. In this paper, we proposed spatial panel probit model. But if the period becomes long, it takes very long time to estimate the model analytically. Therefore, we can only capture the changes in probability and cannot examine the turning point of business cycle empirically. It is also important to consider the efficient method to estimate the long run panel data. But we think this is the first step to examine the spatial interaction in econometrics using panel data.

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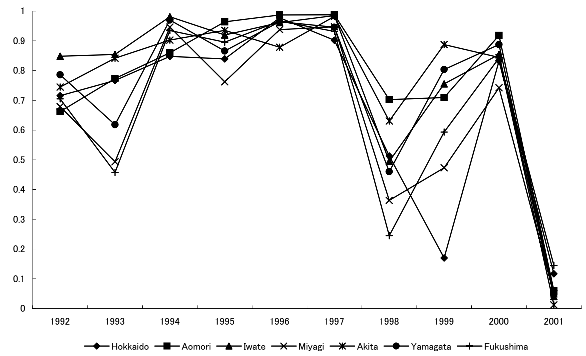
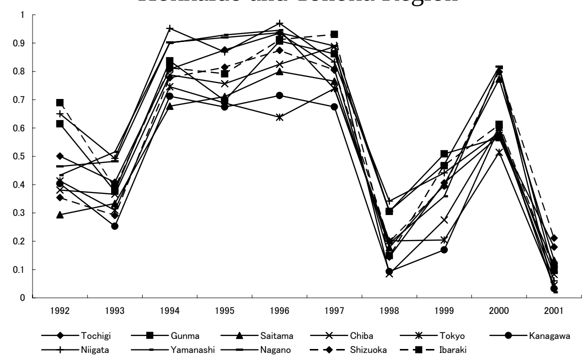
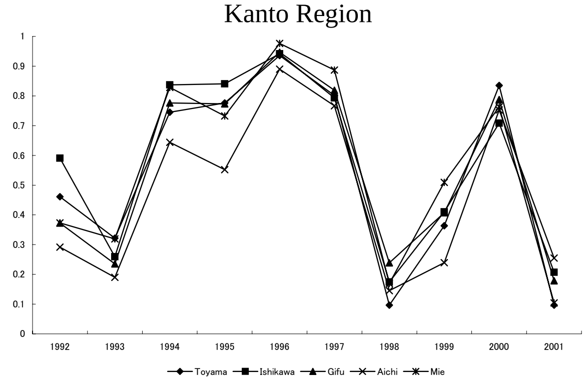


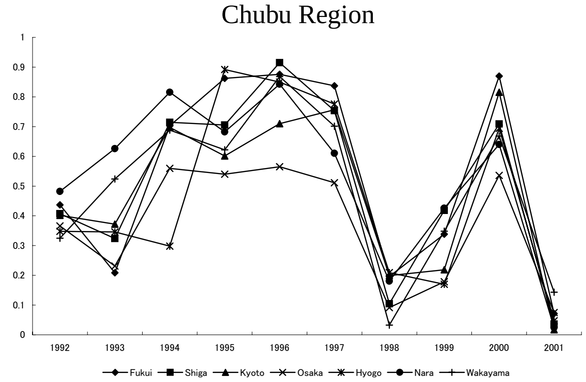
Hokkaido and Tohoku Region



Kanto Region

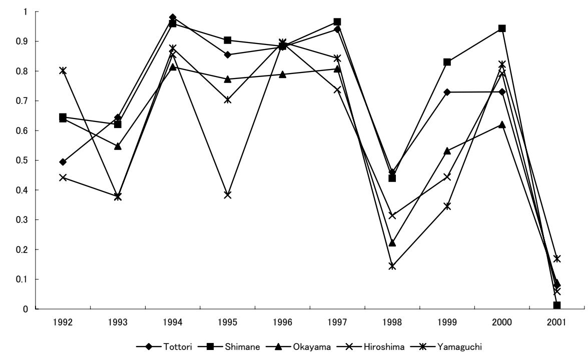


Chubu Region



Kinki Region

Figure 4-1:The changes in the probability



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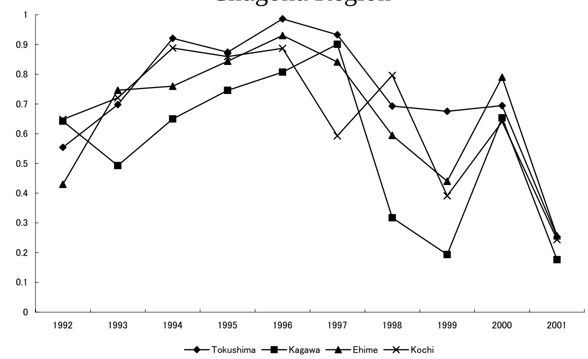
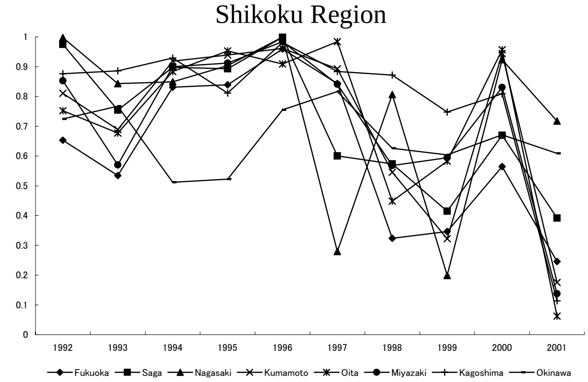


Shikoku Region



Kyushu Region

Figure 4-2:The changes in the probability