Coalitional Effect in a Pure Bargaining Problem: An Example with Nonlinear Utilities

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ABSTRACT

"Coalition" is a key concept in cooperative game analysis. It means the possibility of striking a binding contract among the members. Still, the concept is too broad and several different concepts seem to coexist in the literature. This paper takes a simple pure bargaining problem, in which all the participants must agree to realize a bargaining surplus. It then examines if an "ideal" coalition (as suggested in recent literature) could affect the outcome. Some past research reported a negative result i.e. a coalition might hurt its members. In the setting of linear utilities, in a previous paper, we showed that such claim depended crucially upon what a coalition could do, and that "ideal" coalition would have no meaningful influence in the consequence of bargaining (Imai and Watanabe, 2005).

Along these lines of research, here we take the case of players with nonlinear utility functions. We show that the straight adaptation of our previous approach causes several problems, and neutrality results may not be obtained. Firstly, a simple fixed share of coalitional payoffs may not work. Thus, this leads us to employ sharing schemes, rather than a fixed share. Secondly, among the contingencies covered by a sharing scheme, only one contingency is expected to be realized. This implies a wide range of indifferences on the part of players, and thus a lot of freedom in designing sharing schemes. In turn, this means that members of a coalition can manipulate their preferences in a credible manner. Therefore, the issues here are similar to those in delegated bargaining, or of misrepresentation problem. Aside from the difficulty in finding an optimal solution for such a problem, which we tentatively resolve by adopting a simplistic assumption that an induced preference must be concave, another problem is the effect generated by correlated interests. As is well-known, only the "toughest" player's preference matters in each coalition. Therefore, in this example we will look to the case of coalition formation.

Precisely, the game proceeds as follows. A randomly chosen proposer makes a proposal of a coalition contract specifying the member of a coalition and sharing scheme. Members of the proposed coalition replies either "Yes" or "No" . If some member rejects, then again new proposer is chosen (from scratch). If all the members accept the proposal, than the contract becomes binding and if there are remaining players, then coalition formation stage continues among them. At the end of the stage, a coalition structure emerges, and the game moves into the bargaining stage where a sequential bargaining game is played. In this game, each player acts individually guided by their self interests (possibly transformed by the sharing scheme.) The solution concept is the stationary subgame-perfect equilibrium.

In our example, we take three players, two of whom have nonlinear utilities in money. A player with a linear utility function usually performs better than the one with a strictly nonlinear utility function. However, a coalition of players can transform some players' preference through a sharing scheme. As a result, the coalition of players with nonlinear utility functions can "create" a player with a linear utility function to make bargaining outcome favorable to them. Thus, in our example, players with nonlinear utility functions have a chance to perform better than the ones without the possibility to form a coalition, or achieve better expected payoffs ex ante.

The non-neutrality of coalition formation exhibited here implies the approach we have taken should be reexamined. Accordingly, we indicate a future research direction towards that at the conclusion part of the paper.

Despite the fact that neutrality is not obtained, results of this example may be of some independent interest. This is because there are some informal intuition held among people that a collusion among the participants of (pure) bargaining should be meritorious for colluders. Often this intuition is wrong in the sense that the underlying situation raised in support of such argument usually is not that of a pure bargaining problem. The result obtained here gives an instance where such intuition could be correct in the case of a pure bargaining problem.

1 INTRODUCTION

"Coalition" is a key concept in cooperative game analysis. It means the possibility of striking a binding contract among the members. Still, the concept is too broad and, several different concepts seem to coexist in the literature. This paper takes a simple pure bargaining problem, in which all the participants must agree to realize a bargaining surplus. It then examines if an "ideal" coalition (as suggested in recent literature) could affect the outcome. Some of the past research reported a negative result i.e. a coalition might hurt its members as in Harsanyi's (1977) joint bargaining paradox.

In Imai and Watanabe (2005), we showed that for a pure bargaining game of splitting a dollar among n players, the outcome of bargaining was not affected by formation of coalitions among players and so the paradox was lost. The basic driving force for this result was the assumption that a coalition did not need to act like a single player unless it chose to do so. The model was based on a sequential bargaining game (a la Stahl (1972) and Rubinstein (1982)) with random proposers (c.f. Binmore (1987)) and prior to this game, players were allowed to form a coalition. Coalition formation game was adapted from Ray and Vohra (1997) and (1999) (also Bloch (1996)) with random proposers (as in Okada (1996) which extended Chatterjee *et al*. (1993) in this aspect). Players could form a coalition to sign a binding contract for redistribution of the total (coalitional) payoffs among themselves. Taking advantage of the assumption of linear utilities, it was assumed that the contract specifies the share of each player in the total payoff. Each player acted to maximize his/her final payoff. As has already been mentioned above, it was shown that although the grand coalition was formed, the payoff distribution did not differ from the one when coalition formation was not allowed.

We believe that this test of neutrality is important because a "coalition" could mean many different phenomena. Furthermore, some observation like "joint bargaining problem" hinges upon the fact that the members within a coalition cannot act individually under the coalitional contract. To understand or evaluate the extent to which such phenomenon is valid, one needs to clarify the nature of a "coalition" contributing to it. In this regard, the extension of the abovementioned result to general cases where utility functions are not necessarily linear is desirable. However, there are several issues arising from such an attempt. In this paper, we aim to illustrate these issues via some examples.

Firstly, a simple fixed share of coalitional payoffs may not work. Thus, this leads us to employ sharing schemes, rather than a fixed share. Secondly, among the contingencies covered by a sharing scheme, only one contingency is expected to be realized. This implies a wide range of indifferences on the part of players, and thus a lot of freedom in designing sharing schemes. In turn, this means that members of a coalition can manipulate their preferences in a credible manner. Therefore, the problem involves the one similar to delegated bargaining, or of misrepresentation. Aside from the difficulty in finding an optimal solution for such a problem, which we tentatively resolve by adopting a simplistic assumption that an induced preference must be concave, another problem is the effect generated by correlated interests. As is well-known, only the "toughest" player's preference matters in each coalition. Therefore, in this example we will look to the case of coalition formation.

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In our example, we take three players, two of whom have nonlinear utilities in money. A player with a linear utility function usually performs better than the one with a strictly nonlinear utility function. However, a coalition of players can transform some players' preference through a sharing scheme. As a result, the coalition of players with nonlinear utility functions can "create" a player with a linear utility function to make bargaining outcome favorable to them. Thus, in our example, players with nonlinear utility functions have a chance to perform better than the ones without the possibility to form a coalition, or achieve better expected payoffs ex ante.

2 EXAMPLE

Let us consider a three person example with the set of players $N = \{1, 2, 3\}$ and the set of money allocations is given by $X = \left\{ x \in \mathbb{R}^3_+ : \sum_{i=1}^{3} x_i = 1 \right\}$. Utility functions are given by $u_1(x_1) = x_1,\ u_2(x_2) = x_2^{\frac{1}{2}},\ u_3(x_3) = x_3^{\frac{1}{2}}$. If one computes the Nash bargaining solution with disagreement point (0,0,0), the solution is $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$ [1)]. If u_i's represent the stationary time preferences too, then given the common discount factor $\delta \in (0,1)$, the stationary subgame perfect equilibrium yields the payoff distribution (ex ante). To verify this, let v_i be i's stationary subgame-perfect equilibrium payoffs at the beginning of the period (i.e. prior to the choice of the proposer) and let v_{ij} be the stationary subgame perfect equilibrium payoff of i when j is chosen to be the proposer.

From

$$v_1 = \frac{1}{3}v_{11} + \frac{2}{3}v_{12}$$

and

$$v_{12} = \delta v_1,$$

we have

$$v_1 = \frac{1}{3}v_{11} + \frac{2\delta}{3}v_1,\ v_1 = \frac{v_{11}}{3-2\delta}$$

and so we have

$$\begin{aligned} v_{11} &= u_1(1-2x_2) \\ &= u_1\left(1-2\delta^2 v_2^2\right) \\ &= 1-2\delta^2 v_2^2. \end{aligned}$$

Also we have

$$v_2 = \frac{1}{3}v_{22} + \frac{2}{3}v_{21},\ v_{21} = \delta v_2,$$

and so

$$v_2 = \frac{v_{22}}{3-2\delta}$$

to yield

$$\begin{aligned} v_{22} &= u_2(1-x_1-x_2) \\ &= u_2\left(1-\delta v_1-\delta^2 v_2^2\right) \\ &= \left(1-\delta v_1-\delta^2 v_2^2\right)^{\frac{1}{2}}, \end{aligned}$$

or

$$v_{22}^2 = 1-\delta v_1 - \delta^2 v_2^2.$$

[1)]The Nash solution is given as the solution of

$$\max(u_1u_2u_3) = \max\left\{x_1 x_2^{\frac{1}{2}} x_3^{\frac{1}{2}} : x_1+x_2+x_3 = 1\right\}$$

and $\arg\max u_1u_2u_3 = \arg\max x_1^2 x_2(1-x_1-x_2)$ whose first order condition is

$$\begin{aligned} \frac{2}{x_1} - \frac{1}{(1-x_1-x_2)} &= 0 \\ \frac{1}{x_2} - \frac{1}{(1-x_1-x_2)} &= 0, \end{aligned}$$

which yields

$$\begin{aligned} x_1 &= \frac{1}{2} \\ x_2 &= \frac{1}{4}. \end{aligned}$$

Combining these, we have

$$\begin{aligned} (3-2\delta)\, v_1 &= 1-2\delta^2 v_2^2, \\ (3-2\delta)^2 v_2^2 &= 1-\delta v_1 - \delta^2 v_2^2 \end{aligned}$$

and hence

$$\begin{aligned} \left\{(3-2\delta)^2+\delta^2\right\} v_2^2 &= 1-\delta v_1 \\ &= 1-\frac{\delta\left(1-2\delta^2 v_2^2\right)}{3-2\delta} \\ &= 1-\frac{\delta}{3-2\delta}+\frac{2\delta^3}{3-2\delta}v_2^2, \end{aligned}$$

to yield

$$\begin{aligned} v_2^2 &= \tfrac{3-3\delta}{(3-2\delta)\left\{(3-2\delta)^2+\delta^2\right\}-2\delta^3} = \tfrac{3(1-\delta)}{(3-2\delta)^3+3\delta^2-4\delta^3} \\ &= \tfrac{3(1-\delta)}{27-54\delta+39\delta^2-12\delta^3} = \tfrac{1-\delta}{9-18\delta+13\delta^2-4\delta^3} \\ &= \tfrac{1}{9-9\delta+4\delta^2} \end{aligned}$$

and

$$\begin{aligned} v_1 &= \frac{1-2\delta^2 v_2^2}{3-2\delta} = \frac{1-\frac{2\delta^2}{9-9\delta+4\delta^2}}{3-2\delta} \\ &= \frac{9-9\delta+2\delta^2}{(3-2\delta)(9-9\delta+4\delta^2)} = \frac{3-\delta}{9-9\delta+4\delta^2}. \end{aligned}$$

Next, consider the coalition formation problem prior to this bargaining game. For example, players 2 and 3 may form a coalition. If this is the case, equal splitting of the coalitional payoffs make sense. If two players choose to be represented by one player, then a direct application of the theory would yield $\frac{1}{3}$ (in terms of money) for this coalition if the Nash solution is applied [2)] or $\left(\frac{1}{4-2\delta+\delta^2}\right)\left(\frac{1}{9-9\delta+4\delta^2}\right)$(for δ close to 1) if the stationary subgame perfect equilibrium of the sequential bargaining game is computed. (For the verification of the latter result, note that the conditions are

$$v_1 = \frac{1}{2}v_{11} + \frac{1}{2}v_{12} = \frac{1}{2}v_{11} + \frac{1}{2}\delta v_1$$

which becomes

$$v_1 = \frac{v_{11}}{2-\delta},\ v_{11} = u_1(1-x_2) = 1-\delta^2 v_2^2$$

[2)]Since the solution is given by

$$\max\ x_1 x_2^{\frac{1}{2}} = \max\ x_1(1-x_1)^{\frac{1}{2}},$$

or

$$\max\ x_1^2(1-x_1)$$

and the first order condition yields

$$2x_1 - 3x_1^2 = 0,$$

so

$$x_1 = \frac{2}{3}.$$

and

$$v_2 = \frac{1}{2}v_{22} + \frac{1}{2}v_{21},\ v_2 = \frac{v_{22}}{2-\delta},$$

to yield

$$v_{22} = u_2\left(1 - x_1\right) = \left(1 - \delta v_1\right)^{\frac{1}{2}},$$

or

$$v_{22}^2 = 1 - \delta v_1.$$

Thus from

$$\left(2-\delta\right)v_1 = 1 - \delta^2 v_2^2$$

we have

$$\left(2-\delta\right)^2 v_2^2 = 1 - \delta v_1 = 1 - \frac{\delta\left(1 - \delta^2 v_2^2\right)}{2-\delta}$$

or

$$\left(2-\delta\right)^3 v_2^2 = 2 - 2\delta + \delta^3 v_2^2,$$

and so we have

$$\begin{aligned} v_2^2 &= \frac{2\left(1-\delta\right)}{\left(2-\delta\right)^3 - \delta^3} \\ &= \frac{2\left(1-\delta\right)}{2\left(1-\delta\right)\left(\left(2-\delta\right)^2 + \delta\left(2-\delta\right) + \delta^2\right)} \\ &= \frac{1}{4 - 4\delta + \delta^2 + 2\delta - \delta^2 + \delta^2} = \frac{1}{4 - 2\delta + \delta^2} \end{aligned}$$

and

$$\begin{aligned} v_1 &= \frac{1 - \frac{\delta^2}{4-2\delta+\delta^2}}{2-\delta} \\ &= \frac{4-2\delta}{\left(2-\delta\right)\left(4-2\delta+\delta^2\right)} = \frac{2}{4-2\delta+\delta^2}.) \end{aligned}$$

This result exhibits a joint bargaining paradox[3)], since the total share of the coalition in terms of money shrank compared to the result before the formation of the coalition. However, this shows how special such a coalition is, because a coalition does not always imply representation by a single player. In an ideal coalition, any contract among players could be made binding. Thus, for each player to act individually must be one alternative, and by such strategy, representation by a single player must be dominated. We tried to express this idea through an adaptation of the models developed by Ray and Vohra (1997), (1999) and Bloch (1996)[4)]. In the straightforward application of their approach, a strategic choice is constrained to constitute a noncooperative equilibrium given a coalition structure, whereas we try to allow individual actions. We thus employ the approach in which players can agree upon a reallocation scheme (represented by a vector of shares) for the coalition when they form it. In the above coalition structure, one may adopt the similar approach and may expect that an equal share for both players 2 and 3 would result for this coalition, and each player acts to maximize the individual payoff (after reallocation), so that the paradox is lost (at least in the sense of allocation), for the coalition $\{2, 3\}$.

As for the coalition $\{1, 2\}\,(or\,\{1, 3\})$, the simple assumption of a fixed (and single) share may not be appropriate. Due to differing preferences, at least for each amount of money the coalition members have obtained, players must agree upon their shares. Let us write $a_1^S\left(y\right)$ and $a_2^S\left(y\right)$ for the amount (not a share) each player receive when the coalition $S = \{1, 2\}$ earned y dollar. Thus, if player 1 or 2 proposes to form the coalition $S = \{1, 2\}$, the sharing scheme $\left(a_1\left(y\right),\ a_2\left(y\right)\right)$ for all $y \in [0, 1]$ must be in the proposal.

Nevertheless, this approach contains some intriguing issues. First, in a pure strategy equilibrium, only one coalitional worth y (in terms of money) is realized, so that all other parts of sharing schemes are representing off-the-path events. Hence, if a proposal is made, which is "wrong" for the off-the-path events, a responder would not have an incentive to reject it. This further implies that the proposer can propose a sharing scheme which would distort members' preferences through a sharing scheme to affect bargained outcome. For instance, in the coalition $\{2, 3\}$, one player may propose a sharing scheme with $a_j\left(y\right) = ky^2$ so that $u_j\left(a_j\left(y\right)\right) = k^{\frac{1}{2}}y$, which converts player j's preference into a much tougher one.

Suppose that 2 proposes $a_3\left(y\right) = ky^2$ then $a_2\left(y\right) = y - ky^2$ (and hence if $k < 1, a_2\left(y\right) \geq 0$ for $y \in [0, 1]$ and if $k < \frac{1}{2},\ a_2\left(y\right)$ is increasing on $[0, 1]$). If this proposal is accepted, then the ensuing sequential bargaining game is the one as if 2 and 3's preferences are given by $u_i \circ a_i$, which we shall refer to as a pseudo-utility function.

To derive a stationary subgame perfect equilibrium, one must notice that there is another non-standard element in this game, which was concealed under linear assumptions in our previous work. What matters in bargaining between player 1 and players 2 and 3 who signed a coalitional agreement, is the fact that player 2 and 3's interests are correlated via the sharing scheme. Therefore knowing 2's payoff determines 3's payoff as well. Thus, it could be the case that whenever 3 accepts the offer, 2 always

3)The paradox is due to Harsanji (1977). Relevent discussion on the number of players in two party bargaining can be found in Chae and Heidhues (2004) and Chae and Moulin (2004).

4)As a matter of fact, these literatures contain example involving the coalition effect, which is similar to the joint bargaining paradox. For other such examples, see Salant et.al. (1983), Cho, Jewell, and Vohra (2002) and Ray and Vohra (2001).

accepts it. In this case, what matters is 3's preference and 2's preference does not matter. This point was raised in Imai and Salonen (2000) under a slightly different rule[5]. In fact, here player 3 is "tougher" than player 2 in the sense that whenever 3 says yes to a proposal 2 always chooses "Yes".

Let us illustrate this point: given a coalitional worth y (in terms of money),

$$u_1(1-y) = 1-y$$
$$u_2(a_2(y)) = \left(y-ky^2\right)^{\frac{1}{2}}$$
$$u_3(a_3(y)) = k^{\frac{1}{2}}y.$$

Let v_i be i's stationary subgame-perfect equilibrium payoffs evaluated at the beginning of the period (i.e. prior to the choice of the proposer) and let v_{ij} be the stationary subgame perfect equilibrium payoff of i when j is chosen to be the proposer. We have $v_i = \frac{1}{3}\sum_{i=1}^{3} v_{ij}$ due to stationarity.

Next, we find the minimal monetary reward at which they would accept the offer, i.e. $u_i^{-1}(\delta v_i)$.

$$x_1 = u_1^{-1}(\delta v_1) = \delta v_1$$
$$x_2 = u_2^{-1}(\delta v_2) = \delta^2 v_2^2$$
$$x_3 = u_3^{-1}(\delta v_3) = \delta^2 v_3^2$$

In terms of y, solving $a_i(y_i) = x_i \quad (i = 2, 3)$ yields

$$y_1 = 1-\delta v_1$$
$$y_2 = \frac{1}{2k} - \left(\frac{1}{4k^2} - \frac{\delta^2 y_2^2}{k}\right)^{\frac{1}{2}}$$
$$y_3 = \frac{\delta v_3}{k^{\frac{1}{2}}}$$

Let $y_{23} = \max\{y_2, y_3\}$. Then we have

$$v_{11} = 1 - y_{23}$$
$$v_{22} = u_2(a_2(y_1)) \quad or \quad v_{22}^2 = y_1 - ky_1^2$$
$$v_{33} = u_3(a_3(y_1)) \quad or \quad v_{33}^2 = ky_1^2$$

Now, compare

$$\widetilde{y}_2 = \left(\frac{1}{2k} - \left(\frac{1}{4k^2} - \frac{\delta^2\left(y-ky^2\right)}{k(3-2\delta)^2}\right)^{\frac{1}{2}}\right)$$

and

$$\widetilde{y}_3 = \frac{\delta k^{\frac{1}{2}} y}{k^{\frac{1}{2}}(3-2\delta)}.$$

For $y = 0$, $\widetilde{y}_2 = \widetilde{y}_3 = 0$ and for $0 < y < 1$, $\widetilde{y}_2'/\widetilde{y}_3' < 1$[6]. Thus we conclude that $y_{23} = y_3$. Then from $v_{11} = 1 - y_3 = (3-2\delta)v_1$, and $v_{33} = k^{\frac{1}{2}}y_1 = (3-2\delta)v_3$, we have

$$\frac{1-\delta v_3}{k^{\frac{1}{2}}} = (3-2\delta)v_1$$

$$k^{\frac{1}{2}}(1-\delta v_1) = (3-2\delta)v_3$$

Finally, note that $v_{ij} = \delta v_1$ for $j \neq 1$, while $v_{31} = \delta v_3$ but $v_{32} = v_{33}$. Thus $v_1 = \frac{v_{11}}{(3-2\delta)}$ and $v_3 = \frac{(3-\delta)}{2}v_{33}$. From these relationships, we can solve for v_1 and v_3 to conclude that $v_1 = \frac{1}{3}$ and $v_3 = \frac{2k^{\frac{1}{2}}}{3}$. This implies $y_1 = y_3 = \frac{2}{3}$ and $v_2 = \left[\frac{2}{3}\left(1-k\frac{2}{3}\right)\right]^{\frac{1}{2}}$.

Therefore choosing $k = \frac{1}{4}$ for example, both players 2 and 3 are better off compared to the case without any nontrivial coalition formation.

This example shows a potentially positive effect of coalition. Through commitment to a sharing scheme, players could create an effect similar to a delegation to a tough negotiator. The next question may be if there is an optimal sharing scheme.

If one could impose somewhat unnatural restriction that the resulting pseudo utility function of the coalitional payoff should be concave (and continuous), then linear pseudo-utility is optimal. Since a sharing contract that makes one player's pseudo-utility linear is always feasible for a coalition with more than or equal to two players, the result would represent each nontrivial coalition by a player with a linear pseudo-utility function. (Yet, whether individual rationality constraint is always satisfied has to be checked.)

Consider the following strategies (sharing schemes are the same as above):

Player1: proposes $\{1, 2\}$

Player2: proposes $\{2, 3\}$

[5] In Imai and Salonen (2000), the first rejector can make a counter proposal. This makes the result essentially that of two person bargaining, which is quite different if a proposer is determined by a fixed order. Here, with a random proposer, the effect is quite similar to the case of a fixed order so that the number of players does matter.

[6] We have $\widetilde{y}_2' = \frac{\delta^2(1-2ky)}{2\left(\frac{1}{4k^2} - \frac{\delta^2\left(y-ky^2\right)}{k(3-2\delta)^2}\right)^{\frac{1}{2}} k(3-2\delta)^2}$ and $\widetilde{y}_3' = \frac{\delta k}{k^{\frac{1}{2}}(3-2\delta)}$, and thus

$$\frac{\widetilde{y}_2'}{\widetilde{y}_3'} = \frac{\delta^2(1-2ky)}{2\left(\frac{1}{4k^2} - \frac{\delta^2\left(y-ky^2\right)}{k(3-2\delta)^2}\right)^{\frac{1}{2}} k(3-2\delta)}$$
$$< \frac{\delta^2(1-2ky)}{3-2\delta} < 1.$$

Player3: proposes $\{2,3\}$

so that the proposal yield the other player j in the proposed coalition δv_j in the equilibrium. This gives rise to a system of equations,

$$v_1 = \frac{1}{3} \times \left(\frac{4}{5} - \delta^2 v_2^2\right) + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}$$
$$v_2 = \frac{1}{3}\delta v_2 + \frac{1}{3}\left(\frac{2}{3} - \delta^2 v_3^2\right)^{\frac{1}{2}} + \frac{1}{3}\delta v_2$$
$$v_3 = \frac{1}{3} \cdot \frac{1}{\sqrt{5}} + \frac{1}{3}\delta v_3 + \frac{1}{3}\left(\frac{2}{3} - \delta^2 v_2^2\right)^{\frac{1}{2}}.$$

From the optimality condition for player 1, $v_2 = v_3$ must hold. Thus we have $\delta v_2 = 1/\sqrt{5}$,

As a result, we have

$$v_1 = \frac{1}{5} + \frac{2}{9} = \frac{19}{45}.$$

To confirm the optimality, observe that

$$v_{11} = \frac{4}{5} - \frac{1}{5}$$
$$\geq 1 - \frac{2}{5}\left(1 - \delta^2 v_2^2 - \delta^2 v_3^2\right)(v_{22})^2 = \frac{2}{3} - \frac{1}{5} = \frac{7}{15}$$
$$\geq \frac{4}{5} - \delta\frac{19}{45}\left(= \frac{4}{5} - \delta v_1\right)\left(= 1 - \delta v_1 - \delta^2 v_3^2\right)$$

and the similar inequalities for $(v_{33})^2$.

Thus, in pure bargaining, coalition formation could matter and efficiency cannot be expected. Also a formed coalition brings benefit to the proposer, while the partner in the coalition may not be better off compared to the case where no coalition is formed.

3 REMARKS

This analysis hinges upon the restraint that sharing schemes must leave pseudo-utility functions concave. Without such limitation, characterization of the optimal sharing schemes would be difficult. Another eminent aspect in this approach is the disadvantage of being alone, since there is no room to manipulate that players' preference through a sharing scheme. One could imagine that bringing a third party into the game should have the same effect.

In view of supporting our conjecture for neutrality, one may criticize the unnatural characters of ex ante commitment to the sharing schemes for the contingencies with 0 probability. To this end, deferring resolution on redistribution among members of a coalition to the last period may make more sense, even though the nature of a coalitional contract differs to some extent. This direction is left to be analyzed in a future study.

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5 REFERENCE

Binmore, K. (1987), Perfect Equilibria in Bargaining Models, in K. Binmore and P. Dasgupta (eds.), *The Economics of Bargaining*, Basil Blackwell, Oxford.

Bloch, F. (1996), Sequential Formation of Coalitions in Games with Externalities and Fixed Payoff Division, *Games and Economic Behavior* Vol.14, pp90-123.

Chae, S. and P. Heidhues (2004), Nash Bargaining Solution with Coalitions and the Joint-Bargaining Paradox: a Group Bargaining Solution, *Mathematical Social Sciences* Vol.48, pp37-53.

Chae, S. and H. Moulin (2004), Bargaining among Groups: an Axiomatic Viewpoint, *mimeo* Rice University.

Chatterjee, K., B. Dutta, D. Ray and K. Sengupta (1993), A Noncooperative Theory of Coalitional Bargaining, *Review of Economic Studies* Vol.60, pp463-477.

Cho, I-K., K. Jewell and R. Vohra (2002), A Simple Model of Coalitional Bidding, *Economic Theory* Vol.19, pp435-457.

Harsanyi, J.C. (1977), *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*, Cambridge University Press, Cambridge.

Imai, H. and H. Salonen (2000), The Representative Nash Solution for Two-sided Bargaining Problems, *Mathematical Social Sciences* Vol.39, pp349-365.

Imai, H. and N. Watanabe (2005), On the Neutrality of Coalition Formation in a Pure Bargaining Game, *Japanese Economic Review, forthcoming*.

Okada, A.,(1996), A Noncooperative Coalitional Bargaining Game with Random Proposers, *Games and Economic Behavior* Vol.16, pp97-108.

Ray, D. and R. Vohra (1997), Equilibrium Binding Agreement *Journal of Economic Theory* Vol.26, pp30-78.

Ray, D. and R. Vohra (1999), A Theory of Endogenous Coalition Structures, *Games and Economic*

Behavior vol.26, pp286-336.

Ray, D. and R. Vohra (2001), Coalitional Power and Public Goods, *Journal of Political Economy* vol.109, pp1355-1384.

Rubinstein, A. (1982), Perfect Equilitrium in a Bargaining Model, *Econometrica* 50, pp97-109.

Salant, S.W., S. Switzer and R. J. Reynolds (1983), A Losses from Horizontal Merger: the Effect of an Exogenous Change in Industry Structure in Cournot-Nash Equilibrium, *Quarterly Journal of Economics* vol.98, pp185-199.

Stahl, I. (1972), *Bargaining Theory*. Stockholm: Economics Research Institute, Stockholm School of Economics.