It Pays to Violate: Model Choice and Critical Value Assumption for Forecasting Value-at-Risk Thresholds

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EXTENDED ABSTRACT

The internals models amendment to the Basel Accord allows banks to use internal models to forecast Value-at-Risk (VaR) thresholds which are used to calculate the required capital banks must hold in reserves as a protection against negative changes in the value of their trading portfolios. As capital reserves lead to an opportunity cost to banks it is likely that banks could be temped to use models that underpredict risk and hence lead to low capital charges. In order to avoid this problem the Basel Accord introduced backtesting procedure whereby banks using models that led to excessive violations would be penalised through higher capital chares. This paper investigates the performance of five popular volatility models that can be used to forecast VaR thresholds under a variety of distributional assumptions. The results suggest that within the current constraints and penalty structure set out in the Basel Accord the lowest capital charges arise when using models that lead to excessive violations, suggesting the current penalty structure is not severe enough.

1 Introduction

On June 26, 1974 Herstatt, a German bank, had received large payments of DEM in Frankfurt in exchange for USD payments that were to be made in New York later that day due to time zone differences. However, before the USD payments were made, Herstatt was forced into liquidation by German regulators. The Herstatt fiasco led the G-10 countries to form a committed called the Basel Committee on Banking Supervision which was initially intended to deal with the role of regulators in cross-jurisdictional situations and investigate ways of harmonizing international banking regulations.

In 1988 the Basel Committee issued the Basel Capital Accord, which prescribe minimum capital requirements that Authorized Deposit Taking Institutions (ADI's) must meet as a protection against credit risk, this became law in all G-10 countries by 1992 with the exception of Japan that was granted an extended transition period.

In 1993 the Basel Accord was amended to require ADI's to also hold capital in reserve against market risk based on the Value-at-Risk (VaR) approach. Value-at-Risk is a procedure designed to forecast the maximum expected loss over a target horizon, given a statistical confidence limit (see Jorion (2000) for a detailed discussion of VaR methods). Initially, the Basel Accord stipulated a standardized approach which all institutions were required to adopt in calculating their VaR thresholds. This approach suffered from several deficiencies, the most notable of which were its assumption of no diversification benefits which led to conservatism (or lost opportunities) and its failure to reward institutions with superior risk management expertise. In view of these drawbacks a further amendment, called the Market Risk amendment, was proposed in 1995 and subsequently adopted in 1996.

The Market Risk amendment to the Basel accord allows Authorized Deposit Taking Institutions (ADI's) to use internal models to measure and forecast market risk The forecasted market risk, or volatility, forms a basis for the calculation of the Value-at-Risk. However, in order to maintain discipline and ensure that ADI's have in place adequate models of market risk a backtesting procedure is used to count the number of times the actual losses exceeded the forecasted VaR over the previous 250 business days. As VaR models are designed to provide 99% coverage (or lead to violations 1% of the time) the Basel accord specifies penalties that increase the required capital charge if too many violations are detected.

A three-zone approach is used to measure the accuracy of the forecasting model as shown in table 1. ADI's that fall in the Green zone are deemed to have models that are adequately accurate and do not incur penalties from regulators. Once in the Yellow zone regulators will impose a penalty which will increase the required capital charge and will be required to justify the excessive number of violations, the greater the number of violation the more likely it is that ADI's will be penalized and required to revise their model. Finally, once an ADI enters the Red zone the model used is deemed to be unacceptably inaccurate and the ADI will be required to adopt a more stringent model that will lead to fewer violations and larger capital charges.

Within the constraints of the Basel accord ADI's should choose the model that leads to the lowest possible capital charge, conditional on the model not leading to the ADI falling in the Red zone (or upper Yellow zone). Such approach will ensure that the opportunity cost associated with capital charges are minimized while maximizing the benefits associated with minimal regulatory intervention, furthermore ADI's that have good risk management systems in place will benefit from a superior reputation, lowed cost of debt and perhaps stronger demand for its deposit facilities.

Table 1: Basel Accord Penalty Zones

Zone	Number of Violations	Increase in *k*
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85

Red	10+	1.00
Note: The number of violations is given for 250 business days.		

This paper evaluates the VaR forecasting performance of five popular conditional volatility models namely the ARCH model of Engle (1982), GARCH model of Bollerslev (1986), EGARCH model of Nelson (1991), the GJR model of Glosten Jagannathan and Runkle (1993) and the restricted EWMA model proposed by Riskmetrics^{TM} (1996) which is the industry standard (see McAleer (2005) for a detailed discussion of conditional volatility models).

For the purpose of forecasting VaR thresholds is necessary to assume a distribution for t returns so that the appropriate critical values c be chosen. In this paper three distributior assumptions are made namely that returns a normally distributed, that the returns follow a distribution where the appropriate degrees freedom are estimated and that the returns follc a generalised error distribution (GED) where t appropriate GED parameter is also estimate Finally, as an alternative to assuming a particul distribution the critical values are also obtain through bootstrapping.

2 Data

The data used in this paper is a long series of the S&P500 index daily returns ranging from 14 January 1986 to 28 March 2005. The S&P500 index was chosen as it is commonly regarded as the industry proxy for US stock market performance.

Figure 1 plots the S&P500 index returns for the period. As can be seen the series displays considerable clustering that needs to be captured by an appropriate model. Figure 2 plots the histogram of returns and gives the descriptive statistics. The S&P500 has a mean return of 0.035%, maximum of 8.709% and minimum of -22.833% which occurred during the 1987 stock market crash. Furthermore, the series is negatively skewed, has extremely high excess kurtosis and the Jarque-Bera statistic strongly rejects the null hypothesis of normality.

Figure 1: S&P500 Returns

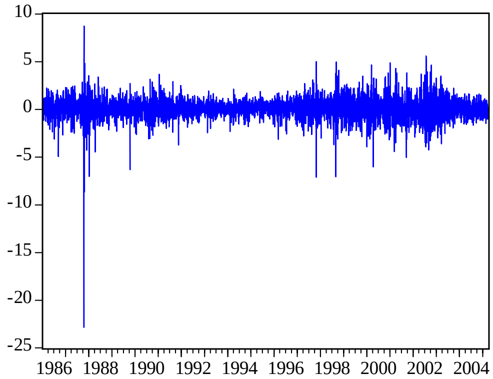
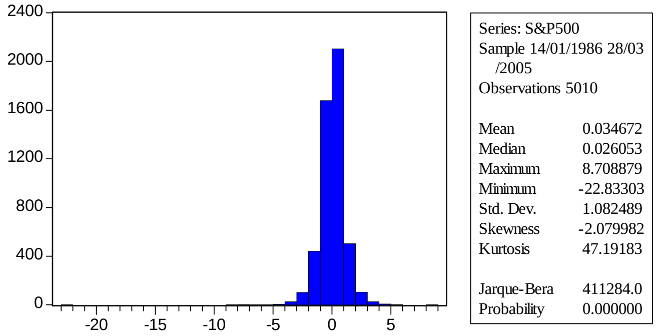


Figure 2: S&P500 Returns Histogram and Descriptive Statistics



3 Models

2.1 EWMA

Riskmetrics^{TM} (1996) developed a model which estimates the conditional variances and covariances based on the exponentially weighted moving average (EWMA) method, which is, in effect, a restricted version of the ARCH(∞) model of Engle (1982). This approach forecasts the conditional variance at time t as a linear combination of the lagged conditional variance and the squared unconditional shock at time $t-1$. The EWMA model calibrates the conditional variance as:

$$h_t = \lambda h_{t-1} + (1-\lambda)\varepsilon_{t-1}^2 \qquad (1)$$

where λ is a decay parameter. Riskmetrics^{TM} (1996) suggests that λ should be set at 0.94 for purposes of analysing daily data.

2.3 ARCH

Engle (1982) proposed the Autoregressive Conditional Heteroskedasticity of order p, or ARCH(p), model as follows:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 . \qquad (2)$$

For the case $p=1$, $\omega > 0$, $\alpha_1 > 0$ are sufficient conditions to ensure a strictly positive conditional variance, $h_t > 0$. The ARCH (or α_1) effect captures the short run persistence of shocks.

2.4 GARCH

Bollerslev (1986) generalized ARCH(p) to the GARCH(p,q) model, which is given by:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{q} \beta h_{t-i} . \qquad (3)$$

For the case $p=1$, $\omega > 0$, $\alpha_1 > 0$, $\beta_1 \geq 0$ are sufficient conditions to ensure a strictly positive conditional variance, $h_t > 0$. The ARCH (or α_1) effect captures the short run persistence of shocks, and the GARCH (or β_1) effect indicates the contribution of shocks to long run persistence ($\alpha_1 + \beta_1$).

In ARCH and GARCH models, the parameters are typically estimated using the maximum likelihood estimation (MLE) method. In the absence of normality of the standardized residuals, η_t, the parameters are estimated by the Quasi-Maximum Likelihood Estimation (QMLE) method (see, for example, Li, Ling and McAleer (2002)).

2.5 GJR

Glosten, Jagannathan and Runkle (1992) extended the GARCH model to capture possible asymmetries between the effects of positive and negative shocks of the same magnitude on the conditional variance through changes in the debt-equity ratio. The GJR(p,q) model is given by:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \gamma I(\eta_{t-1}) \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \qquad (4)$$

where the indicator variable, $I(\eta_t)$, is defined as:

$$I(\eta_t) = \begin{cases} 1, & \varepsilon_t \leq 0 \\ 0, & \varepsilon_t > 0 \end{cases} . \qquad (5)$$

For the case $p=1$, $\omega > 0$, $\alpha_1 > 0$, $\alpha_1 + \gamma_1 > 0$, $\beta_1 \geq 0$ are sufficient conditions to ensure a strictly positive conditional variance, $h_t > 0$. The indicator variable distinguishes between positive and negative shocks, where the asymmetric effect ($\gamma_1 > 0$) measures the contribution of shocks to both short run persistence ($\alpha_1 + \gamma_1 / 2$) and long run persistence ($\alpha_1 + \beta_1 + \gamma_1 / 2$).

Several important theoretical results are relevant for the GARCH model. Ling and McAleer (2002a) established the necessary and sufficient conditions for strict stationarity and ergodicity, as well as for the existence of all moments, for the univariate GARCH(p,q) model, and Ling and McAleer (2003) demonstrated that the QMLE for GARCH(p,q) is consistent if the second moment is finite, $E(\varepsilon_t^2) < \infty$, and asymptotically normal if the fourth moment is finite, $E(\varepsilon_t^4) < \infty$. The necessary and sufficient condition for the existence of the second moment of ε_t for the GARCH(1,1) model is $\alpha_1 + \beta_1 < 1$.

Another important result is that the log-moment condition for the QMLE of GARCH(1,1), which is a weak sufficient condition for the QMLE to be consistent and asymptotically normal, is given by $E(\log(\alpha_1 \eta_t^2 + \beta_1)) < 0$. The log-moment condition was derived in Elie and Jeantheau (1995) and Jeantheau (1998) for consistency, and in Boussama (2000) for asymptotic normality. In practice, it is more straightforward to verify the second moment condition than the weaker log-moment condition, as the latter is a function of unknown parameters and the mean of the logarithmic transformation of a random variable.

The GJR model has also had some important theoretical developments. In the case of symmetry of η_t, the regularity condition for the existence of the second moment of GJR(1,1) is $\alpha_1 + \beta_1 + \gamma_1 / 2 < 1$ (see Ling and McAleer (2002b)). Moreover, the weak log-moment condition for GJR(1,1), $E(\log[(\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1]) < 0$, is sufficient for the consistency and asymptotic normality of the QMLE (see McAleer, Chan and Marinova (2002)).

2.6 EGARCH

Nelson (1991) proposed the Exponential GARCH (EGARCH) model, which is given as:

$$\log(h_t) = \omega + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{h_{t-i}} \right| . + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{h_{t-k}} + \sum_{j=1}^{q} \beta_j \log(h_{t-j}) \quad (6)$$

As the range of $\log(h_t)$ is the real number line, the EGARCH model does not require any parametric restrictions to ensure that the conditional variances are positive. Furthermore, the EGARCH specification is able to capture several stylised facts, such as small positive shocks having a greater impact on conditional volatility than small negative shocks, and large negative shocks having a greater impact on conditional volatility than large positive shocks. Such features in financial returns and risk are often cited in the literature to support the use of EGARCH to model the conditional variances.

Unlike the EWMA, ARCH, GARCH and GJR models, EGARCH uses the standardized rather than the unconditional shocks. Moreover, as the standardized shocks have finite moments, the moment conditions of EGARCH are straightforward and may be used as diagnostic checks of the underlying models. However, the statistical properties of EGARCH have not yet been developed formally. If the standardized shocks are independently and identically distributed, the statistical properties of EGARCH are likely to be natural extensions of (possibly vector) ARMA time series processes (for further details, see McAleer (2005)).

4 Forecasts

A rolling window approach is used to forecast the 1-day ahead 1% VaR thresholds using the five conditional volatility models described in

Table 2: Forecast Results

	Normal	t-dist	GED	Bootstrap
GARCH				
	Distributional Assumption			
Number of Violation	47	13	22	21
Capital Charge	7.928	10.353	8.308	8.591
Proportion of Time Spent out of the Green Zone	42%	0%	9%	6%
Risk Metrics				
	Distributional Assumption			
Number of Violation	59	14	35	31
Capital Charge	7.882	10.095	8.271	8.483
Proportion of Time Spent out of the Green Zone	57%	0%	19%	12%
EGARCH				
	Distributional Assumption			
Number of Violation	42	15	28	25
Capital Charge	7.536	9.710	8.123	8.301
Proportion of Time Spent out of the Green Zone	35%	0%	15%	8%
ARCH				
	Distributional Assumption			
Number of Violation	80	11	43	42
Capital Charge	8.099	11.542	8.542	8.448
Proportion of Time Spent out of the Green Zone	67%	3%	41%	41%
GJR				
	Distributional Assumption			
Number of Violation	45	13	30	27
Capital Charge	7.656	9.818	8.307	8.509
Proportion of Time Spent out of the Green Zone	30%	0%	21%	15%

Notes:
1) The daily capital charge is given as the negative of the higher of the previous day's VaR or the average VaR over the last 60 business days times (3+*k*), where *k* is the penalty.
2) The expected number of violations is 30 at the 1% level

section 1. A rolling window approach is one where the first n observations are used to estimate the model and forecast the $n^{th}+1$ observation. the sample is then rolled forward by 1 observation so that it ranges from the 2^{nd} to the nth+1 observation and the $n^{th}+2$ observation s forecasted. This process is repeated until the end of the sample. In order to strike a balance between efficiency in estimation and a viable number of forecasts a rolling window size of 2000 observations is chosen, which leaves 3010 observations to be forecasted.

Table 2. gives the results of the forecasting exercise .As can be seen assuming a t-distribution always leads to the lowest number of violations and the highest average capital charge, while assuming a normal distribution always leads to the highest number of violations and the lowest average capital charge. It is interesting to note that the EGARCH model leads to the lowest average capital charge across all distributional assumptions, while the ARCH model leads to the highest capital charge in all cases except where the critical values are obtained through bootstrapping.

It is interesting to note that difference in VaR forecasting performance is much greater across the various distributional assumptions for a given model, than across the various models for a given distributional assumption. This result suggests that the distributional assumption is more important than the choice of conditional volatility model.

Finally, as can be seen assuming a t-distribution always leads to the lowest number of violations and the highest average capital charge, while assuming a normal distribution always leads to the highest number of violations and the lowest average capital charge. As the VaR thresholds are estimated assuming a 99% confidence level and there are 3010 forecasts the expected number of violations is approximately 30.

The results then suggest that the assumption of normality is inadequate, as it leads to more violations than could be reasonably expected; while the assumption that the returns follow a t-distribution is also inadequate as it leads to far fewer violations than could be reasonably expected. The results suggest that assuming the returns follow a GED distribution or bootstrapping the critical values lead to VaR threshold forecasts that yield the correct number of violations in most cases.

Within the framework set out in the Basel Accord banks should choose the VaR model that leads to the Lowest capital charge, while not yielding backtesting results that fall in the red zone. Out of all the model/critical value combinations considered in this paper only the ARCH-normal, ARCH-GED, ARCH-bootstrap and GJR-normal lead to backtesting results that fall in the red zone over the entire forecasting period. Hence all other models satisfy the Basel Accord constraints and hence are eligible to be used for the purpose of calculating the banks capital charges.

Two important observations arise from this study. First, across all model/critical value combinations that satisfy the Basel Accord the EARCH-normal model gives the lowest daily capital charge at 7.536%, while the ARCH-t gives the highest at 11.542%, suggesting that the capital charges can be significantly reduced by choosing the appropriate model/critical value combination.

Second, for each model considered the assuming a t-distribution always leads to the highest capital charges, while assuming that the returns follow a normal distribution always leads to the lowest capital charges. This results has serious implications for regulators as it suggests that given the current penalty structure proposed in the Basel Accord, banks have an incentive to choose models that lead to excessive violations.

5 Conclusion

This paper analysed the performance of five popular conditional volatility models in forecasting VaR thresholds. The Basel Accord stipulates that banks must hold capital in reserves to cover their exposure to market risk, or the risk that a banks portfolio will experience a severe negative return. The need for banks to have in place adequate risk management systems stems from the intrinsically systemic nature of the banking industry, where bank failures can quickly spread and harm the entire financial system.

Originally the market risk amendment set out a standardised model that all banks were required to use when calculating their VaR thresholds. This model was heavily criticised by industry participants as being too conservative and hence leading high capital charges. Furthermore it was argued that this approach did not reward institutions with superior risk management and did not promote research into more sophisticated VaR models.

The internal modes amendment to the Basel accord was intended to allow bans to use internal models, provided a series of quantitative and qualitative criteria were met. An obvious concern of regulators was that this amendment would encourage banks to pick models that underpredicted risk and hence led to lower capital charges than models that correctly predicted risk. Hence a backtesting procedure was developed to assess the performance of each model and to penalise models that underpredicted risk through higher capital charges.

The results suggest that the EGARCH model dominates all other models, as it gives the lowest capital charge while never entering the red zone. The ARCH model is always the worst performing model, giving the highest capital charge and almost always falling in the red zone.

Of all the distributional assumptions, the assumption of normality always leads to the most number of violations, which are much higher than expected given the confidence level chosen, and the lowest capital charges. While assuming a t-distribution always leads to the lowest number of violations, which are much lower than expected given the confidence level chosen, and the highest capital charges.

When the critical values are obtained through bootstrapping or the assumption that the returns follow a Generalised Error distribution the results lead to the correct number of violations and capital chares that are higher than under the assumption of normality but lower than under the assumption that the returns follow a t-distribution.

These results suggest that the penalty structure proposed under the Basel Accord is not severe enough to discourage banks from choosing models and critical values that clearly underpredict risk.

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