Multiobjective Optimisation of Active and Semi-Active Suspension Systems with Application of Evolutionary Algorithm

[2]Bourmistrova, A., [1]Storey, I. , [2]Subic, A.

[1]RMIT Business Information Systems, [2]RMIT Mechanical and Aerospace Engineering, E-Mail: ian.storey@rmit.edu.au

Keywords: *Evolutionary Algorithms; Control; Suspension.*

EXTENDED ABSTRACT

Suspension system design is a challenging task with multiple control parameters, complex (often conflicting) objectives and stochastic disturbances. It is essential to develop a design environment, in the form of a mathematical model, which will help engineering efforts, not only in algorithm design but also in the investigation of various research questions. This paper examines issues relevant to semi-active suspension control system design optimization. It also presents a numerical approach for such optimization.

Evolutionary algorithms (EAs) are applied to the optimization of the control system parameters. EAs are computer-based techniques that mirror natural genetic evolution, and they have been found to be successful in application to a wide range of problems that are difficult to solve analytically.

The algorithms use randomly chosen road surfaces as input to a quarter-car computer model and develop the design of a number of non-linear, semi-active suspension control systems. These are then compared using a fitness function as a measure.

For a suspension system the goal of ride comfort conflicts with the restriction of staying within the limits of suspension travel, or rattle space (the distance between the car body and the tyre which is restricted to the suspension travel limit). Thus the EAs examined in this paper use a multi-objective fitness function which is a weighted sum of car body rate-of-change of acceleration and suspension travel. Two separate fitness measures are analysed and developed, and are combined in a weighted sum. Various different weights are used and the effect of the weighting is analysed.

An EA was applied to the development of a semi-active suspension system using a computer simulation model. The modelled control system uses sensor measurements of car-body acceleration and suspension travel to provide indicators that will allow an EA to determine the best mix of strategies. The goal is to simultaneously maintain the suspension travel within the rattle space and to minimize car-body rate-of-change of acceleration. A number of car-suspension control algorithms mentioned in the literature are analysed using an EA, such as passive and skyhook controls (Gordon and Best, 1994; Savaresi et al., 2003; Caponetto et al., 2003; Jalili, 2002) as well as the “on-off skyhook control policy” (Caponetto et al., 2003; Jalili, 2002). Some minor variations on these designs are also analysed.

The parameters developed in the EAs are compared and the corresponding fitness functions are also compared. The results are given below, indicating strengths and weakness of the various control strategies using the components of the fitness measure. The results are developed for verification and validation purposes. These indicated that the passive system was quite robust and that adapting the skyhook system to a semi-active system was counter productive for comfort and for keeping the suspension within the rattle space.

It is also clear that genes covering more factors in combination are needed for a deeper investigation using this technique, and that it is necessary to go beyond the skyhook control. Experiments using a combination of factors, and using member-set functions, are currently being planned. Further work will use Pareto optimums. Pareto optimum measures compare fitness using a number of separate objectives simultaneously, and the various objectives can be compared independently.

Experiments are being planned to investigate real road surfaces, especially the frequency profile of road noise. The simulation test bed is being extended to include active suspensions as well as more degrees of freedom.

1. INTRODUCTION

Suspension system design is a multidisciplinary task which presents a computational and modelling challenge. Passive and active control systems have been developed over a number of years, mainly by analysing linear filters (Son et al., 2001; Jalili, 2002) or by a combination of experimentation and numerical methods using piecewise-linear analysis (Majjad, 1997; Huang and Lin, 2004). Studies attempting to frame analysis in terms of nonlinear analysis (Gordon and Best, 1994) prove impractical. Such analyses are extremely sensitive to the choice of optimality measure: some measures result in physically unrealizable controls, such as infinitely small impulses of infinite force, and they often consist of discontinuous pulses. "The application of optimal control to practical problems is an art." (MacCluer, 2005)

A number of suspension system designs have been developed using non-analytical optimization, such as fuzzy logic control (Caponetto et al., 2003), and genetic algorithms applied to the optimization of fuzzy control algorithms (Hashiyama et al., 1995a; Nicolas et al., 1997) as well as neural networks (Guo et al., 2004).

Evolutionary Algorithms are computer algorithms mirroring natural evolution in which a string of values is used to represent the parameters of a problem, in this case the parameters of a suspension control system. A series of generations are produced each one selecting genes from the previous generation. Mutation is applied and crossover is also sometimes applied. EAs have been found to be effective in problems where analytical solutions are computationally intractable.

EAs are generally applied to the control design problem to produce a fast and efficient control algorithm, rather than being applied in real time as the optimal control itself (Caponetto et al., 2003; Nicolas et al., 1997; Hashiyama et al., 1995a). The controls examined here are quite modest with only a handful of parameters for consideration: spring constants and a small number of parameters affecting damper stiffness. Results of these experiments are given below (section 5).

2. MATHEMATICAL MODEL

The simple, linear mathematical model of spring and damper systems in laboratory experiments is accurate to within a fraction of a percent. Where more accurate models are needed there are associated costs when an engineer sets out to verify and validate the design in loop with a real system. The time lag in the suspension system's response to control can be included into a linear model, as in equation 1 (Savaresi et al., 2003) where $c_{in}(t)$ is the desired response, c is the actual damper control response and β is a constant representing the speed of response of the damper to the control signal.

$$\dot{c}(t) = -\beta c(t) + \beta c_{in}(t), \quad c(t) \geq 0 \forall t, \qquad (1)$$

Simple, quarter-car and half-car models have only a small number of degrees of freedom (from 1 to 6). More degrees of freedom are often required, as in the modelling of large trucks with suspended cabins, or articulated vehicles (Valasek and Kortum, 2002) At the very limit, the flexibility in the vehicle's body can be modelled, introducing an extraordinary large number of degrees of freedom and immense computational complexity. This level of detail is used for aircraft with high-tech, flexible fuselages (Valasek and Kortum, 2002) or railway vehicles (Foo and Goodall, 1998).

There also examples of more comprehensive mathematical models which include the nonlinearity of the damper (Wu and Xu, 1999; Gordon and Best, 1994; Caponetto et al., 2003). Such nonlinearities can be modelled in an EA and so the impact of nonlinearities and response times on suspension control performance can be taken into account in the design, although only the simple linear model was used in the results below.

Models of the physical control mechanisms themselves are also used and there is a wide variety of such systems. This includes magnetorheological (MR) dampers (Guo et al., 2004; Jalili, 2002), dampers using fluid and electronically controlled valves (Nicolas et al., 1997; Jalili, 2002), and "hydropneumatic suspension systems" (Deprez et al., 2002) which employ "a hydraulic cylinder, two nitrogen bulbs and a current driven hydraulic valve". In the case of active controls this includes regenerative dampers (Okada and Harada, 1996), and hydraulic systems powered by gas pressure (Williams and Best, 1994).

The road surface is a stochastic environmental disturbance. Some papers examine simple, regular road profiles (Lin and Kanellakopoulos, 1997; Huang and Lin, 2004; Nehl et al., 1996); others use transients (Gordon and Best, 1994; Okada and Harada, 1996), sine-wave holes (Caponetto et al., 2003), actual measurements of road profiles (Deprez et al., 2002), Gaussian white noise (Foo and Goodall, 1998; Yokoyama et al., 2001), and even “chirp” functions to model random road profiles.

The approach used here is to take random sums of sinusoidal functions with randomly chosen amplitude, phase and frequency. Frequency is in proportion to the inverse of amplitude squared, on average.

Figure 1 shows a schematic diagram of a quarter-car semi-active suspension control system.

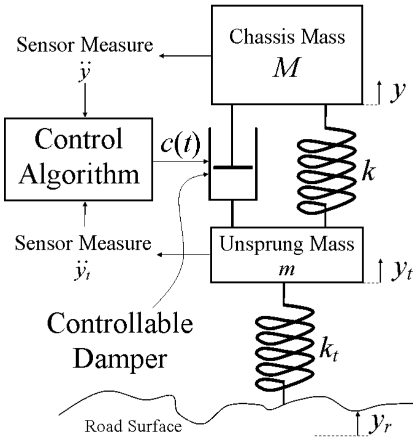

Figure 1. Semi-Active Suspension

The equations of motion are given in equation 2.

$$\begin{aligned} M\ddot{y} &= -k(y - y_t) - c(t)(\dot{y} - \dot{y}_t), \\ m\ddot{y}_t &= k(y - y_t) + c(t)(\dot{y} - \dot{y}_t) - k_t(y_t - h). \end{aligned} \qquad (2)$$

The controlled parameter is $c(t)$, the damper stiffness, which is here controlled directly. (Gordon and Best, 1994; Jalili, 2002; Guo et al., 2004; Majjad, 1997). In the experiment below a chassis mass of M=575kg was used. The tyre mass, m, was assumed to be low at this stage of the investigation. The variable k is the spring constant of the suspension spring and k_t is the spring constant of the tyre.

The measurable parameters are the vertical motion of the chassis body using an accelerometer placed on the body, and a measure of the tyre height. The resulting measurements provide the car-body displacement, y, the tyre displacement, y_t, and their first derivatives (velocity), second derivatives (acceleration), and third derivatives (jerk). Each of these can either be measured directly or can be calculated by fast, efficient algorithms on board (Caponetto et al., 2003; Deprez et al., 2002; Majjad, 1997).

Instabilities in active suspensions can be expressed even when the car is stationary (Williams and Best, 1994). Passive dampers can increase the kinetic energy of the car body when the road is pushing the car in the same direction as its vertical velocity. The skyhook damper absorbs road energy under such conditions. Given that energy build-up is minimized with respect to the average road travel this could mean that a skyhook system is safer than a passive system.

An interesting control approach is to set the suspension lower and presumably stiffer under more demanding driving conditions: high speed, hard braking, cornering, etc. (Middleton, 2000)

The approach here is to use EAs to alter parameters for a number of algorithms to find a compromise between “comfort” and staying safely within the rattle-space limits.

3. FITNESS FUNCTIONS

There are a number of possible measures for possible competing objectives for a suspension system:

- Vertical ground-tyre force, as an indicator of road handling and performance (Caponetto et al., 2003),
- Energy (potential and kinetic) and acceleration as indicators of safety (Jalili, 2002),
- “Rattle space” limit, i.e., the physical limit of the extension of the suspension, $|y - y_t|$ (Lin and Kanellakopoulos, 1997),
- Magnitude of the frequency transfer function $(FTF(\omega) = |FTF(s)|_{s=j\omega})$ (Jalili, 2002),
- “Describing function”, defined by Savaresi et. al. (2003), as a better measure than the variance gain for nonlinear systems,
- Legal limitations (http://www.dotars.gov.au/transreg/vsb/PDF/vsb_11.pdf, 2004; Deprez et al., 2002), and
- Vertical chassis displacement (discussed below).

Most investigations have concentrated solely on acceleration. A typical measure of performance has the form shown in equation 3, where the integral is taken over a given time period, T

(Caponetto et al., 2003; Savaresi et al., 2003; Wu and Xu, 1999; Yedavalli and Liu, 1994).

$$J = \int_0^T \ddot{y}(t)^2 \, dt, \qquad (3)$$

The official European measure of comfort, Vibration Dose Value, uses fourth-order powers (equation 4). (Deprez et al., 2002)

$$VDV = \left[\frac{T}{N} \sum_{n=1}^{N} a_n{}^4 \right]^{\frac{1}{4}}. \qquad (4)$$

Car-body travel distance and/or velocity, as in equations 5 or 6 below, are also used (Gordon and Best, 1994; Wu and Xu, 1999).

$$J = \int_0^T y(t)^2 \, dt, \qquad (5)$$

$$J = \int_0^T \dot{y}(t)^2 \, dt. \qquad (6)$$

The two acceleration profiles shown in Figure 2 give exactly the same measure of passenger comfort using equation 3, since they have exactly the same envelope, and yet the first, Figure 2(a), will clearly be much less comfortable, given that the passenger is constantly thrust from one extreme of acceleration to another. The graph of Figure 2(a) is similar to the kind of profile one would expect from a bang-bang control (MacCluer, 2005; Smith, 1998).

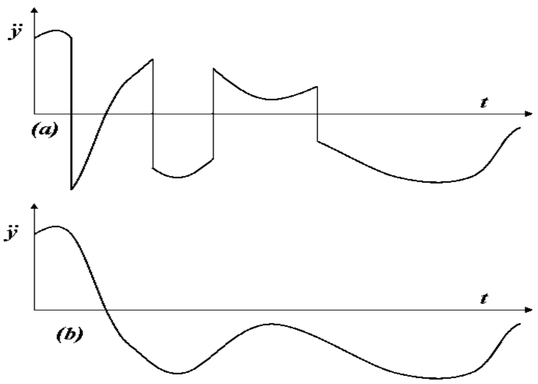

Figure 2. Acceleration profiles

These considerations indicate that jerk (rate-of-change of acceleration) can be used as a measure of comfort, using equation 7 (Hashiyama et al., 1995a; Hashiyama et al., 1995b).

$$J = \int_0^T \dddot{y}(t)^2 \, dt, \qquad (7)$$

Two competing goals are considered here: one is "comfort" described above, and the other is the rattle-space limit. Lin and Kanellakopoulos use a fitness measure, φ, that penalizes controls that pass too close to the rattle-space limits. See Figure 3 and equation 8 (Lin and Kanellakopoulos, 1997).

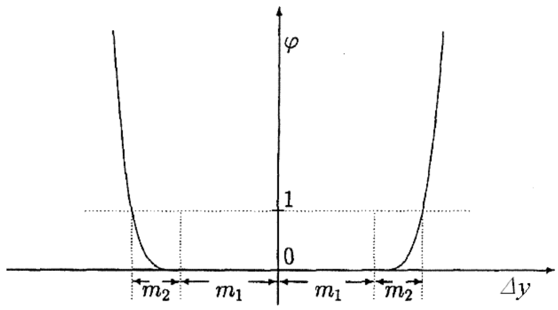

Figure 3. Rattle-Space Weighting Function

$$\varphi(\Delta y) = \begin{cases} \left(\dfrac{|\Delta y| - m_1}{m_2} \right)^4, |\Delta y| > m_1 \\ 0, |\Delta y| \le m_1 \end{cases} \qquad (8)$$

where $\Delta y = y - y_t$. m_1 and m_2 are parameters which are used to penalise a suspension that is too close to its rattle space limits: at a distance of m_1+m_2 it has reached its safety limit; within a distance of m_1 it is inside the safe travel limit. The integral of φ, equation 9, is used to indicate the extent to which the system stays within the limits of the rattle space during a simulation.

$$\int_0^T \varphi(|y - y_t|) dt. \qquad (9)$$

The two fitness measure used in the experiment below was a weighted sum of the measures in equations 7 and 9, as shown in equation 10 below. The change in outcomes can be compared as the weighting, w, is varied from zero to one.

$$w \int_0^T \dddot{y}(t)^2 \, dt, + (1 - w) \int_0^T \varphi(|y - y_t|) dt \qquad (10)$$

4. EVOLUTIONARY ALGORITHM

The EA used here was a generational, ranking algorithm with truncation (Dumitrescu et al., 2000) and programmed in java (1.5). The genomes were binary strings representing design parameters for the suspension system. A number of different genomes were used for experimentation.

Passive: This genome simply represents a passive suspension (Kreyszig, 1993).

Skyhook Emulation: The skyhook control (Jalili, 2002; Savaresi et al., 2003) is a conceptual system. This genome emulates a skyhook within the control limits of the damper. The genome contains two extra real values to represent the simulated skyhook spring constant, k_{sim}, and the simulated skyhook damping constant, c_{sim}. This requires that the damper be controlled according to equation 11 below. The actual value applied is between zero and the maximum damping stiffness that can be applied by the damper.

$$c(t) = \frac{(k_{sim} - k)(y - y_t) + c_{sim}\dot{y}}{\dot{y} - \dot{y}_t}. \qquad (11)$$

Skyhook Emulation with Acceleration Decay: This is a variant of the previous genome except that the damping constant is controlled so that the acceleration decays exponentially to the same value as that of the simulated skyhook.

Skyhook Emulation with Limited Jerk: This is again similar to the skyhook emulation except that the jerk on the chassis body is limited. This requires an extra gene parameter to represent the jerk limit.

Passive Switch: Physical analysis shows that this control minimizes the rate of increase of the energy of the system (spring potential energy and chassis kinetic energy). The control here is to set the damper to maximum if $y(\dot{y} - \dot{y}_t) > 0$, otherwise set it to zero.

Passive Switch with Limited Decay: This is a variant of the previous that uses a simple linear decay of the damping constant to zero as $y(\dot{y} - \dot{y}_t)$ decreases. Again, outside the damper limits the damping constant is either zero or the maximum.

Each gene in a generation ran a number of simulations over different road profiles. Suspensions were also started from a height to include a transient response within the analysis. All simulations ran for 15 seconds, using Runge-Kutta with a step size of 0.02 seconds. A sample simulation is shown in Figure 4.

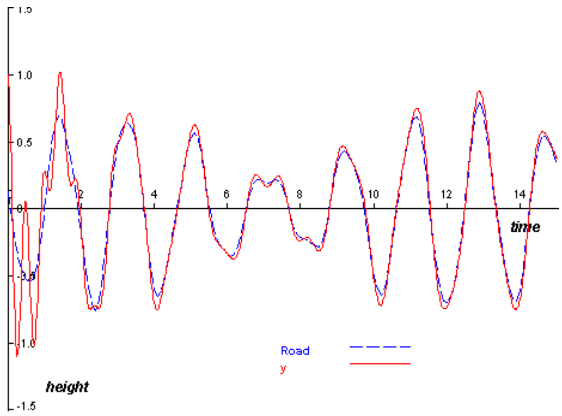

Figure 4. Sample Simulation

Each separate fitness function was run with the schedule of generations shown in Table 1. A weighted average of the data from the 10 highest ranked genes from the final generation, weighted by fitness, was used for the final result.

5. RESULTS

Results for the values of c, k and simulated c and k for the Passive, Passive Switch and the Emulated Skyhook are shown in Figure 5 (for convenience values of c and k are included on the same graph). When the weighting, w, is close to one, the spring constant and the damping stiffness are low, because the fitness measure prioritizes a softer, more comfortable suspension. There is a noticeable shift in evolved parameters between w=0.3 and w=0.5 which seems to indicate that the weighted sum method of multiobjective optimization is somewhat erratic, perhaps due to genetic drift.

The graph in Figure 6 compares the fitness of the various controls. It is clear that the skyhook control using a semi-active system is not as good at optimizing both comfort (measured using jerk) and rattle-space safety (using equation 9) as the simple passive system.

Table 1 Generations Schedule

Num Generations	Num Per Generation	Num Roads/ Gene	Mutation Rate	Crossover Rate	Skew To Higher Fitness	Proportion Of Gen. Kept
1	1000	3	0.2	0.5	2	0.5
2	500	5	0.2	0.5	2	0.6
10	150	8	0.05	0.1	2	0.8
3	80	10	0.005	0.02	2	0.9
2	40	25	0.0005	0.001	1	0.9

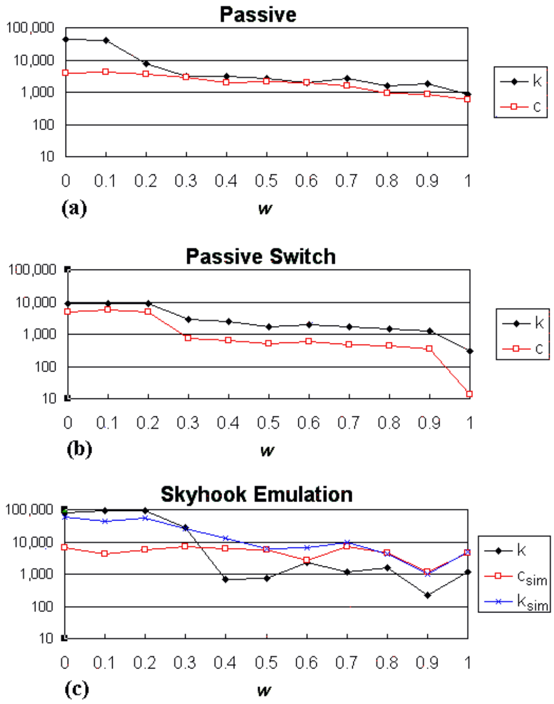

Figure 5. Values for *c*, *k*, and simulated *c* and *k*

It might be expected that the EA would use the higher available forces of a larger spring constant to provide greater control but would simulate a smaller spring constant. In Figure 5(c) we see that the opposite is the case: at higher values of *w* the simulated spring constant is actually higher than the actual value. A semi-active suspension system has the drawback that it can only control in one direction, opposite to the velocity of the damper's extension. This places limits on the controls and causes discontinuities in the simple algorithms used here. A softer actual spring may ameliorate this effect.

Very simple modifications were used to to smooth the response of the system as it approaches the point that the semi-active system control shifts direction. In fact, the modifications to the skyhook emulation, adding acceleration decay and limiting jerk, significantly degraded its performance, as can be seen in Figure 6. The modification to the passive switch, however, was quite successful.

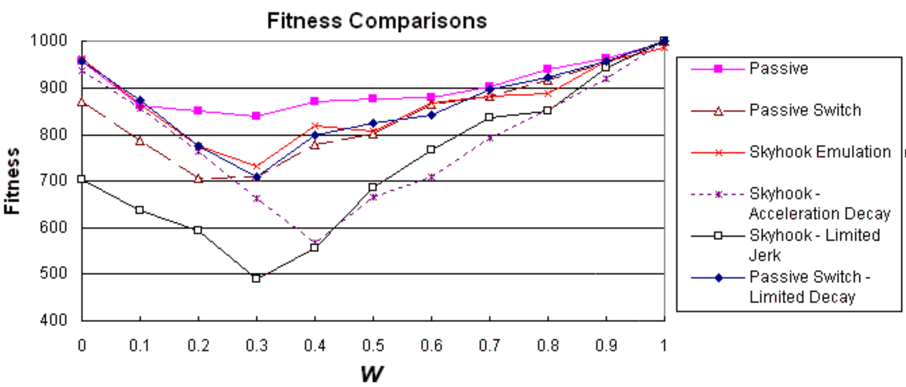

Figure 6. Fitness Measures for All Controls

6. CONCLUSIONS

This paper gave an overview of the simulation model which is developed in order to create a research environment for a variety of suspension systems. The research is in its initial stages. Results present evaluations of a number of simple semi-active control designs derived from the literature.

The parallel evolution of these fitness measures tended to cause rattle-space limitations to overpower the comfort factor. This may be why there is to be a sudden shift in evolved parameters between 0.3 and 0.5 in Figure 5, and why there is a dip in the combined fitness at these values.

The system of weighting fitness provides a good comparison between effects but it could be better employed in a Pareto ranking multi-objective algorithm (Obayashi, 1996). Furthermore the weighting system requires that the entire EA be rerun with each separate value of *w* (11 times in this case). A system using Pareto ranking would only need to run once to give similar comparative results and so would be more efficient.

The simple experimental attempts to control the semi-active suspension when it approaches zero had limited success in improving the passive switch algorithm. More sophisticated algorithms based on combination of factors are needed for more conclusive results. An experiment is being prepared in which an improved control using a broader number of parameters will be included: y, y_t, and the first, second and third derivatives of these. Furthermore EAs will be used to derive member functions that best control these parameters.

7. REFERENCES

(2004) *Certification of Road-Friendly Suspension Systems,* http://www.dotars.gov.au/transreg/vsb/PDF/vsb_11.pdf, 10 Aug 05.

Caponetto, R., Diamante, O., Fargione, G., Ristano, A. and Tringali, D. (2003), A Soft Computing Approach to Fuzzy Sky-Hook Control of SemiActive Suspension, *IEEE Transactions on Control Systems Technology,* 11(No. 6).

Deprez, K., Maertens, K. and Ramon, H. (2002) In *American Control Conference, 2002. Proceedings of the 2002*, Vol. 2, pp. 1497-1501 vol.2.

Dumitrescu, D., Lazzerini, B., Jain, L. C. and Dumitrescu, A. (2000) *Evolutionary Computation,* CRC Press.

Foo, E. and Goodall, R. (1998) In *Control '98. UKACC International Conference on (Conf. Publ. No. 455),* pp. 1300-1305 vol.2.

Gordon, T. J. and Best, M. C. (1994) In *Control '94*.

Guo, D. L., Hu, H. Y. and Yi, J. Q. (2004), Neural Network Control for a Semi-Active Vehicle Suspension with a Magnetorheological Damper, *Journal of Vibration and Control,* 10(3), 461-71.

Hashiyama, T., Behrendt, S., Furuhashi, T. and Uchikawa, Y. (1995a) In *Systems, Man and Cybernetics, 1995. 'Intelligent Systems for the 21st Century'., IEEE International Conference on*, Vol. 5, pp. 4361-4366 vol.5.

Hashiyama, T., Furuhashi, T. and Uchikawa, Y. (1995b) In *Evolutionary Computation, 1995., IEEE International Conference on*, Vol. 1, pp. 279.

Huang, C.-J. and Lin, J.-S. (2004) In *Networking, Sensing and Control, 2004 IEEE International Conference on*, Vol. 2, pp. 719-724 Vol.2.

Jalili, N. (2002) In *The Mechanical Systems Design Handbook: Modeling, Measurement, and Control*(Eds, Nwokah, O. D. I. and Hurmuzlu, Y.) CRC Press, Dallas Texas.

Kreyszig, E. (1993) *Advanced Engineering Mathematics,* John Wiley & Sons, Inc., Singapore.

Lin, J.-S. and Kanellakopoulos, J. (1997) In *American Control Conference, 1997. Proceedings of the 1997*, Vol. 1, pp. 714-718 vol.1.

MacCluer, C. R. (2005) *Calculus of Variations: mechanics, control, and other applications,* Prentice Hall, Upper Saddle River, NJ.

Majjad, R. (1997) In *Procedings of the 1997 IEEE International Conference on Control Application*Hartford CT.

Middleton, G. (2000) *Volkswagen Touareg,* http://www.overlander.com.au/pg/vehicle_tests.php?id=220, 31/July.

Nehl, T. W., Betts, J. A. and Mihalko, L. S. (1996), An integrated relative velocity sensor for real-time damping applications, *IEEE Transactions on Industry Applications,* 32(4), 873(9).

Nicolas, C. F., Landaluze, J., Castrillo, E., Gaston, M. and Reyero, R. (1997) In *Fuzzy Systems, 1997., Proceedings of the Sixth IEEE International Conference on*, Vol. 2, pp. 987-993 vol.2.

Obayashi, S. (1996) In *Industrial Technology, 1996. (ICIT '96), Proceedings of The IEEE International Conference on*, pp. 421-425.

Okada, Y. and Harada, H. (1996) In *35th Conference on Decision and Control*Kobe, Japan.

Savaresi, S. M., Silani, E., Bittanti, S. and Porciani, N. (2003) In *Decision and Control, 2003. Proceedings. 42nd IEEE Conference on*, Vol. 3, pp. 2264-2269.

Smith, D. R. (1998) *Variational Methods in Optimization,* General Publishing Company, Ltd., Toronto, Canada.

Son, H. Y., Jeong, S. G., Choi, J. Y., Kim, J. K., Cheon, Y. S., Paek, Y. I., Kwon, S. H. and Lee, M. H. (2001) In *Industrial Electronics, 2001. Proceedings. ISIE 2001. IEEE International Symposium on*, Vol. 3, pp. 1458-1461 vol.3.

Valasek, M. and Kortum, W. (2002) In *The Mechanical Systems Design Handbook: Modeling, Measurement, and Control*(Eds, Nwokah, O. D. I. and Hurmuzlu, Y.) CRC Press, Dallas Texas.

Williams, R. A. and Best, A. (1994) In *Control, 1994. Control '94. Volume 1., International Conference on*, pp. 338-343 vol.1.

Wu, Y. and Xu, B. (1999) In *Vehicle Electronics Conference, 1999. (IVEC '99) Proceedings of the IEEE International*, Vol. vol.1 Changchun, China, pp. 66-69.

Yedavalli, R. K. and Liu, Y. (1994) In *American Control Conference*Baltimore, Maryland.

Yokoyama, M., Hedrick, J. K. and Toyama, S. (2001) In *American Control Conference, 2001. Proceedings of the 2001*, Vol. 4, pp. 2652-2657 vol.4.