Optimal Capacitor Allocation Using Fuzzy Reasoning and Genetic Algorithms for Distribution Systems

Ching-Tzong Su **Guor-Rurng Lii** **Chih-Cheng Tsai**
Institute of Electrical Engineering, National Chung Cheng University, Chiayi 621, TAIWAN
Fax : 886-52752013 Email: ieects@ccunix.ccu.edu.tw

Abstract- This paper presents an optimal capacitor allocation method which uses Fuzzy Reasoning and Genetic Algorithm for the primary distribution system. In the method, capacitor allocation is applied to correct voltage deviation and reduce power loss for a given load pattern. The problem of capacitor allocation includes determining the location, type(fixed and switched), and size of capacitor. Fuzzy Reasoning finds the sensitive nodes which are used as the candidate locations for capacitors placement. Genetic Algorithms determine the size, number and type of capacitors to be placed in the system. It is a combinatorial optimization problem having objectives composed of peak power losses and capacitors' installation costs subject to bus voltage constraints. Three membership functions for voltage deviation, real power loss, and reactive power loss are defined to enable the fuzzy technique to be applied. The proposed approach is demonstrated using a nine-section feeder. Computational results show that the proposed method can quickly achieve a global optimal or near-optimal solution.

Keywords: Distribution system, Feeder, Optimization, Fuzzy-Reasoning, Genetic Algorithm.

1. INTRODUCTION

Capacitors have been very commonly employed to provide reactive power compensation in distribution systems. They are used to minimize power and energy losses and to enhance the voltage profile. The capacitor placement problem of distribution systems is a well-researched topic and had been addressed by many authors in the past "it was shown by Bae [1978]". The problem was commonly resolved by using mathematical programming technique. In this research, a bus voltage constraint is considered, and both fixed and switchable capacitors are employed. We present an optimization method which uses Fuzzy Reasoning and Genetic Algorithm. Fuzzy Reasoning finds the sensitive nodes. Sensitive nodes are those nodes to which, when additional capacitors are added, will have greater effect in reducing peak power and energy losses. Three membership functions are defined for real power loss, reactive power loss and voltage deviation, respectively. A high-power-loss section of the feeder is given a low membership value, while a low-power-loss section is given a high membership value; on the other hand, the voltage deviation can be similarly defined, a bus with high voltage deviation is designated a low membership value, whereas a bus with low voltage deviation is designated a high membership value. The α-cut operation of Fuzzy set is used to obtain the candidate locations for capacitor installation. The Genetic Algorithm determines the capacitor size and type for installation, is computationally simple and provides robust search in complex problem spaces. In Genetic Algorithm application, the fitness function for each string of the population is defined as the objective function of the system model, which is composed of the peak power losses and cost of capacitors added.

The solution procedures start off with performing a load flow study to calculate bus voltages and line losses. Then, determination of candidate locations for capacitor siting is performed by way of a membership function approach. Since the voltages along the feeders are required to maintain between their upper and lower limits, if a node whose voltage is not kept within its limits, the solution is discarded; otherwise the solution is accepted. Subsequently the peak power loss and overall annual savings are computed and compared with that of the previous selection. The procedures are repeated until an optimal capacitor allocation scheme is achieved.

2. LINE LOSS EVALUATION

The load flow problem includes the calculation of power flows and voltages of a network for specified terminals or bus conditions. A single-phase representation is adequate because power systems are usually balanced. Associated with each bus there are five quantities: the real and reactive power, the voltage magnitude and phase angle, as well as losses which can be computed. In this paper, we apply the simplified method to evaluate the power loss. We first develop a set of simplified line flow equations which completely avoid the iteration process needed for the exact line flow analysis. Consider the simple feeder shown in Figure 1 "it was shown by Chang and Kuo [1994]".

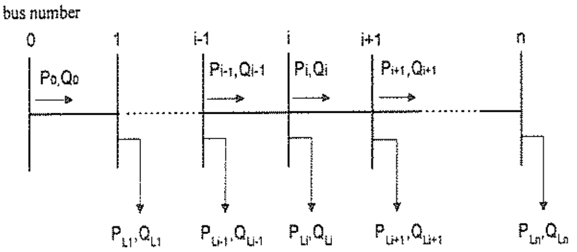

Figure 1 One-line diagram of a main feeder.

The system is assumed to be a balanced three-phase system. The real and reactive power flowing out of bus $i+1$ are represented by P_{i+1} and Q_{i+1}, respectively. The voltage magnitude at bus $i+1$ is denoted by $|V_{i+1}|$. The following set of recursive equations are used for power loss calculation:

$$P_i = P_{i+1} + P_{Li+1} + R_{i,i+1}\left((P_i^2 + Q_i^2)/|V_i|^2\right) \tag{1}$$

$$\approx P_{i+1} + P_{Li+1} + R_{i,i+1}\left[\left(P_{i+1} + P_{Li+1}\right)^2 + \left(Q_{i+1} + Q_{Li+1}\right)^2\right]$$

$$Q_i = Q_{i+1} + Q_{Li+1} + X_{i,i+1}\left((P_i^2 + Q_i^2)/|V_i|^2\right) \tag{2}$$

$$\approx Q_{i+1} + Q_{Li+1} + X_{i,i+1}\left[\left(P_{i+1} + P_{Li+1}\right)^2 + \left(Q_{i+1} + Q_{Li+1}\right)^2\right]$$

$$|V_{i+1}|^2 = |V_i|^2 - 2\cdot(R_{i,i+1}P_i + X_{i,i+1}Q_i) + \frac{(P_i^2 + Q_i^2)}{|V_i|^2} \tag{3}$$

where P_i is the real power flowing out of bus i, Q_i the reactive power flowing out of bus i, P_{Li} the real power load at bus i, Q_{Li} the reactive load power at bus i, V_i the voltage at bus i, $R_{i,i+1}$ the resistance of the line section between buses i and $i+1$, and $X_{i,i+1}$ the reactance of the line section between buses i and $i+1$. The power loss of the line section connecting buses i and $i+1$ may then be computed as:

$$P_{LOSS}(i,i+1) = R_{i,i+1}\cdot\frac{P_i^2 + Q_i^2}{|V_i|^2} \tag{4}$$

3. FUZZY SET OPERATIONS

The fuzzy set, which is a generalization of the classical crisp set, extends the values of set membership from two values {0,1} to the unit interval [0,1]. A fuzzy set can be defined mathematically by assigning to each possible element of the set of value representing its grade of possible element of membership in the set. A basic fuzzy operator used in this study is described in the following "it was shown by Zedeh [1965]":

Let A be a fuzzy set with membership function $u_A(x)$ the membership function of the α-*cut* set of elements that belong to the fuzzy set A at least to the degree of α in the universe of discourse(U) is given below:

$$A\alpha = \left\{x \in X \middle| u_A(x) \geq \alpha, x \in U\right\} \tag{5}$$

4. GENETIC ALGORITHMS

Genetic Algorithms (*GAs*) "it was shown by Glodberg [1989]" are basically search mechanisms based on the Darwinian principle of the natural evolution. They are the results of research done to incorporate the adaptive process of natural systems into design of artificial systems. GAs are computationally simple and provide robust search in complex problem spaces. During each iterative step referred to as generation, the representative strings in the current population are evaluated for their fitness as optimal domain solutions. On a comparative basis of these fitness values, a new population of solution strings is created using the genetic operators. Genetic operators are the stochastic transition rules employed by GA. These operators are applied on each string during each generation to generate a new and improved population from the old one. A simple GA consists of three basic operators: reproduction, crossover and mutation.

Reproduction

Reproduction is a probabilistic selection process in which strings are selected to produce offspring based on their fitness values. This ensures that the expected times or probability a string is chosen to copy on into the next generation is proportional to the string's fitness values.

Crossover

Crossover is the process of choosing a random position in the string and swapping the characters either left or right of this position with another similarly partitioned string. This random position is called the crossover point. For example:

father = 0 1 1 1 1 0 1 ⋮ 0 1 0 1 1
mother = 0 1 0 1 0 1 0 ⋮ 1 0 0 0 1

and suppose the crossover point is selected as shown and if swapping is implemented to the right of this point, the resulting structures would be

offspring 1 = 0 1 1 1 1 0 1 ⋮ **1 0 0 0 1**
offspring 2 = 0 1 0 1 0 1 0 ⋮ **0 1 0 1 1**

Mutation

Mutation is the process of random modification of the value of a string position with a small probability. It is not a primary operator but it ensures that the probability of searching any region in the problem space is never zero and prevents complete loss of genetic material through reproduction and crossover.

5. PROBLEM FORMULATION

limits while minimize the total cost. The total power loss is given by "it was shown by Baghzouz and Ertem [1990]" :

$$P_{T,LOSS} = \sum_{i=0}^{n-1} P_{LOSS(i,i+1)} \tag{6}$$

Considering shunt capacitors, there exists a finite number of standard sizes which are integer multiples of the smallest size Q_o^c. Besides, the cost per kvar varies from one size to another. In general, larger-size capacitors have lower unit prices than smaller ones have. The available capacitor size is usually limited to

$$Q_{\max}^c = LQ_o^c \tag{7}$$

where L is an integer. Therefore for each installation location, we have L capacitor sizes $\{Q_o^c, 2Q_o^c, \ldots, LQ_o^c\}$ to choose from.

Let $\{K_1^c, K_2^c, \ldots, K_L^c\}$ be the corresponding equivalent annual cost per kvar for the L capacitor sizes $\{Q_o^c, 2Q_o^c, \ldots, LQ_o^c\}$. Following the description aforementioned, the total annual cost function due to capacitor placement and power loss change is written as:

$$F = C_L(P_{T,LOSS}) + \sum_{j=1}^{J} K_j^c \cdot Q_j^c \tag{8}$$

where C_L is the equivalent annual cost per unit of power loss (in \$/(kW-Year)), and $j=1,2,\ldots,J$ are the indices of buses selected for compensation. The objective is to minimize the cost function subject to

$$V_{min} \le |V_i| \le V_{max}, \quad i = 1,2,\ldots,n \tag{9}$$

where V_{min} and V_{max} are the permissible minimum and maximum bus voltages, respectively, and n is the total number of buses. The problem mentioned above is a nonlinear optimization problem. In this case, the objective function and constraints can be computed for each possibility. Then, the global optimum solution is simply the one which satisfies the constraints with the least cost. However, as the number of possibilities increases the computational requirements become unacceptable.

6. THE PROPOSED METHOD

Three membership functions are defined to enable the proposed method to apply fuzzy technique. The first membership function U_{V_i} for voltage of bus $i(V_{i,pu})$ is defined as shown in Figure 2.

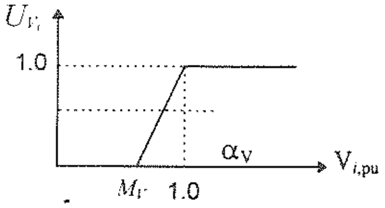

Figure 2 Membership function for U_{V_i}

$$U_{V_i} = \frac{V_{i,pu} - M_V}{1 - M_V}, \quad i = 1,2,\ldots,n;\ M_V \le V_{i,pu} \le 1$$

$$U_{V_i} = 1, \quad i = 1,2,\ldots,n; \quad 1 < V_{i,pu}$$

$$U_{V_i} = 0, \quad i = 1,2,\ldots,n; \quad V_{i,pu} < M_V \tag{10}$$

where M_V is a constant, and α_V is the level for the *α-cut* operation of the fuzzy set.

The second membership function U_{P_i} defined for real power loss of the line section between buses i and $i+1(P_{i,Loss})$ is depicted as Figure 3.

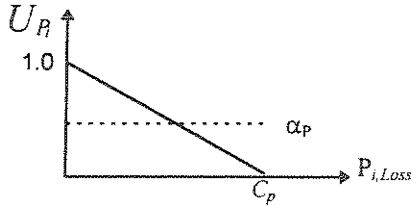

Figure 3 Membership function for U_{P_i}

$$U_{P_i} = -\frac{1}{C_P} P_{i,Loss(i,i+1)} + 1, \quad i = 1,2,\ldots,n;\ P_{i,Loss} \le C_P$$

$$U_{P_i} = 0 \qquad , i = 1,2,\ldots,n;\ C_P < P_{i,Loss} \tag{11}$$

where C_P is a constant, and α_P is the level for the *α-cut* operation of the fuzzy set.

The third membership function U_{Q_i} defined for the reactive power loss of the line section between buses i and $i+1(Q_{i,Loss})$ is depicted as Figure 4.

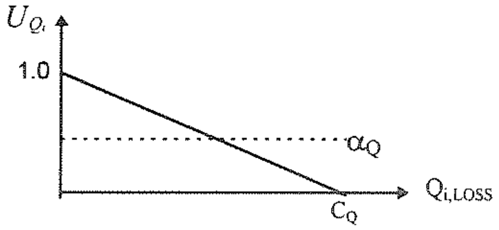

Figure 4 Membership function for U_{Q_i}

$$U_{Q_i} = -\frac{1}{C_Q} Q_{i,Loss(i,i+1)} + 1, \quad i = 1,2,\ldots,n;\ Q_{i,Loss} \le C_Q$$

$$U_{Q_i} = 0 \qquad , i = 1,2,\ldots,n;\ C_Q < Q_{i,Loss} \tag{12}$$

where C_Q is a constant, and α_Q is the level for the *α-cut* operation of the fuzzy set.

A sensitivity analysis is used to select the candidate locations for placing the capacitors which have the greatest effect in reducing the peak power loss. Sensitive nodes are evaluated for the feeder every time a change(e.g. load variation) occurs in the feeder. To find the sensitive nodes by employing the fuzzy set technique, we first calculate the membership values by equations (10), (11) and (12) and specify the α-*cut* levels for this study. If a bus having any membership value of the voltage deviation, real power loss or reactive power loss which is lower than their respective α-*cut* level α_V, α_P or α_Q value, then the bus was selected as sensitive node. Repeat the process until all sensitive nodes are found.

Genetic algorithms determine the optimal size, number and type of capacitors. To apply genetic algorithms, we generate an initial population composed of *k* chromosomes. Each chromosome represents a possible solution, and has a fitness function or objective function. There are L^J total possible solutions, where J is the bus number of the sensitive nodes and L is the number of available capacitor sizes. L^J will become a very large number even for a medium-size distribution system. Accordingly, it will be a heavy computational burden to achieve the optimal solution. In this study, we encode the bus number and the number of available capacitor sizes to binary representation for the previous sensitive nodes selection and GAs operations. Sub- sequently, perform line flow study and cost function evaluation to find the best fitness value in a generation.

We suggest an improvement elitism principle which may be stated below. Let the 50% of chromosomes of the next generation reproduced from the chromosomes having least total cost or total real power losses, and the other 50% of chromosomes obtained by random method, and perform the crossover and mutation operations for all chromosomes. Regarding the crossover operation, if the random number generated for the crossover point is odd , we swapping the left bits of this crossover point; on the contrary, if the random number is even, swapping the right bits of this crossover point. Regarding the mutation operation, the mutation rate is gradually increased during the operation. The purpose of the above operation scheme is to help prevent the solution to be stuck to a single local solution. Each bus voltage is checked to see if it is kept within its upper and lower limits. The overall annual cost is computed and compared with the previous solution, The better solution is stored and the others are discarded.

The main computational procedures of the proposed method may be stated using a flowchart shown in Figure 5.

7. NUMERICAL EXAMPLE AND COMPUTATIONAL RESULTS

In the following illustrative example, we consider a physically existing 23kV nine-section feeder, which is given in Figure 6 "it was shown by Grainger and Lee [1982]".

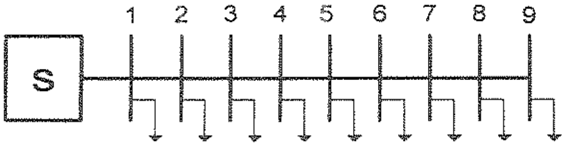

Figure 6 A 9-section feeder

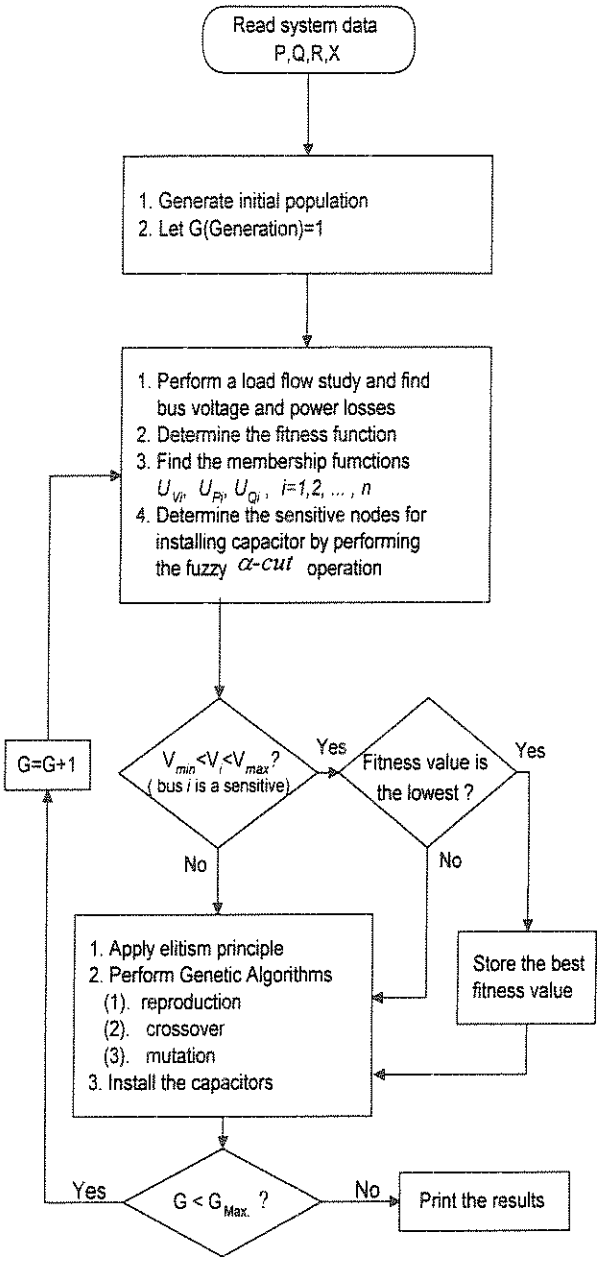

Figure 5 Flowchart for the proposed method

Loads data for the feeder is given in Table 1 and three load cases are assumed. Table 2 shows impedances for the 9 sections of the feeder. The equivalent annual cost per unit of power loss, C_L , was selected to be 168 $/(kW-Year).

The bus voltage limits were selected as follows:

$V_{min}=0.90p.u.$ *and* $V_{max}=1.10p.u.$

For reactive power compensation, the maximum capacitor size $Q^c_{\max}$ should not exceed 4186 Kvar of the reactive load. The 27 possible capacitor sizes along with their corresponding annual costs are given in Table 3.

Other data required are α_V=0.50, α_P=0.45, α_Q=0.48, population size=20 and maximum generation(G_{max}) =2000. The placement, maintenance, and running costs are neglected.

Table 1 Three-phase load data

Bus	Case 1		Case 2		Case 3	
No.	P_L(KW)	Q_L(Kvar)	P_L(KW)	Q_L(Kvar)	P_L(KW)	Q_L(Kvar)
1	1840	460	1050	250	1200	490
2	980	340	950	280	1350	560
3	1790	446	1250	100	1660	910
4	1598	1840	1085	1150	985	1100
5	1610	600	1250	380	1100	470
6	780	110	750	50	760	58
7	1150	60	750	40	1330	340
8	980	130	1080	170	1180	200
9	1640	200	1600	180	1750	260

Table 2 Impedance data at 60 Hz for the feeder

From Bus i	To Bus $i+1$	$R_{i,i+1}$ (Ω)	$X_{i,i+1}$ (Ω)
0	1	0.1233	0.4127
1	2	0.0140	0.6051
2	3	0.7463	1.2050
3	4	0.6984	0.6084
4	5	1.9831	1.7276
5	6	0.9053	0.7886
6	7	2.0552	1.1640
7	8	4.7953	2.7160
8	9	5.3434	3.0264

Table 3 Possible choice of capacitor sizes and cost

j	1	2	3	4	5	6	7
Q_j^c (kvar)	150	300	450	600	750	900	1050
\$/(kvar-year)	.500	.350	.253	.220	.276	.183	.228
j	8	9	10	11	12	13	13
Q_j^c (kvar)	1200	1350	1500	1650	1800	1950	2100
\$/(kvar-year)	.170	.207	.201	.193	.187	.211	.176
j	15	16	17	18	19	20	21
Q_j^c (kvar)	2250	2400	2550	2700	2850	3000	3150
\$/(kvar-year)	.197	.170	.189	.187	.183	.180	.195
j	22	23	24	25	26	27	
Q_j^c (kvar)	3300	3450	3600	3750	3900	4050	
\$/(kvar-year)	.174	.188	.170	.183	.182	.179	

Three cases are investigated for different load data, which are stated in the following. In case 1, buses 2,3,4,5 and 9 were selected for capacitors siting. Figure 7 shows the variation of total cost versus generations, the curve converges rapidly to achieve a global optimal or near optimal solution.

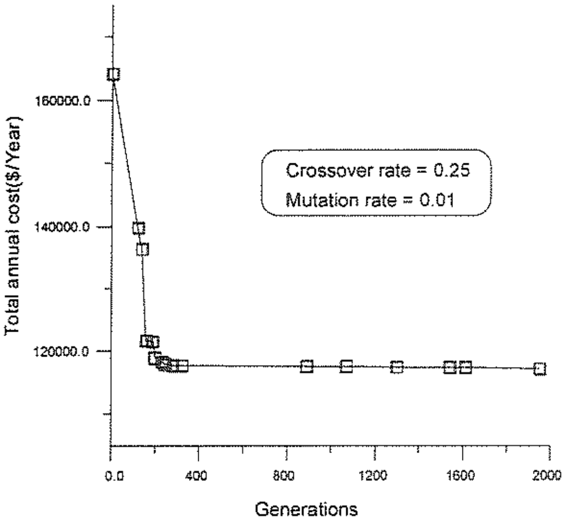

Figure 7 Variation of total cost versus generations computed

Table 4 gives the voltage profile before and after compensation and the reactive power compensation required. It can be seen that no bus voltage is below 0.9 p.u. after compensation. Table 5 shows the power losses and total annual cost before and after compensation.

Table 4 The voltage profile and capacitors placed

Bus No.	Voltage before compensation (p.u.)	Voltage after compensation (p.u.)	Reactive power compensation (kvar)
1	0.9929	1.0009	0.0
2	0.9874	1.0071	4050.0
3	0.9634	0.9978	2100.0
4	0.9480	0.9877	1800.0
5	0.9172	0.9658	1650.0
6	0.9072	0.9572	0.0
7	0.8890	0.9413	0.0
8	0.8587	0.9161	0.0
9	0.8375	0.9000	600.0

Table 5 The total annual cost and power losses

Case 1	Before compensation	After compensation
Power losses (kW)	783.777	686.870
Total annual cost(\$/Year)	131,675	117,276

In case 2, the buses 3,5 and 9 were selected for capacitors siting, and in case 3 buses 2,3,5,7,8 and 9 were selected. Table 6 gives the voltage profile after compensation and reactive power compensation required. Table 7 shows the power losses and total annual cost before and after compensation. For all the three cases, it is observable that the voltage profile is enhanced, and power losses are also reduced by applying the proposed method.

Table 6 The voltage profile after compensation and reactive power compensation required

Bus	Case 2		Case 3	
No.	Q_C(Kvar)	Voltage(p.u.)	Q_C(Kvar)	Voltage(p.u.)
1	0.0	0.9986	0.0	1.0016
2	0.0	1.0002	4050.0	1.0085
3	2700.0	0.9931	4050.0	1.0022
4	0.0	0.9834	0.0	0.9921
5	1200.0	0.9639	1200.0	0.9712
6	0.0	0.9562	0.0	0.9628
7	0.0	0.9418	900.0	0.9462
8	0.0	0.9159	150.0	0.9179
9	600.0	0.9005	600.0	0.9002

Table 7 The total annual cost and power losses

Case 2	Before compensation	After compensation
Power losses (kW)	575.963	533.079
Total annual cost ($/Year)	96,762	90,398

Case 3	Before compensation	After compensation
Power losses (kW)	810.327	710.058
Total annual cost ($/Year)	136,135	121,315

To implement the reactive power compensation, two types of capacitors are usually considered "it was shown by Bala et al. [1997]", they are fixed and switched capacitor banks. Fixed capacitors are operating on the feeder all the time, while switched capacitors are turned "on" or "off" depending on the load level. We may schedule the type of capacitors to be added as shown in Table 8.

Table 8 Schedule of fixed and switched capacitors (S = Switched and F = Fixed)

Bus No.	Case 1 (Kvar)	Case 2 (Kvar)	Case 3 (Kvar)
1	0	0	0
2	4050 (S)	0	4050 (S)
3	2100 (F)	2100(F)+600(S)	2100(F)+1950(S)
4	1800 (S)	0	0
5	1200(F)+450(S)	1200 (F)	1200 (F)
6	0	0	0
7	0	0	900 (S)
8	0	0	150 (S)
9	600 (F)	600 (F)	600 (F)

8. CONCLUSIONS

A new optimal capacitor allocation method which uses Fuzzy Reasoning and Genetic Algorithm to enhance voltage profile and reduce power losses for the primary distribution system is presented. Fuzzy Reasoning finds the sensitive nodes that are used as candidate locations for placing the capacitors. On the other hand, Genetic Algorithm resolves the optimization problem, which is capable of determining the near global optimal compensations with a reasonable computational burden. The method presented has been implemented in a computer program that was applied to a feeder with three cases of load. From the computational results, we observe that the proposed method is effective and flexible in enhancing voltage profile and lowering power losses for a primary distribution system.

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