Evaluation method using chaotic analysis for road transportation system simulator

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Abstract : We have already proposed a simulator of road transportation system, and called it MITRAM. The MITRAM consists of microscopic models for vehicle which have capability of own decision-making through the application of fuzzy logic. This microscopic model is called Fuzzy Model Vehicle (FMV). We have simulated the driving operation of a following vehicle through the FMV, assuming only a leading and a following vehicle existed. In this case, the fuzzy logic inference determined the following vehicle acceleration. Data of the following vehicle speed, the relative speed of the following to the leading vehicle and so on, were obtained as the simulated data of the FMV. To calculate only the mean and the variance of the simulated data was not enough way to evaluate the logic inferences of the FMV. In this paper, we proposed an evaluating method by using a chaotic analysis. In order to compare the simulated data with the measured data of an actual vehicle, correlation exponents and Lyapunov exponents were calculated for the data of the following vehicle speed, the relative speed of the following to the leading vehicle, and the spacing distance between the following and the leading vehicles. We also calculated both the exponents of the data normalized by the standard deviation in order to evaluate the effect of the variance. The correlation exponents and the maximum Lyapunov exponents of the relative speed were different from those of the following vehicle speed and the spacing distance. Differences between the simulation and the measurement data were found at both the correlation exponents and the Lyapunov exponents. The variance had a larger effect on the correlation exponents for the simulation data than for the measurement data.

1. INTRODUCTION

We have already proposed a MITRAM [Satoh et al., 1992]. The MITRAM is a simulator of road transportation system for analyzing traffic jam. The MITRAM consists of microscopic models for vehicle which have capability of own decision-making through the application of fuzzy logic [Honda et al., 1991]. This model is called Fuzzy Model Vehicle (FMV). The FMV is constructed with multistage binomial fuzzy logic inference [Itakura et al., 1992; Yikai et al., 1992; 1993]. Many membership functions of the fuzzy logic are automatically determined by actual data with using neural network [Itakura et al., 1993a; 1993b].
We have simulated driving operations of a following vehicle through the FMV, assuming only two vehicles (a leading and a following vehicles) existed. In this case, the fuzzy logic inference determined the following vehicle acceleration. Data of the following vehicle speed, the relative speed of the following to the leading vehicle and so on, were obtained as the simulated data of the FMV. We figured the results of simulated data to evaluate the simulation models. The results figured in time series expression caused insufficiency of information about constructing and correcting the models. On the other hand, the results figured in distribution charts caused disregard of time dependency of the data. To calculate only the mean and the variance of the simulated data was not enough way to evaluate the FMV [Itakura et al., 1994; 1995].
In this paper, we proposed an evaluating method by using a chaotic analysis on the basis of embedding theorem [Takens, 1981], instead of traditional statistical evaluation. In order to compare the simulated data with the measured data of an actual vehicle, correlation exponents and Lyapunov exponents were calculated for the data of the following vehicle speed (Vf), the relative speed of the following to the leading vehicle ($Vl\text{-}f$), and the spacing distance between the following and the leading vehicles ($Dl\text{-}f$). We also calculated both the exponents of the data normalized by the standard deviation in order to evaluate the effect of the variance on the exponents. The FMV evaluated in this paper was a model in which driving operations could be changed according to various conditions. It was shown by Itakura et al. [1996]. The following sections give full details of the FMV.

2. FUZZY MODEL VEHICLE (FMV)

2.1 Concept of the FMV

We have indicated that various realistic traffic conditions could not be exactly simulated by using a macroscopic model which used statistical probability [Satoh et al., 1992]. In order to realize realistic traffic conditions on a simulator, it is necessary to simulate behavior of individual vehicle with using a microscopic model. Differential equations using in the microscopic simulator [Leutzbach, 1972] is not necessarily suitable to instinctively understand driving operations. On the other hand, fuzzy logic is suitable to understand and construct the logic for some decision of human behavior such as driver's decision-making.
Drivers control the movement of a vehicle through the operations of an accelerator, a brake and a steering wheel according to various control factors. The processing algorithms for the wheel operation is generally very complicated. Therefore, we incorporated the Clothoid curve, which is recently used for the road design, into the MITRAM in order to describe the movement of vehicles. It

means that the movement of vehicles can be almost expressed one-dimensionally, that is, a vehicle on a road can be treated as something like a train running on a rail, and a trace line of the vehicle can be described by joint of straight lines, Clothoid curves, and arcs of a circle. Consequently, the operation of the wheel was not necessary. The operations we simulated on the FMV were almost the operations of the accelerator and the brake.

The FMV could take into consideration of information that is necessary for drivers' decision-making. We, however, must solve the problem how to construct the fuzzy logic of the FMV. If it were constructed with polynomial fuzzy inference consisting of one stage, required logic number would be usually "y" to the power of "x", where "x" equals to the number of input parameters and "y" equals to the number of linguistic truth-values of input parameters. Considering information that is necessary for drivers' decision-making, great number of input parameters are necessary so that the required logic number would be numerous. It is thought that most of the required logic are not always necessary because input parameters which reflect the main part of drivers' decision-making are alternated by the moment and the state. Therefore, it is useful to use a method which has not general inquiry but selective inquiry of input parameters. So we have proposed to construct the FMV consisting of a multistage binomial fuzzy logic that was easy to understand.

2.2 Concrete Model for the FMV

We must identify various fuzzy logic parameters such as membership functions to concrete the FMV consisting of a multistage binomial fuzzy logic. Therefore, we proposed the binomial fuzzy logic constructed by a neural network, shown in Fig.1. When two non-fuzzy values are inputted to the binomial fuzzy logic, a non-fuzzy value can be obtained as its output. The fuzzy membership function could be automatically predetermined with using a back propagation method.

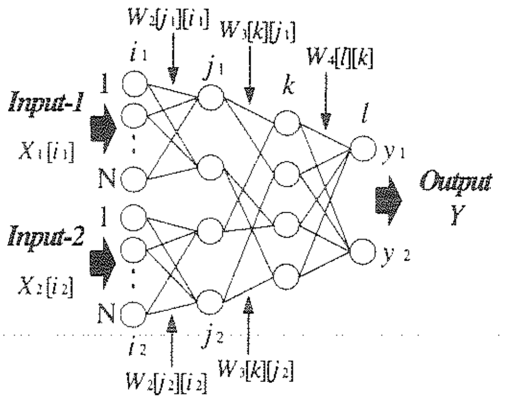

Fig. 1 Neural network for binomial fuzzy logic.

The neural network shown in Fig.1 is correspondence to the binomial fuzzy logic with two membership functions by a input. Data was put into the first layer by digitizing to N steps between the minimum and the maximum values. The second layer is correspondence to the linguistic truth-value. The synapse weights ($W_2[j_1][i_1]$, $W_2[j_2][i_2]$) between the first and the second layers are correspondence to the values of the membership function of the input data. The output of the third layer is compatible degree of the premise (IF-part), and the synapse weights ($W_4[l][k]$) between the third and the fourth layers are parameters of the consequence (THEN-part). The output (Y) is calculated as the center of gravity of the fourth layer's output. The output of second layer, $O_2[j_1]$ and $O_2[j_2]$, are given by the following equations:

$$O_2[j_n] = \sum_{i_n=1}^{N} X_n[i_n]\cdot W_2[j_n][i_n]\text{, for } n=1,2 \tag{1}$$

The output of the fourth layer (y_l) is given as follows:

$$y_l = O_4[l] = \sum_k O_3[k]\cdot W_4[l][k], \tag{2}$$

where

$$O_3[k] = O_2[j_1]\cdot W_3[k][j_1] + O_2[j_2]\cdot W_3[k][j_2]$$
$$k = (j_1 - 1)\times 2 + j_2. \tag{3}$$

When the input data are $X_1[i_1]$ and $X_2[i_2]$, 1 is given to only the i_1 th and the i_2 th units of the first layer, and 0 is given to all other units. As a results, $O_2[j_1]$ and $O_2[j_2]$ are equal to $W_2[j_1][i_1]$ and $W_2[j_2][i_2]$ respectively. Then, the output Y is given as follows:

$$Y = \frac{y_1\cdot(\min) + y_2\cdot(\max)}{y_1 + y_2}, \tag{4}$$

where (min) is the minimum value and (max) is the maximum value in the output data we measured.

In order to determine the values of the membership function, training data, which were obtained through the measurement under actual traffic conditions, were given to the input and the output of the neural network. The values of synapse weights corresponding to the membership functions could be automatically obtained with using a back propagation method.

We have simulated one pair of a leading and a following vehicles by using the neural network shown in Fig.1[Itakura et al. 1994]. ***Input-1*** was the relative speed of the following to the leading vehicle ($V_{l\text{-}f}$), ***Input-2*** was the spacing distance between the following and the leading vehicles ($D_{l\text{-}f}$), and the output was the following vehicle acceleration. However, actual operations of a driver could not be completely modeled on this model. It was thought that a driver could change own operations according to judgment whether the leading vehicle acceleration was positive or negative. In order to give any function of changing the driving operation according to various conditions (the leading vehicle acceleration, etc.,), it is necessary to give selective training method depending on conditions to the FMV. Consequently, we proposed the model shown in Fig.2.

This model consists of four neural networks (NN1, NN2, NN3, and NN4) shown in Fig.1. ***Input-1*** was $V_{l\text{-}f}$, ***Input-2*** was D_{l-f}, ***Condition-1*** was A_l, ***Condition-2*** was D_{l-f}, and ***Output*** Y_m was A_f ,where A_l and A_f are a leading and a following vehicle acceleration respectively. Membership functions ($W11$, $W12$, $W21$, and $W22$) are used to unify Y_1, Y_2, Y_3, and Y_4. The output (Y_m) is given as follows:

$$Y_{12} = W11 \times Y_1 + W12 \times Y_2, \quad Y_{34} = W11 \times Y_3 + W12 \times Y_4,$$
$$Y_m = W21 \times Y_{12} + W22 \times Y_{34}. \tag{5}$$

When the value of ***condition-1*** is small, $W11$ is the larger and $W12$ is the smaller so that the influence of the Y_1 on the Y_{12} is stronger than that of the Y_2. On the other hand, when the value of ***condition-1*** is large, $W12$ is the larger and $W11$ is the smaller so that the influence of the Y_2 on the Y_{12} is stronger than that of the Y_1. The error at the Y_{12} is distributed to NN1 and NN2 at the ratio of $W11$: $W12$ during training. As a result, the logic in the case where the value of the ***condition-1*** is small can be built into the NN1, and the logic in the case where the value of the ***condition-1*** is large can be built into the NN2. We thought that the simulation model shown in Fig.2 was a model in which driving operations could be changed according to ***condition-1*** and ***condition-2***.

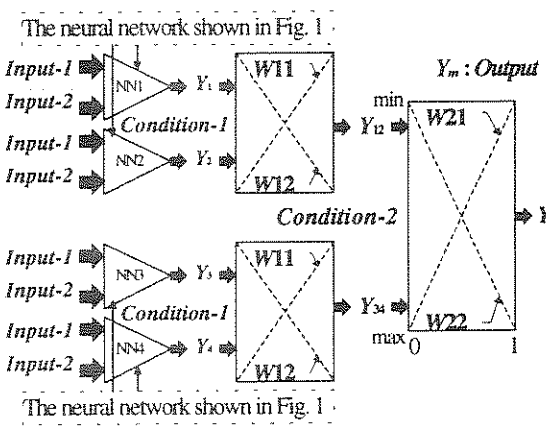

Fig. 2 Model for FMV.

3. CHAOTIC ANALYSIS

3.1 Correlation Exponent

At first, we created a set of d-dimensional vector out of an actual time series measured at the discrete time interval Δt. An embedding dimension d was chosen and a d-dimensional orbit was reconstructed by use of delay coordinates: $X(i)=\{x(i), x(i+t_d), \cdots, x(i+(d-1)t_d)\}$ for $i=1,2,\cdots,N_D-t_d+1$, where t_d is the delay time, N_D is the total number of the data. The dynamic information in the one-dimensional data has been converted to spatial information in the d-dimensional vector set.

A correlation exponent is defined on the basis of a long-time d-dimensional orbit $X(i)$ by considering the correlation integral

$$C(r) = \frac{1}{{N_V}^2} \sum_{i,j(i \neq j)} H(r - |X(i) - X(j)|), \tag{6}$$

where H is the Heaviside function and N_V is the total number of the d-dimensional vector set. This correlation integral counts the number of pairs whose distance $|X(i) - X(j)|$ is smaller than r. When the $C(r)$ behaves as a power of r for small r: $C(r) \propto r^{\nu}$, the exponent ν is defined as the correlation exponent. The correlation exponent is usually determined by a constant slope of the plot of $\log C(r)$ as a function of $\log r$. These algorithm was shown by Grassberger et al. [1983] in detail.

3.2 Lyapunov Exponent

We chose an embedding dimension d and construct a d-dimensional orbit $X(j)$. We have to find any set of point $X(k_i)$ (i=1,2,$\cdots$, N_P) which is the neighbors of $X(j)$, that is, the points Xk_i of the orbit which are contained in a ball of suitable radius r centered at $X(j)$. The displacement vector $y(i)$ between $X(k_i)$ and $X(j)$ as follows:

$$y(i)=X(k_i)-X(j) \quad (|X(k_i)-X(j)|< r). \tag{7}$$

After the evolution of a time interval $\tau=m\Delta t$, the orbital point will proceed to $X(j+m)$ and the point $X(k_i)$ of the neighbors of $X(j)$ to $X(k_i+m)$. The displacement vector $y(i)$ is thereby mapped to

$$z(i)=X(k_i+m)-X(j+m). \tag{8}$$

If the radius r is small enough for the displacement vector $y(i)$ and $z(i)$ to be regarded as good approximation of tangent vectors in the tangent space, evolution of $y(i)$ to $z(i)$ can be represented by some matrix $A(j)$, as $z(i)=A(j)y(i)$. The least-square-error algorithm was used for the optimal estimation of the $A(j)$ from the data sets $\{y(i)\}$ and $\{z(i)\}$. The Lyapunov exponents can be computed as

$$\lambda_i = \lim_{M \to \infty} \left(\frac{1}{M\tau} \right) \sum_{j=1}^{M} \ln |A(j) e_i(j)|, \tag{9}$$

for i=1,2,$\cdots$, d, where $\{e_i(j)\}$ is a set of basis vector of the tangent space at $X(j)$. The maximum λ_i for i=1,2,$\cdots$, d is the maximum Lyapunov exponent. In the numerical procedure, we choose an arbitrary set $e_i(j)$ and operate with the matrix $A(j)$ on $e_i(j)$, and renormalized $A(j)e_i(j)$ to have length one. Using the Gram-Schmidt procedure, maintain mutual orthgonality of the basis. We repeat this procedure for M iterations and compute (9). These algorithm was shown by Sano et al. [1985] and Eckmann et al. [1986].

4. RESULTS AND DISCUSSIONS

We drove two vehicle at roads in a city area. The speed of the vehicles and the spacing distance between the vehicles were measured at discrete intervals of 1 second only when there were no other vehicles between our vehicles. Data sets for analysis were chosen except on the condition both the leading and the following vehicles did not move at the same time. The total number of the data sets was 4252. Acceleration of the following vehicle was calculated by using the proposed models shown in Fig.2 with a back propagation method, and one pair of the leading and the following vehicles was simulated to obtain simulation data.

We show the mean and the standard deviation of the measurement and the simulation data in Table 1. The measurement and the simulation data of the V_f is almost the same in the mean and the standard deviations, although those of the $D_{l\text{-}f}$ and the $V_{l\text{-}f}$ is not.

Table 1 Mean and standard deviation.

	Measurement		Simulation	
	Mean	S.D.	Mean	S.D.
V_f [m/s]	7.4476	4.2435	7.5200	4.4288
$D_{l\text{-}f}$ [m]	14.561	9.7105	29.683	19.752
$V_{l\text{-}f}$ [m/s]	0.0161	1.1990	-0.0813	1.8652

(S.D. : Standard deviation)

We limit the embedding dimension d between 2 and 10. In order to evaluate the influence of the delay time t_d, we vary the t_d between 1 and 10. In order to get a calculation result of Lyapunov exponents, it is necessary that there are many neighbors of $X(j)$ greater than the embedding dimension. The number of the neighbors was determined by a radius r shown in (7). However, we could not always set a suitable radius r so that we could not get all the calculation results of the maximum Lyapunov exponent.

4.1 Correlation Exponents for Data

Fig.3 shows the correlation exponents for the measurement data at the delay time t_d=3 and t_d=6. The relative speed of the following to the leading vehicle ($V_{l\text{-}f}$) shows a relatively flat section in the region above the embedding dimension d=9. Note that the correlation exponents of the $V_{l\text{-}f}$ is larger than those of the following vehicle speed (V_f) and the spacing distance between the following and the leading vehicles ($D_{l\text{-}f}$) in the region above d=3. The exponents of the V_f and the $D_{l\text{-}f}$ show nearly the same value in all the region. The V_f points at t_d=6 are smaller than the V_f points at t_d=3 whereas the $D_{l\text{-}f}$ points at t_d=6 are larger than the $D_{l\text{-}f}$ points at t_d=3. The influence of the delay time on the $V_{l\text{-}f}$ is smaller than that on the V_f or the $D_{l\text{-}f}$.

Fig.4 shows the correlation exponents for the simulation data. The V_f points at t_d=3 are smaller than the V_f points at t_d=6 in Fig.4. Conversely in Fig.3, the V_f points at t_d=6 were smaller than the V_f points at t_d=3. The differences between t_d=3 and t_d=6 for the $V_{l\text{-}f}$ of the simulation data are slightly larger than those of the measurement data. Note that All the correlation exponents in the region above d=3 are small as compared with Fig.3. It was thought that the degrees of freedom of the dynamic system constructed in the simulation model was smaller than that of a actual driving operation.

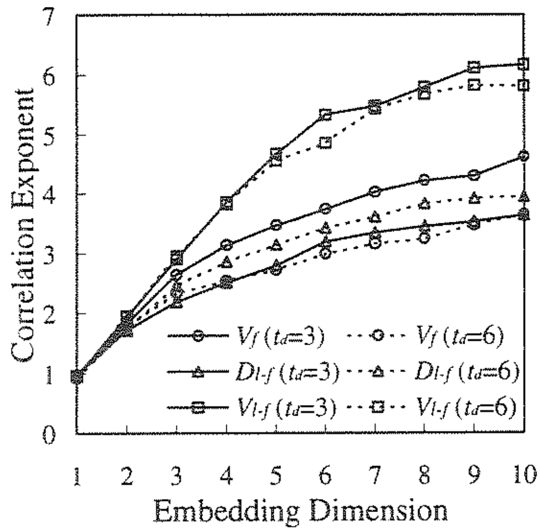

Fig. 3 Correlation exponents for measurement data

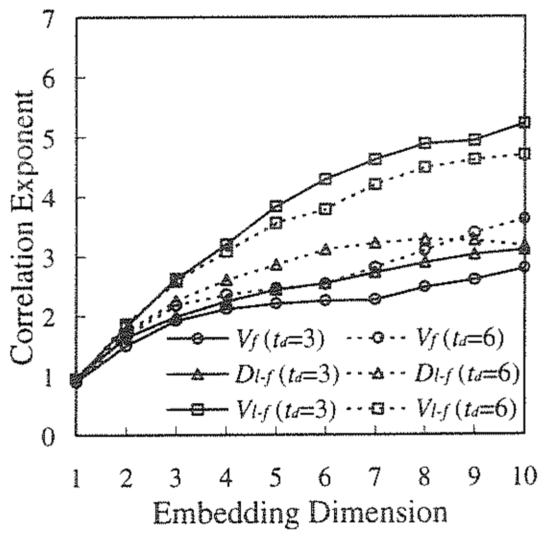

Fig. 4 Correlation exponents for simulation data.

4.2 Lyapunov Exponents for Data

Fig.5 shows the maximum Lyapunov exponents for the measurement data at the delay time t_d=3 and t_d=6. The features of the V_f, the $D_{l\text{-}f}$ and the $V_{l\text{-}f}$ curves are almost the same in all the region except for the $V_{l\text{-}f}$ point at t_d=3 in d=2, the $V_{l\text{-}f}$ points at t_d=6 in d=2, 3, and the $D_{l\text{-}f}$ point at t_d=6 in d=8.

Fig.6 shows the maximum Lyapunov exponents for the simulation data. The features of the $D_{l\text{-}f}$ and the $V_{l\text{-}f}$ curves in Fig.6 are different from those in Fig.5. The V_f points at t_d=6 in the region above d=8 are also different. We could not take a suitable radius r. Therefore, we could not calculate many maximum Lyapunov exponents enough for comparison between the measurement and the simulation data.

Correlation exponents and Lyapunov exponents of data are independent of the mean of data because displacement of the mean corresponds to parallel displacement in the embedding dimensional space. However, both the exponents are dependent on the variance. In order to seek availability to use both the exponents as independent criteria of the variance for evaluation of the simulation data, we normalized data $x(i)$ as follows:

$$x_n(i)=\{x(i)-x_m\}\ /\ x_s\ , \tag{10}$$

where $x_n(i)$ is the normalized data, x_m and x_s are the mean and the standard deviation of the data $x(i)$ respectively. We examined the relation between the variance and both the exponents.

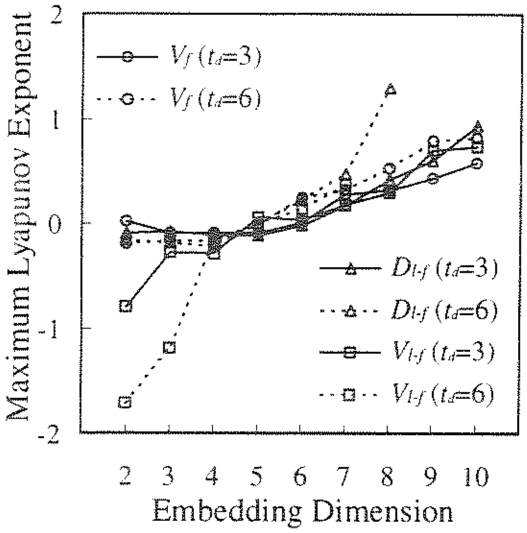

Fig.5 Maximum Lyapunov exponents for measurement data.

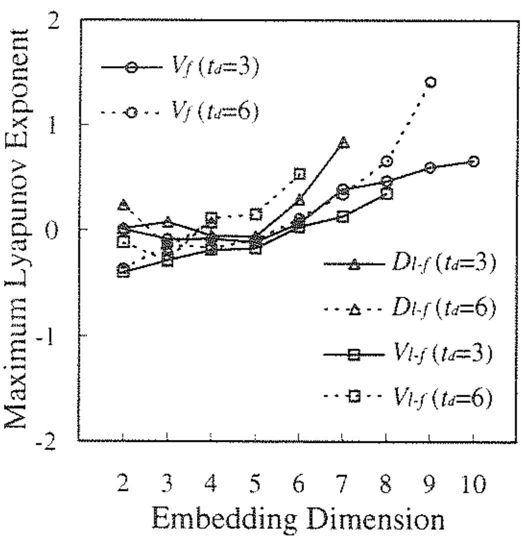

Fig.6 Maximum Lyapunov exponents for simulation data.

4.3 Correlation Exponents for Normalized Data

Fig.7 shows the correlation exponents for the normalized measurement data at the delay time t_d=3 and t_d=6. The features of the curves in Fig.7 hardly change from those in Fig.3. It was thought that the effect of the variance on the correlation exponents was negligible for the measurement data. Note that the influences of the delay time on the V_f, the $D_{l\text{-}f}$, and the $V_{l\text{-}f}$ are small as compared with Fig.3.

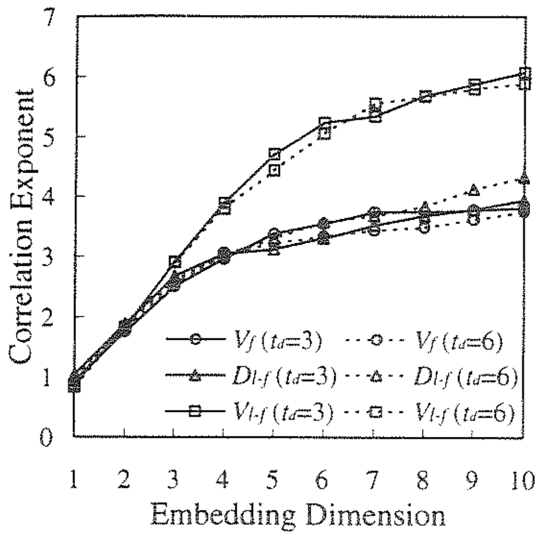

Fig.7 Correlation exponents for normalized measurement data.

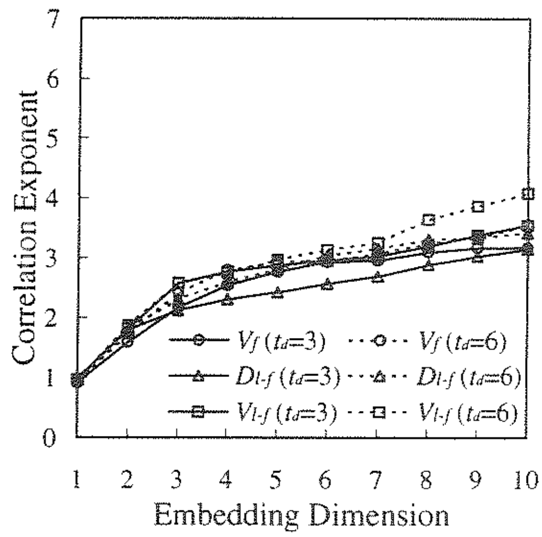

Fig.8 Correlation exponents for normalized simulation data.

Fig.8 shows the correlation exponents for the normalized simulation data. Note that the correlation exponents of the $V_{l\text{-}f}$ in the region above d=3 are smaller than those in Fig.4. It was not observed in the measurement data. The effect of the variance on the correlation exponents of the $V_{l\text{-}f}$ was noticeably large for the simulation data. We proved that the effect of the variance on the correlation exponents was not always small and was dependent on the other factors of data. We thought that the useful suggestions to construct actual driving operations into simulation models might be included in the relation between the variance and the correlation exponents.

4.4 Lyapunov Exponents for Normalized Data

Fig.9 and Fig.10 show the maximum Lyapunov exponents for the normalized measurement data and the normalized simulation data respectively, at the delay time t_d=3 and t_d=6. In Fig.9, the feature of the $V_{l\text{-}f}$ curve in the region below d=5 is different from the V_f and the $D_{l\text{-}f}$ curves. In all the region of Fig.10, the features of the $D_{l\text{-}f}$ and the $V_{l\text{-}f}$ curves are different from those in Fig.6. We proved that the effect of the variance on the maximum Lyapunov exponents was not always constant and dependent on the data.

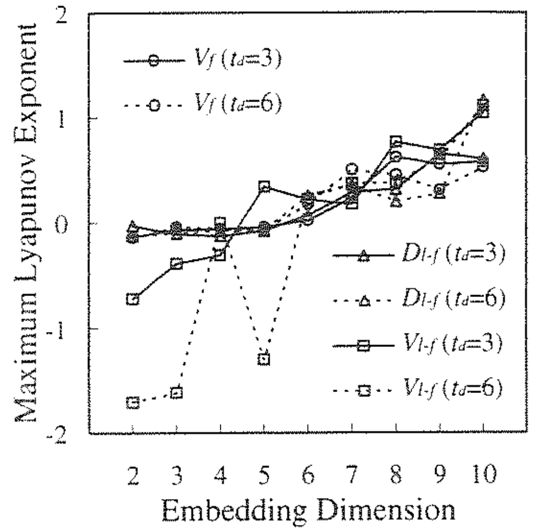

Fig. 9 Maximum Lyapunov exponents for normalized measurement data.

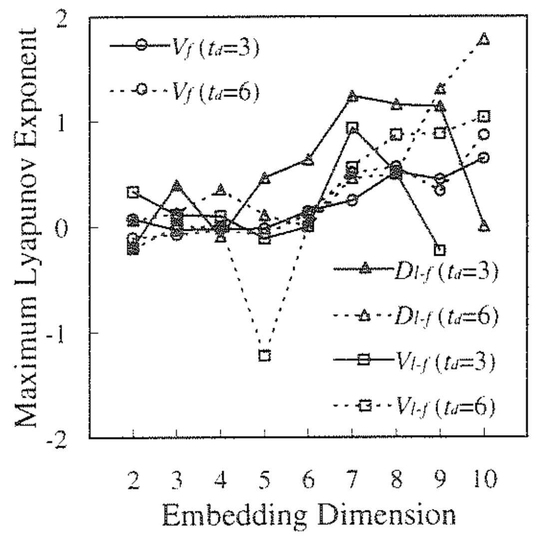

Fig. 10 Maximum Lyapunov exponents for normalized simulation data.

5. CONCLUSIONS

Correlation exponents and Lyapunov exponents were calculated for data of a following vehicle speed (V_f), a relative speed of a following to a leading vehicle ($V_{l\text{-}f}$), and a spacing distance between a following and a leading vehicles ($D_{l\text{-}f}$). We also calculated both the exponents of the data normalized by the standard deviation in order to evaluate the effect of the variance. Simulation data were obtained from a model in which driving operation could be changed according to various conditions. The results are summarized as follows: 1) For the measurement data, the correlation exponents of the $V_{l\text{-}f}$ is larger than those of the V_f and the $D_{l\text{-}f}$ in the region above d=3. 2) All the correlation exponents of the simulation data are smaller than those of the measurement data. 3) The effects of the variance on the correlation exponents and the Lyapunov exponents are dependent on the other factors of data.

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