# A Study of Three-stage Flowshop Scheduling Problem with Alternative Operation Assignments

Yoshihiro FUTATSUISHI Ichie WATANABE Toshio NAKANISHI

Seikei University

**Abstract** The paper describes a study on a problem as follows. We assume to have n processed items which are processed by three machines and either of n processed items is composed of four operations. The three of the four operations have fixed assignments to certain machines. The rest can be processed with either of any machines. If for all the processed items the assignment of fourth operation is determined, the problem becomes three-stage flow shop problem. Now we discuss a problem how to minimize the makespan between the start and completion of processed item under the above-stated conditions. Thus a schedule is determined according to the machine assignment to an elementary operation and the operations sequencing in each machine. Therefore the optimal solution in one problem has to be searched among the $3^n \times n!$ schedules. In general the more processed items we have, the exponentially more the number schedules to be searched will be. We analyzed theoretically a problem where all the processed items are of processed items of the same property ( this is considered to be the simplest condition ) and clarified the characteristics of the optimal schedule. Then we proposed approximation algorithm based on theoretical analysis applicable to more generalized problem. We executed numerical simulation to have comparison with the conventional numerical method. The paper shows the approximate method according to our proposal is of better performance.

## 1. INTRODUCTION

In recent production field in Japan, the curtailment of operations employees, the reduction of investment in plant and equipment, and the shortening of operations hours are showing rapid progress. These trends are spurring the generalization of processing machines, the flexible applicability of operators and the free machine assignment to elementary processes.

The study aims at discussing the problem where the assignment of processes to machines are flexible and the operation time is different from an assignment to another. We are to obtain the optimal schedule which gives the minimum required time in the case where the above-stated operations are processed in the flowshop lines. For example, in the basic board assembling line of multi-item small lot production. Elementary parts like IC and condensers are fixed to the board. There is a problem how to minimize the portal-to-portal (binding) hours of operators under the conditions that the main units are fixed in the specific process and other units can be fixed in any process and finally to have optimal daily production schedule.

For the similar problem in terms of 2 processes, Z.Nakamura and I.Watanabe[1] proposed an optimal solution based on Branch and bound method. As for the 3 processes, I.Watanabe and T.Nakanishi[2] proposed 8 kind of approximation solution. This is an application of the method proposed by Z.Nakamura and I.Watanabe[1]. Y.Futatsuishi and I.Watanabe[3][4] analyzed the simplest problem where the processed items are of the same kind and the processing times of elementary operation the assigned machine of which are free are the same for any machine, and

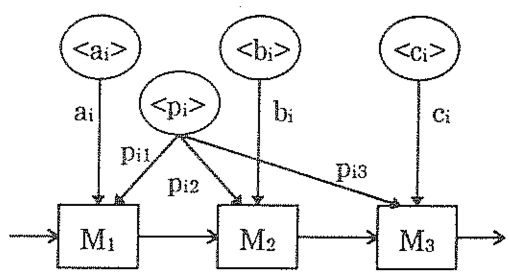


Fig.1 Model deals with in the paper

clarified the several properties of optimal solution. They proposed 3 kinds of approximation solution methods and evaluated them.

Our study supplemented the analysis in the study [3] and[4], and proposed 2 kinds of approximation solutions. In addition, we have made comparison between two methods proposed here and before, and added evaluation to them.

## 2. FORMULATION OF THE PROBLEM

Presuppositions and the signs used in the model (ref. fig.1) of this study shows as follows.

① The n processed items are given and each processed item goes through machine $M_1$, $M_2$ and $M_3$ in this order.

② Each processed item i(i =1,2,⋯,n) has four production elements $<a_i>$, $<b_i>$, $<c_i>$ and $<p_i>$. $<a_i>$, $<b_i>$ and $<c_i>$ are processed by $M_1$, $M_2$ and $M_3$ respectively, and $<p_i>$ can be processed with any of the machine $M_1$, $M_2$ or $M_3$. Processed item i is called Ⅰ-type job when $<p_i>$ is assigned to $M_1$, Ⅱ-type job when $<p_i>$ is assigned to $M_2$, and Ⅲ-type job when $<p_i>$ is assigned to $M_3$.

③ Processing time of production elements $<a_i>$, $<b_i>$, $<c_i>$ and $<p_i>$ for each processed item i is given by $a_i$, $b_i$, $c_i$ and $p_{ij}$ respectively, where $p_{ij}$ designates the processing time of $<p_i>$ on machine $M_j$ (j=1,2,3).

④ Only a job can be in process on one operation at a time. Once an operation starts on a machine, another operation has to wait until the preceding operation is over.

⑤ Each machine can handle at most one operation at a time.

⑥ Each processed item is available at time zero.

Subject to these conditions, the problem is to find an optimal schedule, i.e., to determine the job type of each processed item and the processing sequence of all the processed items which minimize total elapsed time.

## 3. THEORETICAL ANALYSIS UNDER THE SIMPLEST CONDITION

We analyzed theoretically a problem where all the jobs are of jobs of the same property and clarified the characteristics of an optimal schedule. Here we add the following two conditions ⑦ and ⑧ to the conditions ① through ⑥ in section 2.

⑦ The processing time with $<a_i>$, $<b_i>$, $<c_i>$ for every i are a, b, c respectively and definite.

⑧ The processing time of production element $<p_i>$ even if it was processed with any of the machines $M_1$, $M_2$ and $M_3$, is p and definite. We clarified the characteristics of the conditions which restrict the range of the existence of optimal schedule[3]. They are as follows.

We reorganized the properties based on the properties found in the studies [3], [4] and this study. This study shows the properties that lacked in the studies [3] and [4]. Properties 1, 2, 6 and 10 were shown the study [3] and [4], the others were proposed in this study. We regard schedule S as S=(1,2,⋯,n) or S=($\alpha,\beta,\gamma$). Here $\alpha,\beta$ and $\gamma$ are the subsequences of processed items. We define r as follows.

$$r = \max\{a,b,c\}-\min\{a,b,c\}.$$

We state the properties as follows.

Property 1. There exists an optimal schedule among the schedules where 1-st job is Ⅲ-type job and n-st job is Ⅰ-type job.

i) An optimal schedule under the condition $r \geqq p$

Property 2. If $r \geqq p$, then there exists an optimal solution where job 1 is Ⅲ-type job, the other jobs are satisfied with the following conditions.

1) when min{a,b,c}=a and the job is Ⅰ-type job.
2) when min{a,b,c}=b and the job is Ⅱ-type job.
3) when min{a,b,c}=c and the job is Ⅲ-type job.

ii) An optimal schedule under the condition $r < p$

We proceed with the discussion separately with the case when max{a,b,c} is equal to a , the case when max{a,b,c} is equal to b and the case when max{a,b,c} is equal to c.

Property 3. If max{a,b,c}=a and $b \geqq c$ under the condition $r < p$, there exists an optimal schedule among the schedules where S=($\alpha$, $\beta$). Here $\alpha$ is composed of Ⅲ-type jobs or Ⅱ-type jobs and $\beta$ is composed of only Ⅰ-type jobs.

Property 4. If $\max\{a,b,c\}=a$, $b<c$ and $(a-b)+(a-c) \geq p$ under the condition $r<p$, there exist an optimal schedule among the schedules where $S=(\alpha)$. Here $\alpha$ is composed of Ⅲ-type jobs or Ⅱ-type jobs.

Property 5. If $\max\{a,b,c\}=a$, $b<c$ and $(a-b)+(a-c) < p$ under the condition $r<p$, there exist an optimal schedule among the schedules where $S=(\alpha, \beta)$. Here $\alpha$ is composed of Ⅲ-type jobs or Ⅱ-type jobs and $\beta$ is composed of Ⅱ-type jobs or Ⅰ-type jobs.

Property 6. If $\max\{a,b,c\}=b$ under the condition $r<p$, there exist an optimal schedule among the schedules where $S=(\alpha, \beta, \gamma)$. Here $\alpha$ is composed of only Ⅲ-type jobs and $\beta$ is composed of only Ⅱ-type jobs and $\gamma$ is composed of only Ⅰ-type jobs.

Property 7. If $\max\{a,b,c\}=c$ and $a\geq b$ under the condition $r<p$, there exist an optimal schedule among the schedules where $S=(\alpha, \beta)$. Here $\alpha$ is composed of only Ⅲ-type jobs and $\beta$ is composed of Ⅱ-type jobs or Ⅰ-type jobs.

Property 8. If $\max\{a,b,c\}=c$, $a<b$ and $(c-b)+(c-a) \geq p$ under the condition $r<p$, there exist an optimal schedule among the schedules where $S=(\alpha)$. Here $\alpha$ is composed of Ⅱ-type jobs or Ⅰ-type jobs.

Property 9. If $\max\{a,b,c\}=c$, $a<b$ and $(c-b)+(c-a) < p$ under the condition $r<p$, there exist an optimal schedule among the schedules where $S=(\alpha, \beta)$. Here $\alpha$ is composed of Ⅲ-type jobs or Ⅱ-type jobs and $\beta$ is composed of Ⅱ-type jobs or Ⅰ-type jobs.

Property 10. If $r<p$, there exists an optimal schedule among the combination of $n_1$ Ⅰ-type jobs, $n_2$ Ⅱ-type jobs and $n_3$ Ⅲ-type jobs which satisfy the following condition 1.

Condition 1. $\max\{T_1(S),T_2(S),T_3(S)\} - \min\{T_1(S),T_2(S),T_3(S)\} \leq p$.

Here $T_1(S)$, $T_2(S)$ and $T_3(S)$ are defined as follows.

$$T_1(S)=n\times a+b+c+n_1\times p,$$
$$T_2(S)=a+n\times b+c+n_2\times p, \qquad (1)$$
$$T_3(S)=a+b+n\times c+n_3\times p.$$

## 4. APPROXIMATION METHOD APPLIED TO MORE GENERALIZED PROBLEM

In generalized problem, a schedule is defined by how to assign the fourth operation to machine and how to sequence each job in machine processing. Therefore the optimal solution in one problem has to be searched among the $3^n\times n!$ schedules. In general the more jobs we have, the exponentially more the number of schedules to be searched will be.

Here six approximation methods are taken into consideration. (ref. Fig.2) The approximation method no.1 is the best method among those proposed by I.Watanabe[2]. The approximation method no.2, no3. no4 is the method proposed by Y.Futatsuishi and I.Watanabe[3][4]. The approximation method no.5 and no6 proposed here. Approximation method are as follows.

The method no.1: No.1 decide the type of job firstly and then decide the processing sequence is determined according to the NEH method[5] (one of the approximation methods to minimize makespan for m-stage, n-type jobs flowshop scheduling problem). The determination of job type is carried out in parallel with the examination of the processing time rate of difference of processed elements in machines.

The other approximation methods proposed here decide the processing sequence firstly and then decide the type of job. The method of the processing sequence determination is as follows. First, we arranged in order of the processing time with $M_1$ and we arranged for the same with $M_3$ again. Next, the job of less processing time with $M_1$ is processed from the first and the job of less processing time with $M_3$ is processed from the last. The processing sequence is determined. The method of job type determination is different according to the approximation method.

The method no.2: No.2 assumes all the jobs to be processed are Ⅱ-type jobs and then they are changed to Ⅲ-type jobs sequentially from the first job to backward one and changed to Ⅰ-type job from the last job to forward one. As soon as the present solution value get worse than the preceding one, the repetition comes to end.

The method no.3: No.3 assumes all the jobs to be processed are of Ⅱ-type, and computes $T_1(S)$,

$T_3(S)$ according to the definition formula (1). If $T_1(S)$ is smaller than $T_3(S)$, the type of the last processed job is changed to Ⅰ-type and if the $T_3(S)$ is smaller than $T_1(S)$, the type of the first processed job is changed to Ⅲ-type. This procedure is repeated until the present solution value get worse than the total required time of preceding solution.

The method no.4: No.4 applies to both the no.2 and no.3 in parallel and adopts the better one.

The method no.5: No.5 assigns <$p_i$> initially to all machines, and calculates $T_1(S)$, $T_2(S)$ and $T_3(S)$ according to the definition formula (1). If $T_2(S)$ and $T_3(S)$ are larger than $T_1(S)$, the last processed item the type of which has not been decided is selected as Ⅰ-type job. If $T_1(S)$ and $T_2(S)$ are larger than $T_3(S)$, the first processed item the type of which has not been decided is set to Ⅲ-type job. If $T_1(S)$ and $T_3(S)$ are larger than $T_2(S)$, the processed item in between is chosen Ⅱ-type job. This procedure continued until all the job types of processed items have been determined.

The method no.6: No.6 doesn't assign <$p_i$> to any machine and calculates $T_1(S)$, $T_2(S)$ and $T_3(S)$ according to the definition formula(1). If $T_1(S)$ is the smallest, the last processed item the type of which is not decides is set to Ⅰ-type job. If $T_3(S)$ is the smallest, the first processed item the type of which is not decides is set to Ⅲ-type job. If $T_2(S)$ is the smallest, the schedule where the last processed item the type of which has not been decided is chosen Ⅱ-type job and the schedule where the first processed item the type of which has not been decided is chosen Ⅱ-type job are compared with regard to total required time, and the item with smaller total required time is taken up as Ⅱ-type job. This procedure is continued until all the job types of processed items have been determined.

The relation between approximation methods and properties proposed in this study is as follows. The method no. 5 is in terms of the problem which satisfies properties 1,2,6,10 and the method no.6 is in terms of the problem which satisfies properties 3,4,5,7,8,9,10.

## 5. NUMERICAL SIMULATION AND ITS RESULTS

### 5.1 Numerical Simulation

(i) EXPERIMENTAL PURPOSE:

Assuming several patterns of processing time for processed items, the result of the approximation methods proposed in this paper is compared with

| Method Number | First Iteration | The order of Job Selection | The Criterion of Job Selection |
|---|---|---|---|
| 1 | Ⅱ-type job for all items | ① Ⅱ-type job→ Ⅰ-type job<br>② Ⅱ-type job→ Ⅲ-type job | ① smallest{pi1/pi2}<br>② smallest{pi3/pi2} |
| 2 | Ⅱ-type job for all items | ① Ⅱ-type job→ Ⅲ-type job<br>② Ⅱ-type job→ Ⅰ-type job | ① from the first job to backward one<br>② from the last job to forward one |
| 3 | Ⅱ-type job for all items | ① Ⅱ-type job→ Ⅲ-type job<br>② Ⅱ-type job→ Ⅰ-type job | ① in case T3(S)<min{T1(S),T2(S)}, from the first job to backward one<br>② in case T1(S)<min{T2(S),T3(S)}, from the first job to backward one |
| 4 | Ⅱ-type job for all items | ① Ⅱ-type job→ Ⅰ-type job<br>② Ⅱ-type job→ Ⅲ-type job | adopts the better method among no.2 and no.3 |
| 5 | assigns <pi> initially to all machines | ① T1(S)<min{T2(S),T3(S)}<br>② T3(S)<min{T1(S),T2(S)}<br>③ T2(S)<min{T1(S),T3(S)} | ① the last processed item the type of which has not been decided is selected as Ⅰ-type job<br>② the first processed item the type of which has not been decided is set to Ⅲ-type job<br>③ the processed item in between is chosen Ⅱ-type job |
| 6 | doesn't assign <pi> to any machine | ① T1(S)<min{T2(S),T3(S)}<br>② T3(S)<min{T1(S),T2(S)}<br>③ T2(S)<min{T1(S),T3(S)} | ① the last processed item the type of which is not decides is set to Ⅰ-type job<br>② the first processed item the type of which is not decides is set to Ⅲ-type job<br>③ the last processed item the type of which has not been decided is chosen Ⅱ-type job |

Fig.2 Approximation methods

those of the conventional approximation methods and the best approximation method is decided.

(ii) INSTITUTION OF PROBLEM

The number of items:10.

Here e and g mean as follows.

e: Uniformly distributed integral random number between 20 and 49.

g: Uniformly distributed integral random number between 95 and 124.

As for the processing time, 8 patterns ([eee], [gee], etc.) were set up for each (a, b, c) and ($p_{i1}$, $p_{i2}$, $p_{i3}$). Thus the total patterns of processing time reached 64(=8×8).

10 problems were made with each pattern and processing time, thus the total number of numerical simulation reached 640.

(iii) EVALUATION VALUE

The evaluation was made through the mean value of the difference of the total required time between the theoretically best solutions and numerical solutions. The best solution is obtained from examining makespans of jobs of all the types. With regard to each case, the processing sequence was determined according to the NEH method.

### 5.2 Computation Results

Computation results are shown in Fig.3. The left most column shows 8 patterns of processing time for $a_i$, $b_i$ and $c_i$. The first row shows the 8 patterns of processing time for $p_{i1}$, $p_{i2}$ and $p_{i3}$. The second column from the left shows approximation methods. The evaluation values are shown at the crossing of ($a_i$, $b_i$ and $c_i$) and ($p_{i1}$, $p_{i2}$ and $p_{i3}$). Total time is the sum of each row or each column. The right most column and the last row show the method number that has the smallest total time.

This result shows that the method no.4 can have better result on the average than the result shows in figure 3. But the method of this study can have better result according to problem pattern. For example, when $a_i$, $b_i$ and $c_i$ are of [eee] pattern and $p_{i1}$, $p_{i2}$ and $p_{i3}$ are of [gge]pattern, the evaluation value of the method no.5 shows the smallest. And when $a_i$, $b_i$ and $c_i$ are of [gge] or [egg] pattern, the evaluation value of the method no.6 shows the smallest.

## 6. CONCLUSIONS AND FUTURE PROBLEMS

If $a_i+p_{i1}$ is almost equal to $c_i+p_{i3}$, the method no.2 is better, since he number of Ⅰ-type job and Ⅲ-type job are the same for the method no.2. In other cases the method no.3 is better , since the method no.3 computes $T_1(S)$, $T_2(S)$, $T_3(S)$ and determines type of job. When $a_i$, $b_i$ and $c_i$ are of [gge] or [egg] pattern, the method no.6 of this study gave good result. This is because the problem of [gge] and [egg] pattern conforms to the problem to which the method no.6 satisfies properties 3～5 and 7～9 is applied. As a result, in the problem of one pattern have some properties the approximation method in terms of the problem which satisfies the properties gave better result than the other methods.

We plan to analyze the problem which has more processed items and more machines.

## 7. References


[1]Z. Nakamura and I. Watanabe, "An Optimal Sequencing Problem for a Two-Stage Flow shop with Alternative Job Assignments," Journal of the Operations Research Society of Japan, Vol. 31, No. 1, 1988. pp. 1-18.

[2]Ichie Watanabe, Toshio Nakanishi, "A Study of Numerical Simulation for the Analysis of Three-Stage Flowshop Scheduling Problem." , Summer Computer Simulation Conference Proceedings, pp. 571-575, 1992

[3]Futatsuishi, Y., Watanabe, I., "A Study of Multistage Flow Shop Scheduling Problem with Alternative Job Assignments", Journal of Japan Industrial Management Association

[4]Y. Futatsuishi, I. Watanabe and T. Nakanishi, "The Numerical Simulation for Three-stage Flowshop Scheduling Problem with Alternative Operation Assignments," International Congress on Modeling and Simulation, Vol. 4, pp355-358, 1995

[5]Nawaz, M. E., Enscore Jr., E. E. and Ham, I., "A Heuristic Algorithm for the m-Machine, n-Job Flow-shop Sequencing Problem" , OMEGA The International Journal of Management Science, vol, 11, No. 1, 1983

| Production time of ai, bi and ci | Method Number | Production time of pi1, pi2 and pi3: eee | gee | ege | eeg | gge | geg | egg | ggg | Total Point | Better Methods |
|---|---|---|---|---|---|---|---|---|---|---|---|
| eee | 1 | 42 | 42 | 63 | 54 | 130 | 131 | 139 | 152 | 753 | |
| | 2 | 26 | 68 | 46 | 73 | 122 | 15 | 168 | 44 | 562 | |
| | 3 | 33 | 37 | 110 | 26 | 75 | 17 | 47 | 37 | 382 | |
| | 4 | 24 | 30 | 44 | 26 | 75 | 15 | 45 | 29 | 288 | 4 |
| | 5 | 20 | 89 | 70 | 53 | 53 | 65 | 31 | 30 | 411 | |
| | 6 | 36 | 50 | 83 | 44 | 66 | 35 | 55 | 44 | 413 | |
| gee | 1 | 0 | 0 | 6 | 1 | 0 | 0 | 98 | 44 | 149 | |
| | 2 | 3 | 9 | 45 | 3 | 153 | 1 | 54 | 68 | 336 | |
| | 3 | 9 | 11 | 14 | 8 | 23 | 11 | 88 | 24 | 188 | |
| | 4 | 3 | 9 | 14 | 3 | 23 | 1 | 50 | 19 | 122 | 4 |
| | 5 | 47 | 60 | 38 | 48 | 21 | 30 | 71 | 121 | 436 | |
| | 6 | 10 | 23 | 35 | 24 | 17 | 19 | 35 | 22 | 185 | |
| ege | 1 | 13 | 28 | 9 | 27 | 30 | 112 | 28 | 23 | 270 | |
| | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| | 3 | 0 | 0 | 12 | 0 | 11 | 0 | 0 | 0 | 23 | |
| | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| | 5 | 44 | 29 | 4 | 33 | 0 | 71 | 28 | 19 | 228 | |
| | 6 | 11 | 42 | 23 | 45 | 21 | 55 | 35 | 30 | 262 | |
| eeg | 1 | 1 | 2 | 2 | 0 | 80 | 0 | 0 | 24 | 109 | 1 |
| | 2 | 7 | 7 | 71 | 11 | 53 | 9 | 192 | 86 | 436 | |
| | 3 | 14 | 11 | 13 | 11 | 138 | 9 | 7 | 65 | 268 | |
| | 4 | 6 | 7 | 13 | 11 | 53 | 9 | 7 | 50 | 156 | |
| | 5 | 7 | 20 | 32 | 14 | 64 | 30 | 31 | 49 | 247 | |
| | 6 | 5 | 31 | 23 | 21 | 22 | 11 | 22 | 32 | 167 | |
| gge | 1 | 8 | 3 | 6 | 93 | 0 | 86 | 101 | 249 | 546 | |
| | 2 | 78 | 123 | 109 | 46 | 292 | 86 | 56 | 172 | 962 | |
| | 3 | 7 | 7 | 12 | 26 | 12 | 27 | 130 | 33 | 254 | |
| | 4 | 7 | 7 | 12 | 24 | 12 | 27 | 54 | 33 | 176 | |
| | 5 | 57 | 20 | 62 | 56 | 16 | 131 | 94 | 158 | 594 | |
| | 6 | 15 | 12 | 11 | 13 | 10 | 22 | 33 | 30 | 146 | 6 |
| geg | 1 | 1 | 0 | 55 | 0 | 41 | 0 | 44 | 189 | 330 | |
| | 2 | 8 | 5 | 37 | 9 | 77 | 11 | 81 | 44 | 272 | |
| | 3 | 9 | 9 | 84 | 9 | 70 | 11 | 76 | 58 | 326 | |
| | 4 | 8 | 5 | 37 | 9 | 58 | 11 | 64 | 44 | 236 | 4 |
| | 5 | 41 | 39 | 46 | 67 | 85 | 30 | 66 | 120 | 494 | |
| | 6 | 21 | 10 | 51 | 33 | 66 | 41 | 58 | 72 | 352 | |
| egg | 1 | 9 | 74 | 9 | 6 | 84 | 73 | 0 | 246 | 501 | |
| | 2 | 94 | 66 | 138 | 157 | 81 | 105 | 345 | 208 | 1194 | |
| | 3 | 17 | 67 | 19 | 9 | 153 | 31 | 10 | 71 | 377 | |
| | 4 | 17 | 57 | 19 | 9 | 81 | 29 | 10 | 71 | 293 | |
| | 5 | 49 | 61 | 61 | 27 | 80 | 85 | 24 | 154 | 541 | |
| | 6 | 21 | 23 | 8 | 15 | 36 | 22 | 15 | 43 | 183 | 6 |
| ggg | 1 | 44 | 43 | 67 | 55 | 144 | 142 | 134 | 114 | 743 | |
| | 2 | 34 | 63 | 28 | 76 | 111 | 27 | 188 | 45 | 572 | |
| | 3 | 38 | 26 | 111 | 34 | 69 | 22 | 42 | 42 | 384 | |
| | 4 | 31 | 26 | 28 | 34 | 58 | 21 | 42 | 32 | 272 | 4 |
| | 5 | 36 | 86 | 76 | 61 | 99 | 115 | 89 | 43 | 605 | |
| | 6 | 45 | 30 | 72 | 45 | 84 | 64 | 58 | 77 | 475 | |
| Total Point | 1 | 118 | 192 | 217 | 236 | 509 | 544 | 544 | 1041 | 3401 | |
| | 2 | 250 | 341 | 474 | 375 | 889 | 254 | 1084 | 667 | 4334 | |
| | 3 | 127 | 168 | 375 | 123 | 551 | 128 | 400 | 330 | 2202 | |
| | 4 | 96 | 141 | 167 | 116 | 360 | 113 | 272 | 278 | 1543 | 4 |
| | 5 | 301 | 404 | 389 | 359 | 418 | 557 | 434 | 694 | 3556 | |
| | 6 | 164 | 221 | 306 | 240 | 322 | 269 | 311 | 350 | 2183 | |
| Better Methods | | 4 | 4 | 4 | 4 | 6 | 4 | 4 | 4 | 4 | 4 |

Fig.3 The Scores of the Approximate methods for Given 10 Data