Constant Conditional Correlation In A Bivariate GARCH Model: Evidence from The Stock Markets Of China

Albert K. Tsui and Qiao Yu
Department of Economics and Statistics
National University of Singapore

Abstract In this paper we examine the constancy of conditional correlation in two emerging stock markets in China. These are the Shanghai and Shenzhen markets. We employ the bivariate GARCH model of Bollerslev (1990) to capture the co-movements of stock returns between the markets. However, the information matrix test statistic does not support the hypothesis of a constant conditional correlation.

1. Introduction

Modelling stock market volatilities by the generalized autoregressive conditional heteroscedasticity (GARCH) models has been an important research area in the last decade. Bera and Higgins (1993) provide an excellent survey on the application of the GARCH models to the studies of many financial assets. The general consensus is that financial asset return volatilities have a predictable component which is dependent on the past volatilities and return shocks. As economic variables are often inter-related, and affected by the same set of available information, it is natural to extend the univariate GARCH models to multivariate GARCH models. Such generalizations would inevitably increase the number of parameters to be estimated and complicate specifications of the conditional variance and covariance matrix. Details are discussed in Engle et al. (1995).

In an attempt to circumvent the tedious estimation and inference procedures, Bollerslev (1990) introduced a bivariate GARCH model assuming constancy of the conditional correlation. However, the validity of the constant correlation assumption remains an empirical question. More recently, Bera and Kim (1996) have derived a formal information matrix test for constancy of the conditional correlation, and applied to well-developed stock markets in USA, Japan, Germany, UK, France and Italy. They found strong rejection of constancy of conditional correlation in all cases. In this paper, we examine the constancy of conditional correlation in two emerging stock markets in China. They are the Shanghai and Shenzhen stock exchange markets. Given that these markets are relatively new and share similar set of information, it would be interesting to see how the stock returns react with each other across markets.

The remainder of this paper is as follows. In Section 2, we describe the methodology of the bivariate GARCH model. In Section 3, we discuss the data used in this study. Section 4 reports the estimation and inference results. Section 5 provides some concluding remarks.

2. The Bivariate GARCH Model

Since the seminal work of Engle (1982), the generalized autoregressive conditional heteroscedasticity (GARCH) class of models has been instrumental in modelling time-varying volatility in financial time series. Bollerslev (1990) extended the univariate GARCH model to bivariate GARCH models with time varying conditional variance and covariance, but constant conditional correlation. Such a structure significantly simplifies the estimation and inference procedures. In this paper, we adopt the bivariate GARCH(1,1) model proposed by Bollerslev (1990) with the following structure:

[1] *A simple mean-adjusted equation*

Denote S_{it} as the daily price index ($i = 1$ for the Shanghai market, and 2 for Shenzhen) and define $r_{it} = 100 \times ln(S_{it}/S_{it-1})$ as the daily return on a continously compounding basis. We assume that the mean equation of the return is captured by five dummy variables representing different expected returns for various days of the week. In addition, the residues follow an autoregressive process of order h. Thus, the mean-adjusted equation can be written as

$$r_{it} = \sum_{j=1}^{5} \mu_{ij} D_{tj} + u_{it} \qquad (2.1)$$

and

$$u_{it} = \sum_{k=1}^{h} \theta_{ik} u_{it-k} + \epsilon_{it}$$

where r_{it} is the return defined as the difference of the logarithmic prices, and D_{tj} for $j = 1, ..., 5$ are the dummy variables with value 1 if the return is on the jth day of the week and value 0 otherwise. Hence, μ_{ij} represents the expected return of the jth day of the weeek. Also, θ_{ik} are the respective autoregressive parameters.

[2] *Bivariate GARCH(1,1) Model*

We assume that under the null hypothesis the conditional correlation is constant over time so that all the variations over time in the conditional covariances are caused by changes in each of the corresponding two conditional variances. Let $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$, then

$$\epsilon_t | \Psi_{t-1} \sim N(0, H_t) \qquad (2.2)$$

where Ψ_{t-1} is the σ-algebra generated by all the available information up through time $t-1$, and

$$H_t = \begin{pmatrix} h_{1t} & h_{12t} \\ h_{21t} & h_{2t} \end{pmatrix} \qquad (2.3)$$

Note that the conditional covariance matrix, H_t, is almost surely positive definite for all t, and

$$h_{it} = \alpha_{i0} + \alpha_{i1} \epsilon_{it-1}^2 + \beta_i h_{it-1} \qquad (2.4)$$

and

$$h_{12t} = \rho \sqrt{h_{1t}} \sqrt{h_{2t}} \qquad (2.5)$$

The likelihood function of all unknown parameter ω is

$$l_t(\omega) = -\frac{1}{2} log|H_t| - \frac{1}{2} (\epsilon_{1t} \; \epsilon_{2t}) H_t^{-1} \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \qquad (2.6)$$

The maximum likelihood estimation method is used to estimate all parameters of the bivariate GARCH(1,1) model.

[3] *The test statistic*

To test the validity of the constant correlation assumption, Bollerslev (1990) applied the Ljund-Box portmanteau test on the cross products of standardized residues. However, serially uncorrelated cross product of residues does not necessary imply constancy of the conditional correlation over time. As such, we adopt the information matrix (IM) test formally derived by Bera and Kim (1996) for the constancy of ρ. The test statistic is

$$IM_\rho = \frac{\left[\sum_{t=1}^{N} (\hat{v}_{1t}^{\star 2} \hat{v}_{2t}^{\star 2} - 1 - 2\hat{\rho}^2\right]^2}{4N(1 + 4\hat{\rho}^2 + \hat{\rho}^4)}$$

where

$$\hat{v}_{1t}^\star = \frac{\hat{\epsilon}_{1t}^\star - \hat{\rho}\hat{\epsilon}_{2t}^\star}{\sqrt{1 - \hat{\rho}}}$$

$$\hat{v}_{2t}^\star = \frac{\hat{\epsilon}_{2t}^\star - \hat{\rho}\hat{\epsilon}_{1t}^\star}{\sqrt{1 - \hat{\rho}}}$$

$$\hat{\rho} = \sum_{t=1}^{N} \hat{\epsilon}_{1t}^{*}\hat{\epsilon}_{2t}^{*}/N$$

and

$$\hat{\epsilon}_{it}^{*} = \hat{\epsilon}_{it}/\sqrt{h_{it}} \qquad (2.7)$$

Under the null hypothesis of constant correlation, IM_ρ asymptotically follows a Chi-squared distribution with one degree of freedom.

3. The Data

We examine the conditional volatility of the Shanghai and Shenzhen stock markets of China. Shanghai is the most important industrial and financial centre in China, whereas Shenzhen is the pioneering market-oriented special economic zone in China. The Shanghai market commenced on 19 December 1990, and the Shenzhen market started trading on 3 April 1991. However, the Shanghai market set a daily price limit during the period of 19 December 1990 to 20 May 1992. Initially, an upper and lower bound of 5% change of the share price was set, with subsequent adjustments. The price restriction was finally lifted on 21 May 1992. As such, we disregard the observations within the period of price control. Our data set comprises 862 daily price indices of the Shanghai and Shenzhen stock exchanges from 21 May 1992 to 13 October 1995. All indices are price-weighted series of all listed stocks on the exchange. Figures 1 to 4 plot the S_t and r_t series for the two markets. Particular periods of tranquility and turbulence can be observed in these figures, indicating that the market volatility is varying over the period of study. Table 1 gives the summary statistics of the stock returns. It is noted that both the Shanghai and Shenzhen markets experience negative returns, with Shenzhen suffering a greater average loss of 0.097%. Based on the magnitude of the unconditional standard deviations, the Shanghai market is more volatile. In addition, all markets demonstrate very high kurtosis, and the kurtosis of the Shanghai market is more striking.

Figure 1: Shanghai Market Index

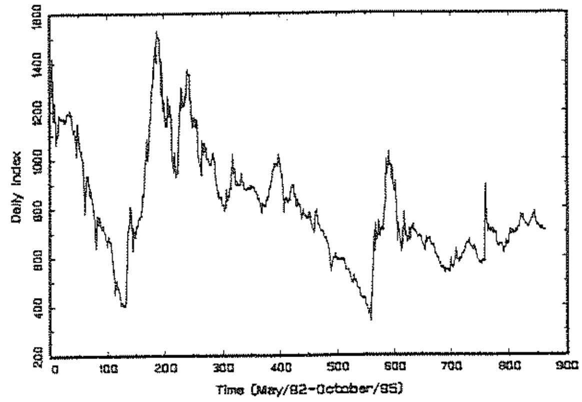

Figure 2: Shenzhen Market Index

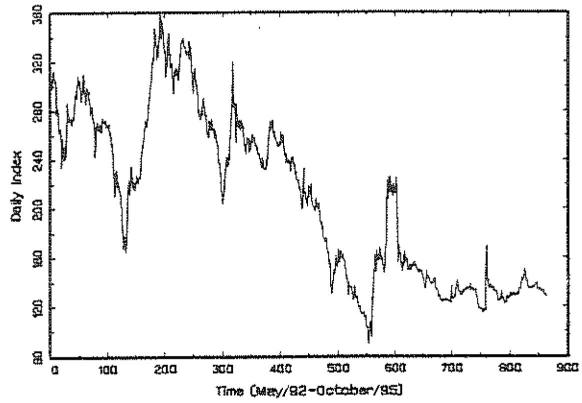

Figure 3: Shanghai Market Return

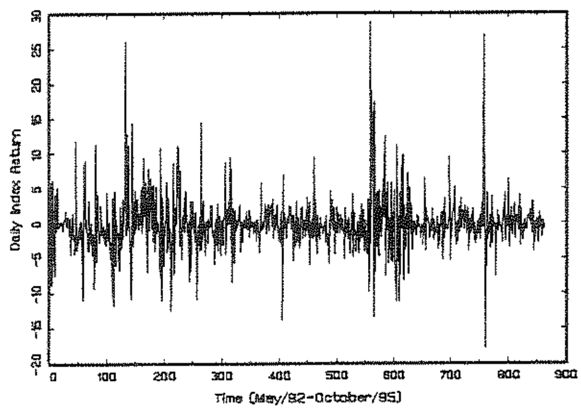

Figure 4: Shenzhen Market Return

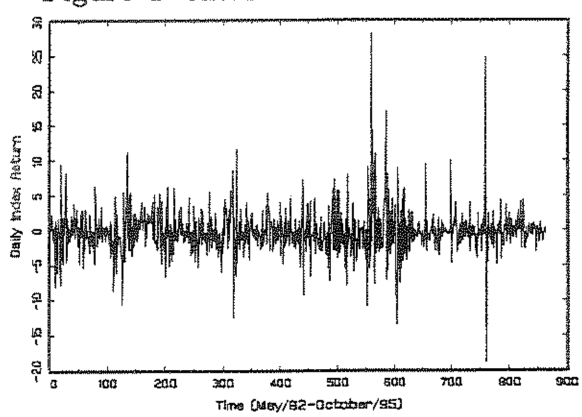

Table 1: Summary Statistics of r_{it}

	Shanghai	Shenzhen
Statistics		
Mean	-0.0683	-0.0970
S.D.	4.1128	3.3672
Minimum	-17.9051	-18.8828
Maximum	28.8618	28.2541
Skewness	373.6940	165.4875
Kurtosis	11.8308	14.9468
Runs Tests in		
r_t	2.1596	0.8435
$\|r_t\|$	-6.0175	-6.2411
r_t^2	-8.0849	-6.4727

Table 2: Mean-Adjusted Equations

	Shanghai	Shenzhen
Dummy Variables		
μ_1	-0.6650	-0.4639
	(0.3280)	(0.0866)
μ_2	-0.4460	-0.3661
	(0.3243)	(0.0865)
μ_3	0.1301	-0.0113
	(0.3369)	(0.0989)
μ_4	0.1843	-0.0049
	(0.3243)	(0.0810)
μ_5	0.5072	0.3325
	(0.3268)	(0.0865)
Autoregressive Structure		
θ_1	0.0310	0.0104
	(0.0341)	(0.0314)
θ_2	0.0507	0.0447
	(0.0340)	(0.0320)
θ_3	0.0358	-0.0501
	(0.0339)	(0.0320)
θ_4	0.0447	0.0575
	(0.0338)	(0.0319)
θ_5	0.0381	0.0359
	(0.0338)	(0.0319)
σ	16.3614	11.1772
	(0.7899)	(0.0902)

Note: The model parameters are estimated using maximum likelihood estimation method. The figures in the parentheses are the standard errors.

To examine the serial correlation in r_t, both in the mean and variance, we perform the nonparametric runs tests on r_t, $|r_t|$ and r_t^2, respectively. The results in Table 1 show that there is significant serial correlation in $|r_t|$ and r_t^2, but not in r_t. We mention in passing that the correlation-integral statistic proposed by Brock, Dechert and Scheinkman (1987) (denoted as BDS statistic) can be used to test whether a series is independently and identically distributed. If there is no deterministic chaotic structure in the data, the BDS statistic has an approximate normal distribution.

4. Estimation and Results

In Table 2 we summarise the estimation results of the mean-adjusted equations. Both the Shanghai and Shenzhen markets exhibit significant serial correlation in their returns. Apparently, the order of the serial correlation is higher for the Shenzhen market.

In addition, the day-of-the-week effect (from Monday to Friday) is represented by the estimated values of $\mu_1, ..., \mu_5$. Both markets show negative mean returns on Monday and Tuesday, and positive return on Friday. However, not all of them are significantly different from zero at the 5% level. In particular, both markets have high and significant negative mean returns on the first trading day of the week. This is consistent with the observed negative returns on Monday in well-developed stock markets of the US, Canada, Singapore, and Hong Kong. See Wong et al. (1992). Unlike the Shanghai market but like those in Japan and Australia, the Shenzhen market suffers from significant negative mean return on Tuesday. This matches with the observations reported by Jaffe et al. (1985). Moreover, the Shenzhen market also enjoys a high positive return on Friday (again like the US market). While acknowledging the fact that the day-of-the-week effect is rather unique to each stock market, the settlement procedure may explain part of the observed stock return pattern.

Table 3: Bivariate GARCH Model

Parameters	Estimates	Std. error
α_{10}	1.7974	0.2592
α_{11}	0.0969	0.0134
β_1	0.5811	0.0487
α_{20}	1.3442	0.1955
α_{21}	0.0856	0.0140
β_2	0.5853	0.0516
ρ	0.4753	0.0175

Table 4: Specification Tests

	Shanghai	Shenzhen
m_3	5.752	6.709
m_4	15.226	18.310
e_t	0.893	0.613
$\|e_t\|$	-1.932	-2.815
e_t^2	-1.659	-2.642
$Q^2(10)$	2.160	3.986
$Q^2(20)$	5.119	5.620
$Cross - Q(10)$	2.721	2.721
$Cross - Q(20)$	4.207	4.207
$Cross - e_t$	-3.037	-3.037
$Cross - \|e_t\|$	-2.906	-2.906
IM	612	612

Note: The model parameters are estimated using maximum likelihood estimation method.

The residual returns of the mean-adjusted equations are employed to estimate the bivariate GARCH model. The results are summarised in Table 3. As can be observed, all estimated coefficients for the variance equations are significant, thereby indicating the presence of GARCH effects. Table 4 provides various diagnostic checks, including the nonparametric runs tests, Ljund-Box (1978) portmanteau tests, and the information matrix test. The descriptive statistics m_3 and m_4 represent the usual sample estimates of skewness and kurtosis in the standardized residuals $\hat{\epsilon}_{it}/\sqrt{\hat{h}_{it}}$. Under the normality assumption, m_3 and m_4 should be 0 and 3 respectively. However, excessive kurtosis and skewness are still present in all the residual series. Such departures from normality are consistent with most of the empirical results on financial data.

In addition, e_t, $|e_t|$ and e_t^2 represent the runs tests for the serial correlation in $\hat{\epsilon}_{it}/\sqrt{\hat{h}_{it}}$, $|\hat{\epsilon}_{it}/\sqrt{\hat{h}_{it}}|$ and $\hat{\epsilon}_{it}^2/\hat{h}_{it}$, respectively. The evidence suggests a tremenduous reduction in the serial correlation due to the conditional variance. The lack of serial correlation in the standardized residuals is more obvious for the Shanghai stock market. Moreover, results of the runs tests are consistent with those of the Ljund-Box test results recorded in Table 4. Here, $Q^2(10)$ and $Q^2(20)$ represent the Ljund-Box tests for up to the 10^{th} and 20^{th} order serial correlation in the squared standardized residuals, $\hat{\epsilon}_{it}^2/\hat{h}_{it}$. They are all less than the 5% critical values for both the Shanghai and Shenzhen markets. As such, we may conclude that the serial correlation caused by the conditional variance has been basically removed.

Under the null hypothesis of constant conditional correlation, the cross products of the standardized residuals should be serially uncorrelated. This is supported by the Ljund-Box tests which do not reject the hypothesis of serial uncorrelation. However, both the runs test and the information test formally derived by Bera and Kim (1996) do not support the constant conditional correlation hypothesis.

5. Concluding Remarks

We have studied the behaviour of stock returns in the emerging Shanghai and Shenzhen stock exchange markets in China. We found that both markets suffer from negative mean returns on Monday and Tuesday, but enjoying a high positive return on Friday. This is consistent with most of the well-developed stock markets. In addition, we apply the information matrix test to detect the existence of constant conditional correlation in the two markets. However, our result does not support such a constancy.

References

Bera, A.N., and M.L. Higgins, ARCH models:properties, estimation and testing, *Journal of Economic Survey*, 7, 305-366, 1993.

Bera A.N., and S.W. Kim, Testing constancy of conditional correlation in bivariate GARCH model, *Working paper*, Department of Economics, University of Illinois, 1996.

Bollerslev, T., Modelling the coherence in short-term nominal exchange rates: A multivariate generalized ARCH approach, *Review of Economics and Statisticss*, 72, 498-505, 1990.

Brock, W., W. Dechert, and J. Scheinkman, A test for independence based on the correlation dimension, *Working paper*, University of Wisconsin at Madison, 1987.

Engle, R.F., Autoregressive conditional heteroskedasticity with estimates of variance of United Kingdom Inflation, *Econometrica*, 50, 987-1007, 1982.

Engle, R.F., and F.K. Kroner, Multivariate simultaneous generalized ARCH, *Econometric Theory*, 11, 122-150, 1995.

Jaffe, J. and R. Westerfield, The week-end effect in common stock returns: the international evidence, *Journal of Finance*, 40, 433-454, 1985.

Ljund, G.M., and G.E.P. Box, On a measure of lack of fit in time series models, *Biometrika*, 65, 67-72, 1978.

Wong, K.A., T.K. Hui, and Y.C. Choy, Day-of-the-week effects: evidence from developing stock markets, *Applied Financial Economics*, 2, 49-56, 1992.