LM TESTS FOR UNIT ROOTS IN THE PRESENCE OF MISSING OBSERVATIONS: SMALL SAMPLE EVIDENCE

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Abstract The purpose of this paper is to provide small sample evidence on the properties of Lagrange Multiplier (LM) tests for unit roots in time series models in the presence of missing observations. LM type tests for a unit root in a first order autoregressive process for two types of null and alternative hypotheses are considered: a unit root without drift vs level stationarity, and a unit root with drift vs trend stationarity. The small sample size and power properties of the tests are investigated using a Monte Carlo simulation.

1. INTRODUCTION

Testing for the presence/absence of unit roots has now become an integral part of empirical work using time series data and there is a wide variety of tests available for this purpose (see, for example, Campbell and Perron (1991)). One important limitation of these tests is that they all implicitly assume that there are no missing observations on the variable being examined. However, when the data being analyzed is collected on a weekly or daily basis, there is a high likelihood that missing observations will occur regularly or irregularly owing to National holidays like Christmas (see, for example, Lim and McKenzie (1992) using Australian data).

The Lagrange Multiplier (LM) testing principle is applied to the problem of testing for a unit root in the presence of missing observations in a first order autoregressive process because it only requires parameter estimates under the null hypotheses which are easily computed in this case. Moreover, despite the presence of missing observations, the test statistics have well–known asymptotic distributions which can easily be obtained from appropriately defined regression equations.

Section 2 presents LM tests for a unit root in a first order autoregressive process in the presence of missing observations and gives their asymptotic distributions. The small sample properties of the LM tests are investigated through a Monte Carlo simulation in section 3. Section 4 contains some concluding remarks.

2. LM TESTS FOR UNIT ROOTS

Following Schmidt and Phillips (1992), let y(t) be a univariate time series with an assumed data generating process:

$$y(t) = \alpha + \beta t + x(t),\ t = 0,..,T, \tag{1}$$

where α and β are unknown constants, and x(t) is a first order autoregressive process:

$$x(t) = \rho x(t-1) + \epsilon(t),\ \epsilon(t) \sim iid(0, \sigma^2),\ t=1,..,T. \tag{2}$$

The process (2) is initialized at t=1 with the initial value x(0). In this formulation, the null and alternative hypotheses are H_0: $\rho=1$ and H_1: $\rho<1$. In this paper, two types of null and alternative hypotheses, are considered, namely,

Case 1 H_0^1: y(t) has a unit root without drift vs H_1^1: y(t) is stationary; and

Case 2 H_0^2: y(t) has a unit root with drift vs H_1^2: y(t) is trend stationary.

In Case 1, β in (1) is set equal to zero.

Suppose that y(t) is observed only at t(0), t(1),..,t(K), where K is the number of data points for the dependent variable that are observed, and $\{t(k)\}_{k=0}^{K}$ is a subsequence of $\{t\}_{t=0}^{T}$. The case of no missing observations corresponds to $\Delta t(k) \equiv t(k)-t(k-1) = 1$ for all k.

Toda and McKenzie (1994) show that a one sided LM test for H_0: $\rho=1$ in Case 2 can be obtained as the usual t–statistic of φ, τ_2, in the regression

$$\Delta S(t) = \varphi S(t-1) + \text{error}, \tag{3}$$

where S(t) is defined as

$$S(t) = \begin{cases} y(t(k)) - y(0) - \tilde{\beta}t(k) & \text{if } t=t(k), \\ y(t(k\text{-}1)) - y(0) - \tilde{\beta}t(k\text{-}1) & \text{if } t=t(k\text{-}1)+1,..,t(k)-1, \end{cases} \tag{4}$$

and $\tilde{\beta} = T^{-1}(y(t(K)) - y(t(0))$, the restricted maximum likelihood estimate of β. The regression analogy also suggests the use of a 'coefficient test' in (3), namely, the use of the following statistic $\rho_2 = T\hat{\varphi}$, where $\hat{\varphi}$ is the OLS estimate of φ in (3). In addition, following Schmidt and Phillips (1992) demeaned versions of τ_2 and ρ_2, $\underline{\tau}_2$ and $\underline{\rho}_2$, can be defined as the t– and coefficient statistics of φ from the regression

$$\Delta S(t) = \text{constant} + \varphi S(t-1) + \text{error}. \tag{5}$$

If there are no missing observations so that $\Delta t(k)=1$ for all k and K=T, then τ_2, ρ_2, $\underline{\tau}_2$ and $\underline{\rho}_2$ reduce to Schmidt and Phillips' (1992) $\bar{\tau}$, $\bar{\rho}$, $\tilde{\tau}$ and $\tilde{\rho}$, respectively.

If a bounding assumption is placed on $\Delta t(k)$, then Toda and McKenzie (1994) show that:

(a) $\tau_2 \Rightarrow (-1/2)(\int_0^1 V(r)^2 dr)^{-1/2}$; (b) $\rho_2 \Rightarrow (-1/2)(\int_0^1 V(r)^2 dr)^{-1}$;

(c) $\underline{\tau}_2 \Rightarrow (-1/2)(\int_0^1 \underline{V}(r)^2 dr)^{-1/2}$; (d) $\underline{\rho}_2 \Rightarrow (-1/2)(\int_0^1 \underline{V}(r)^2 dr)^{-1}$,

where V(r) is a standard Brownian bridge on [0,1], that is, $V(r) = W(r)-rW(1)$ with W(r) denoting a standard Brownian motion on [0,1], $\underline{V}(r)$ is a demeaned standard Brownian bridge, that is, $\underline{V}(r) = V(r) - \int_0^1 V(r)dr$, and "=>" denotes weak convergence. These limit distributions are special cases of those of $\bar{\tau}$, $\bar{\rho}$, $\tilde{\tau}$ and $\tilde{\rho}$, respectively, given in Schmidt and Lee (1991) and Schmidt and Phillips (1992). The critical values for the asymptotic distributions of τ_2 and ρ_2 are found in Schmidt and Lee (1991, Table 1) and those for $\underline{\tau}_2$ and $\underline{\rho}_2$ are found in Schmidt and Phillips (1992, Table 1A).

The one sided LM test of the null hypothesis for Case 1 is given by the one sided t–test, τ_1, in (3) with S(t) defined as in (4) except that $\tilde{\beta}=0$. Using this new S(t), the corresponding 'coefficient test' from (4) is denoted by ρ_1, and demeaned versions of τ_1 and ρ_1, $\underline{\tau}_1$ and $\underline{\rho}_1$, may be defined as the usual t– and coefficient statistics of φ in (5). Toda and McKenzie (1994) show that:

(a) $\tau_1 => \int_0^1 W(r)dW(r)\{\int_0^1 W(r)^2dr\}^{-1/2}$; (b) $\rho_1 => \int_0^1 W(r)dW(r)\{\int_0^1 W(r)^2dr\}^{-1}$;

(c) $\underline{\tau}_1 => \{\int_0^1 \underline{W}(r)dW(r)\{\int_0^1 \underline{W}(r)^2dr\}^{-1/2}$; (d) $\underline{\rho}_1 => \int_0^1 \underline{W}(r)dW(r)\{\int_0^1 \underline{W}(r)^2dr\}^{-1}$,

where W(r) is a standard Brownian motion on [0,1], and $\underline{W}(r)$ is a demeaned standard Brownian motion, that is, $\underline{W}(r) = W(r) - \int_0^1 W(r)dr$. These distributions are just the usual Dickey–Fuller (1979) $\hat{\tau}$, $\hat{\rho}$, $\hat{\tau}_\mu$ and $\hat{\rho}_\mu$ distributions, respectively. It should be noted that τ_1 and ρ_1 have the same limit distributions as Dickey–Fuller's (1979) τ and ρ tests which were originally derived for testing H_0: a unit root without drift vs H_1: mean–zero stationarity.

3. SMALL SAMPLE PROPERTIES

In this section, the small sample properties of the unit root tests stated in section 2 are investigated through a Monte Carlo simulation. The data generating process used in the experiments is (1) and (2) with $\alpha=\beta=0$. The $\epsilon(t)$'s are generated as independently and normally distributed variables with mean zero and unit variance. Three sample sizes T=49, 98 and 196 are considered. The reason for choosing sample sizes divisible by seven relates to the way the missing observations are generated. Missing observations are generated to occur regularly by assuming the last n observations in each block of seven observations is missing where n=0,1,2,3 and the first block of seven observations starts at t=0. This is then like a model on daily data where the data are observed on some days of the week and not on other days. The effect of this assumption is that for any given sample size, T, the number of blocks of missing observations is fixed but the number of missing observations in each block differs. For T=49, 98 and 196 the number of blocks of missing observations is 7, 14 and 28, respectively.

Tables 1–2 report the simulation results on the small sample performance of both the coefficient and t–tests for a unit root based on 10,000 replications in each case. Figures in all tables indicate the rejection frequencies (%) of the null hypothesis of a unit root for various sample sizes, numbers of missing observations and values of ρ. The nominal size of each test is set equal to 5%. All calculations were performed using the GAUSS matrix programming language.

Table 1 shows the actual sizes and powers of the τ_1, $\underline{\tau}_1$, ρ_1 and $\underline{\rho}_1$ tests. Rejections frequencies for $\rho=1.0$ correspond to sizes. All tests have actual sizes reasonably close to the nominal size of 5% even for a sample size of 49 with three missing observations in every seven. However, the sizes of the two coefficient tests, ρ_1 and $\underline{\rho}_1$, are slightly more distorted than the t–tests especially for T=49 and to a lesser extent for T=98. For T=49, as the number of missing observations increases the size distortion of any given test worsens.

For $\rho\neq1.0$, the rejection frequencies in Table 1 are the (size uncorrected) powers of the τ_1, $\underline{\tau}_1$, ρ_1 and $\underline{\rho}_1$ tests. The power of the tests is affected substantially by the value of ρ and T in the expected ways, namely, as ρ increases or T falls, power falls. For a given T and ρ, the power of any test falls as the number of missing observations in a block increases. As might be expected, the impact of missing observations on power is largest for T=49 and decreases as the sample size increases. For a given value of ρ, a comparison of the results for T=49 and n=0 (implying K=49) with those for T=98 and n=3 (K=56) provides an insight into how increasing the time span of the data series, T, while holding the number of data points observed, K, roughly constant affects power. This comparison suggests that increasing the time span of the series appears to lead to large increases in power reinforcing the point that power is affected by both the time span of the data series as well as how many data points we have (see Campbell and Perron (1991)). Comparing τ_1 and $\underline{\tau}_1$, we observe that which test is more powerful depends on T, ρ and the number of missing observations. However, when power is low, τ_1 is the more powerful test and when power is high $\underline{\tau}_1$ is the more powerful test. Similarly, when comparing ρ_1 and $\underline{\rho}_1$, ρ_1 is the more powerful test when power is low and $\underline{\rho}_1$ is the more powerful test when power is high. One of the coefficient tests, $\underline{\rho}_1$, is sometimes more powerful than both t–tests while for other configurations of the parameters one or both the t–tests is more powerful than both the coefficient tests. This comparison, however, should be treated with a little caution given that the coefficient tests suffer larger size distortions than the t–tests.

Table 2 shows the rejection frequencies for the τ_2, $\underline{\tau}_2$, ρ_2 and $\underline{\rho}_2$ tests; again for ρ=1.0 these are sizes and for $\rho\neq1.0$ they are (size uncorrected) powers. As with Table 1, no test suffers from a serious size distortion but again the coefficient tests suffer greater size distortions than the t–tests at T=49 (and to a lesser extent at T=98). The impact of changes of ρ, T and n on power is as for Table 1. For all but one of the combinations of T, ρ and n considered, $\underline{\tau}_2$ is more powerful than τ_2. In comparing ρ_2 and $\underline{\rho}_2$, the more powerful test depends on the choice of ρ, T and n. Of the four tests, $\underline{\tau}_2$ is the most powerful in all but one case. However, this conclusion also needs to be tempered by the fact that the lower (size unadjusted) power of the coefficient tests in some cases may be explained by their greater downward size distortions.

4. CONCLUSION

Various approaches have been followed in the literature to deal with missing observations, for example, by generating estimates of the missing observations by interpolation or some other method, or by ignoring the problem. It would be of interest to see how these approaches impact on the problem of testing for unit roots and to investigate the small sample performance of such tests compared to tests like those considered here that deal with the missing observations in a model consistent way.

5. REFERENCES

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TABLE 1: SIZES AND POWERS OF CASE 1 TESTS

		n	τ_1	$\underline{\tau}_1$	ρ_1	$\underline{\rho}_1$
T=49	ρ=1.0	0	4.8	5.9	4.1	3.4
		1	4.6	5.3	3.9	3.4
		2	4.5	4.8	3.6	3.1
		3	4.3	4.1	3.6	2.8
	ρ=0.9	0	18.6	13.4	15.7	12.4
		1	17.6	13.0	14.8	12.0
		2	16.9	11.7	14.3	10.7
		3	15.3	9.3	12.7	9.0
	ρ=0.8	0	43.4	35.9	38.7	36.9
		1	41.3	33.1	37.0	34.3
		2	38.1	28.8	33.5	29.4
		3	33.8	22.7	28.9	23.8
	ρ=0.7	0	64.0	67.8	60.2	70.9
		1	61.5	63.6	57.6	66.9
		2	57.6	54.8	53.5	58.0
		3	50.4	43.1	45.7	45.6
T=98	ρ=1.0	0	5.1	5.3	4.6	4.3
		1	5.1	5.2	4.6	3.9
		2	4.7	5.0	4.5	4.1
		3	4.2	4.3	3.8	3.2
	ρ=0.9	0	42.1	33.5	40.1	40.6
		1	41.8	32.4	39.5	38.9
		2	39.7	30.0	37.4	35.8
		3	37.3	26.1	34.7	31.8
	ρ=0.8	0	75.4	87.6	73.9	93.1
		1	74.9	85.8	73.4	91.5
		2	72.0	81.1	70.4	88.2
		3	68.8	73.4	66.7	82.0
	ρ=0.7	0	88.4	99.7	87.5	99.9
		1	87.1	99.5	86.2	99.9
		2	84.9	98.5	84.0	99.4
		3	81.5	96.2	80.2	98.4
T=196	ρ=1.0	0	4.7	5.0	4.3	4.4
		1	4.8	5.3	4.6	4.7
		2	4.7	4.8	4.6	4.6
		3	5.2	4.5	5.0	4.1
	ρ=0.9	0	75.4	86.5	74.7	93.4
		1	75.2	85.0	74.6	92.9
		2	73.6	82.7	72.9	91.0
		3	71.9	79.8	71.0	88.8
	ρ=0.8	0	94.2	100.0	93.9	100.0
		1	93.7	100.0	93.5	100.0
		2	92.1	100.0	91.8	100.0
		3	90.9	99.9	90.5	100.0
	ρ=0.7	0	98.5	100.0	98.3	100.0
		1	98.0	100.0	97.9	100.0
		2	97.3	100.0	97.2	100.0
		3	96.2	100.0	96.0	100.0

TABLE 2: SIZES AND POWERS OF CASE 2 TESTS

		n	τ_2	$\underline{\tau}_2$	ρ_2	$\underline{\rho}_2$
T=49	ρ=1.0	0	5.2	6.0	3.3	3.1
		1	5.1	5.2	3.0	2.5
		2	4.5	5.1	2.8	2.8
		3	4.1	4.7	2.3	2.4
	ρ=0.9	0	10.3	10.8	6.5	5.7
		1	9.8	10.1	6.0	5.4
		2	8.4	9.0	5.2	4.8
		3	6.8	7.2	4.2	3.7
	ρ=0.8	0	25.4	27.0	17.4	16.5
		1	22.6	23.9	15.4	13.6
		2	19.2	20.5	12.6	11.3
		3	14.4	15.6	8.9	8.1
	ρ=0.7	0	46.6	52.9	36.5	37.0
		1	42.5	47.6	32.5	32.1
		2	35.9	38.9	26.0	24.7
		3	27.0	29.1	18.1	17.4
T=98	ρ=1.0	0	5.0	5.4	4.2	4.3
		1	5.5	5.2	4.6	3.9
		2	4.8	5.5	3.8	3.9
		3	4.5	4.4	3.6	3.1
	ρ=0.9	0	23.5	25.4	19.8	20.2
		1	22.8	23.6	19.1	18.5
		2	20.3	21.6	17.1	17.2
		3	18.2	17.9	15.0	13.8
	ρ=0.8	0	63.4	72.4	58.4	65.1
		1	61.2	69.0	56.5	62.0
		2	55.0	63.1	50.1	55.3
		3	49.7	56.0	44.6	47.2
	ρ=0.7	0	85.9	95.4	83.1	93.7
		1	83.6	94.2	80.7	91.8
		2	80.0	90.5	76.4	87.1
		3	71.7	83.5	67.4	78.1
T=196	ρ=1.0	0	4.8	4.9	4.4	4.3
		1	4.8	5.0	4.4	4.4
		2	4.7	5.0	4.3	4.3
		3	4.7	4.6	4.2	4.0
	ρ=0.9	0	62.1	70.5	60.2	67.7
		1	61.0	69.4	59.1	66.2
		2	58.9	66.1	56.9	63.0
		3	55.0	62.7	53.1	59.2
	ρ=0.8	0	93.1	99.1	92.6	99.0
		1	93.1	98.8	92.5	98.4
		2	91.5	98.4	90.6	98.1
		3	88.7	97.5	87.7	96.9
	ρ=0.7	0	98.5	100.0	98.4	100.0
		1	98.7	100.0	98.5	100.0
		2	97.8	99.9	97.5	99.8
		3	96.5	99.8	96.1	99.8