Properties of the Unit Root Tests Which Allow for a Trend Break When the Break Point is Misspecified

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ABSTRACT Dickey and Fuller proposed some tests for the unit root hypothesis in an uni-variate time series. Perron(1989) extended the t-ratio type unit-root tests so that they allow for a break in the deterministic trend and/ or in the intercept term. Purpose of the paper is to study by simulations the effect of misspecified break point to the tests proposed by Perron and to the likelihood ratio test. Further, the limits of the test statistics by Perron and the likelihood ratio test are derived under the assumption of misspecified break point, and the accuracy of the limit formula is examined by simulation techniques. (JEL Classification Number- C22)

1. INTRODUCTION

The alternative regression for one of the unit root tests which allow for a break in the deterministic trend as well as in the intercept term is

$$(1) \quad \Delta y_t = \phi\, y_{t-1} + \sum_{i=1,2}(\alpha_i + \beta_i\, t)DU_{it} + \gamma D_t + \varepsilon_t .$$

The dummy variables are defined as $DU_{1t}=1$ for $1 \leq t \leq T_B - 1$ but 0 otherwise, $D_t = 1$ for $t = T_B$ which is the break point but 0 otherwise, and $DU_{2t}=1$ for $T_B+1 \leq t \leq T$ but 0 otherwise. These dummy variables are made to be orthogonal to each other in this paper to simplify analyses. The null hypothesis of the test is $\phi=\beta_1=\beta_2=0$, and the null regression is nested by the alternative regression. This regression is different from the original regressions used by Perron (1989) since equation (1) includes the D_t term. The reason and effect of this difference was studied by Morimune and Nakagawa (1997). Other tests include either of the trend or the constant shift but an extension of analyses to these cases are straightforward and is omitted from the paper to save space.

In this paper, the effect of specifying incorrect break point is studied. That means the correct break point is T_C which is unknown and different from T_B by P where P is the interval between the correct and incorrect break points. This misspecification has a direct effect in the distribution of the test statistics through a parameter δ which is defined as $(\beta_2-\beta_1)/\sigma$, i.e., a standardized difference in the trend coefficients. This parameter is estimable. Compared with this parameter, the difference in the intercept $(\alpha_2-\alpha_1)/\sigma$ is of smaller order of magnitude and ineffective in the asymptotic analysis. In the asymptotic theory of this paper, various ratio between the misspecified interval P and T is analyzed but our main result comes from treating P as fixed.

2. EFFECT OF MISSPECIFIED BREAK POINT

The effect of misspecified break point is first studied by simulations. After some intensive work, a typical example is presented by the table 1 where the sample size is 100, the standardized difference between the two trend coefficients (δ) is -2, and the trend as well as the intercept terms have breaks. The number of replications in simulations is 10000. It may be obvious from the tables that the effect in the bias of tests caused by misspecifying the break ratio is asymmetric with respect to the position of the correct break point compared with the misspecified break point. The critical values of the coefficient test and the t-test are closer to 0 than the correct specification cases when the break point over-shoots the correct break point by 10%. This is caused by the trend in the misspecified interval. It may be noticed that, when the break ratio is over-shot, the critical values of the coefficient and t-tests become almost zero then the real sizes are almost zero. When the break point is under-shot, the absolute values of critical values of the coefficient test become smaller but they become greater in the t-test. This causes clear difference in the real sizes of the two tests. The size distortion of the t-test is extremely large in this case. In the likelihood ratio test, the error in the critical values are positive in both under and over-shooting cases. The size distortions are large in both cases, but extreme when the break point is under-shot. In general, errors in the critical points and size distortions are extremely large which can be caused by the large δ value.

2.1 The coefficient test

The bias in the break-ratio is dragging the estimated coefficient toward zero in the both over and under-shooting cases. The trend in the misspecified interval is dominating the estimated coefficient. The over-shooting cases have greater trend values in the misspecified interval than the under-shooting cases. This causes greater bias in the over-shooting case than in the under-shooting case since the denominator has squared trend but the numerator has a linear function of trend in the formula of the least squares estimator. Real sizes are almost zero in both cases then the bias is almost 5%.

2.2 The t-test

The bias in the critical value caused by over-shooting or by under-shooting takes opposite sign around the correct critical values. This naturally causes opposite sign of the bias in the size distortion. Compared with the coefficient test, the denominator of the t-test is the " square root" of the squared trend. Then the effect of the misspecified trend is smaller than the coefficient test. However, real size of the test is almost zero when the break-ratio is over-shot, and enormously large when it is under-shot. The distribution is shifted toward the origin in the over-shooting cases, and toward the minus direction in the under-shooting cases.

2.3 The likelihood ratio test

The likelihood ratio is the sum of three squared ratios as is proven by the equation (13) among which the t-ratio is included. Since the critical values are greater than the correct values both in the over and under-shooting cases, two other ratios than the t-ratio are taking greater values than they supposed to be when the break point is wrongly specified. If these two other terms are not effected by the wrongly specified break point, properties of the critical values of the likelihood ratio test must be similar to those of the t-ratios, i.e., the critical values must be smaller than the correct values in the over-shooting cases. Similarly, they must be greater than those of the correct values in the under-shooting cases.

The t-ratio effect is not negligible since the size distortion is bigger in cases where the break point is under-shot than in cases where it is over-shot. Since critical values of the over-shooting cases are closer to the correct critical values than those of the under-shooting cases, it is safer to over-shoot the break point than under-shooting the break point.

Table 2: The slope difference is one sigma of the error variance in the simulations summarized by this table which is one half of the parameter used by simulations in Table 1. The size distortions are smaller than those in Table 1, but the tests are still useless. The real value of δ as well as the misspecified interval are never known in empirical studies, but this table show that these nuisance parameter are very effective to the test results.
Zivot and Andrews type test avoids specifying the break point prior to the test. However the break point must be determined by the first round test, and there is no guarantee that the chosen break point is correct. If the chosen break point is in error, their test also suffers from the same size distortions. As for the likelihood ratio test, critical values of the over-shooting cases are closer to the correct critical values than those of the under-shooting cases. Then it is safer to over-shoot the break point than under-shooting the break point.

Table 3: If the regression specified by the alternative hypothesis includes a break in trend but the DGP has only the intercept break, the tests are not biased at all even when the intercept break point is misspecified. The choice of the break point is not important. If the test is only about the intercept break, the Dickey and Fuller's unit root test $\hat{\tau}_\tau$ is applicable and will not be biased. This is proven by showing the fact that the deterministic trend in the alternative regression equation dominates the test statistic and the effect of the intercept break becomes negligible (Montanes 1997).

3. THE ASYMPTOTIC DISTRIBUTION

3.1 The t-ratio test

Firstly, the asymptotic formula is derived for the t-ratio of the lagged level variable. This t-ratio is well known to be the unit root test statistics proposed by Perron as a general extension of the $\hat{\tau}_\tau$ test statistics by Dickey and Fuller. The regression equation under the alternative hypothesis is transformed as

$$(2)\quad \Delta y_t = \phi y^*_{t-1} + \sum_{i=1,2}[\alpha^*_i + \beta^*_i (t - \bar{t}_i)]DU_{it} + \gamma^* D_t + \varepsilon_t$$

where $\bar{t}_1$ and $\bar{t}_2$ are the mean of trend in the first and second sub-periods, and

$$(3)\quad y^*_{t-1} = \sum_{i=1,2}[y_{t-1} - \bar{y}_i - \hat{\beta}_i(t - \bar{t}_i)]DU_{it} - \hat{\gamma} D_t .$$

where $\bar{y}_1$ and $\bar{y}_2$ are the mean of trend in the first and second sub-periods. The variable (3) is defined, for example, by regressing $y_{t-1}-\bar{y}_1$ on $(t-\bar{t}_1)$ then y^*_{t-1} is orthogonal to other regressors in the equation (2). The coefficients in the equation (2) are adjusted to this transformation of variable. Under the null hypothesis, the DGP is $\Delta y_t=\beta_1^+DU_{1t}^+ +\beta_2^+DU_{2t}^+ +\gamma^+D_t^+ +\varepsilon_t$ where the asterisked dummy variables have breaks at T_C instead of T_B. It is assumed that $T_C<T_B$ in the following analysis. Then the DGP of the level variable is

$$(4)\quad y_t = \eta_t + \sum_{i=1,2}(\alpha_i^+ + \beta_i^+ t)DU_{it}^+$$

$$= \eta_t + \sum_{i=1,2}(\alpha_i^+ + \beta_i^+ t)DU_{it}$$

$$-[(\alpha_1^+ + \beta_1^+ t)-(\alpha_2^+ + \beta_2^+ t)]DIF_t^+,$$

neglecting D_t^+ term which does not affect the asymptotic results, η_t is the sum of ε_t up to t, α_1^+ and α_2^+ are the initial values, and DIF_t^+ is 1 for $T_C+1\le t\le T_B-1$ since $T_C<T_B$. The null regression is derived as

$$(5)\quad \Delta y_t = \beta_1^+DU_{1t} + \beta_2^+DU_{2t} - (\beta_1^+ - \beta_2^+)DIF_t^+ + \varepsilon_t$$

similarly neglecting D_t^+ term. Using the DGP, the deviation of y_t from the mean is calculated as

$$(6)\quad (y_t - \bar{y}_i)DU_{it} = [(\eta_t - \bar{\eta}_i) + \beta_i^+(t-\bar{t}_i)]DU_{it}$$

$$+(\beta_1^+ - \beta_2^+)DU_{1t}[tDIF_t - \frac{P}{T_1}(T_1 - \frac{P}{2})].$$

for i=1,2. Then the regression coefficients of the trend in (3) are

$$(7)\quad \hat{\beta}_i = \frac{\sum_{t=1,T}(t-\bar{t}_i)DU_{it}(y_{t-1}-\bar{y}_i)}{\sum_{t=1,T}[(t-\bar{t}_i)DU_{it}]^2}$$

$$=\beta_i^+ + \frac{12\sigma}{\sqrt{T_i}}\bar{\beta}_i^+ + O(\frac{1}{T_i}),$$

where $\bar{\beta}_i^+ \Rightarrow \int_0^1 (r-\frac{1}{2})B_i(r)dr$ using the standardized Brownian motion. The misspecified interval does not affect the regression coefficient up to the second term. The transformed regressor is

$$(8)\quad \frac{1}{T}y^*_{t-1} = \sum_{i=1}^{2}\sqrt{\lambda_i}DU_{it}\{\frac{(\eta_{t-1}-\bar{\eta}_i)}{\sqrt{T_i}} - \frac{(t-\bar{t}_i)}{T_i}12\sigma\bar{\beta}_i\}\frac{1}{\sqrt{T}}$$

$$+\frac{1}{T}(\beta_1^+ - \beta_2^+)DIF_{t-1}^+$$

where λ_i is the break fraction T_i/T. Using this and neglecting the higher order terms,

$$(9)\quad \sum_{t=1}^{T}(\frac{y^*_{t-1}}{T})^2$$

$$=\sum_{i=1}^{2}\lambda_i^2\sum_{t=1}^{T}\{\frac{(\eta_{t-1}-\bar{\eta}_i)}{\sqrt{T_i}} - \frac{(t-\bar{t}_i)}{T_i}12\sigma\bar{\beta}_i\}^2 DU_{it}\frac{1}{T_i}$$

$$+\lambda_1^2(\beta_1^+ - \beta_2^+)^2\sum_{t=1}^{T_1}DIF_t^+$$

$$\Rightarrow \lambda_1^2\sigma^2\left\{\int_0^1 \bar{B}_1(r)^2dr + P\delta^2\right\} + \lambda_2^2\sigma^2\left\{\int_0^1 \bar{B}_2(r)^2dr\right\}$$

where $\bar{B}_1(r)$ is the demeaned and detrended Brownian motion and $\delta=(\beta_2^+ - \beta_1^+)/\sigma$ which is the parameter of discrepancy in the misspecified trend function. The numerator of the t-ratio is

$$(10)\quad \frac{1}{T}\sum_{t=1,T}y^*_{t-1}\Delta y_t = \frac{1}{T}\sum_{t=1,T}y^*_{t-1}\{\varepsilon_t - (\beta_1^+ - \beta_2^+)DIF_t^+\}$$

$$=\sigma^2\sum_{i=1,2}\lambda_i\sum_{t=1,T}DU_{it}\{\frac{(\eta_{t-1}-\bar{\eta}_i)}{\sigma\sqrt{T_i}} - \frac{(t-\bar{t}_i)}{T_i}12\bar{\beta}_i - \delta\frac{tDIF_t^+}{\sqrt{T_1}}\}$$

$$\bullet\{\frac{\varepsilon_t^*}{\sqrt{T_i}} + \frac{\delta}{\sqrt{T_i}}DIF_t^+\}$$

$$\Rightarrow \sigma^2\left\{\sum_{i=1,2}\lambda_i\int_0^1 \bar{B}_i(r)d\bar{B}_i(r) - \lambda_1\delta\sum_{t=T_C,T_B}\varepsilon_t^* - \lambda_1P\delta^2\right\}$$

and ε_t^* is the standardized white noise. The t-ratio is

$$(11)\quad \hat{\tau}_C = \frac{\sum_{1,T}y^*_{t-1}\Delta y_t}{\sigma\sqrt{\sum_{1,T}y^{*2}_{t-1}}}$$

$$\Rightarrow \frac{\sum_{i=1,2}\lambda_i\int_0^1\bar{B}_i(r)d\bar{B}_i(r) - \lambda_1\delta\sum_{t=T_1^*,T_1}(\varepsilon_t^* + \delta)}{\sqrt{\sum_{i=1,2}\lambda_i^2\int_0^1\bar{B}_1(r)^2dr + \lambda_1^2P\delta^2}} \equiv \tau_C.$$

If the break point is correctly specified, then the second term in both numerator and denominator disappear. The resulting formula is the same as that by Perron. Since T_C is assumed to be less than T_B in this analysis, the nuisance parameter δ is multiplied by the break fraction λ_1 and the miss-specified interval P in the denominator. In the numerator, a sum of white noises for the same interval is added besides a similar nuisance parameter.

3.2 The likelihood ratio test

In addition to the t-ratio test statistic, asymptotic expression of the likelihood ratio is also derived. It is noted that the likelihood ratio is simply the F-ratio multiplied by a scalar. Using the formula of the test by Dickey and Fuller (1981), the test is defined as

$$(12)\quad \Psi_C = \frac{1}{3\hat{\sigma}^2}(RSS_0 - RSS_A)$$

$$= \frac{1}{3\hat{\sigma}^2}\Delta y'\ [X_A(X_A'X_A)^{-1}X_A' - X_0(X_0'X_0)^{-1}X_0']\Delta y$$

where RSS_0 and RSS_A are the sum of squared residuals under the null and the alternative hypothesis, respectively, $\hat{\sigma}^2$ is RSS_A divided by T and is a consistent estimator of the error variance, Δy, X_0, and X_A are a T by 1 column vector, T by 3 matrix and T by 5 matrix consisting of Δy_t, $\{D_t, DU_{1t}, DU_{2t}\}$, and $\{D_t, DU_{1t}, DU_{2t}, (t-\bar{t}_1)DU_{1t}, (t-\bar{t}_2)DU_{2t}, y^*_{t-1}\}$, respectively. Since all regressors are orthogonal to each other, the Ψ_C test statistic is decomposed into

$$(13)\quad \Psi_C = \frac{1}{3}\{(\hat{\tau}_C)^2 + \sum_{i=1}^{2}\frac{\{\sum_{t=1,T}\Delta y_t(t-\bar{t}_i)\,DU_{it}\}^2}{\sigma^2\sum_{t=1,T}[(t-\bar{t}_i)\,DU_{it}]^2}\}.$$

Using (5), the second term of (13) converges to

$$(14)\quad \left\{\sqrt{12}\int_0^1(r-\frac{1}{2})dB(r) + \delta\sqrt{3}\frac{P}{\sqrt{T_1}}(1-\frac{P}{T_1})\right\}^2$$

when the break point is over-shot. When d is fixed, the F-ratio statistic weakly converges to $\{(\tau_C)^2 + \chi^2(2)\}/3$.

3.3 Alternative convergence rate of P

In the proof, it is easily checked that the t-ratio diverges if the misspecified interval P increases together with the sample size T. This kind of analysis is found in Vogelsang and Perron (1994). In their study, the break fraction is kept constant which implies that the misspecified interval P is a fixed fraction of the sample size T. A simulation study is necessary to find out which assumption is reasonable to characterize the small sample distribution of the t-ratio statistics of the unit root when a break is allowed in the deterministic trend function.

If the misspecified interval is a fraction of the sample size T, the t-ratio diverges asymptotically. This is verified by similar calculations as was shown in Section 3.1. The asymptotic result is summarized as follows. If $P=O(T^i)$, $0<i<1$, the t-ratio normalized by $\sqrt{P}$ converges to $-|\delta|$:

$$(15)\quad \text{plim}_{T\to\infty}\frac{1}{\sqrt{P}}\hat{\tau}_C = -|\delta|.$$

If $P=O(T)$, then the same ratio converges to $-|\delta|$ multiplied by a fraction of polynomials in (P/T). This fraction converges to one if (P/T) converges to 0. Then, (15) is covered by this result. Vogelsang and Perron (1994) proved that $\hat{\tau}_C/\sqrt{T}$ is convergent when P is a fixed fraction of T. As can be seen from (14), the two other terms in the likelihood ratio are affected by the nuisance parameters and converge to a constant if P is increasing with T at any rate. If the test includes only the intercept break, then the $\hat{\tau}_C$ test can be applied without being affected by the choice of the break point. This is proven by assuming that the two trend coefficients are the same in the above analysis. For example, the difference in the two trend coefficients is nullified in the equation (6). Similarly in (8), (9), and (10), and δ is zero in (11). The limit of the standard Dickey-Fuller $\hat{\tau}_\tau$ test is derived using our calculations but without using the two sub-sample periods. Even if the intercept term has a break in the DGP, the $\hat{\tau}_\tau$ statistic has the same limit since the difference in the intercept in the equation (6) does not affect (7), (8), (9), and (10).

The likelihood ratio test has the same convergence rate as the t-ratio. If $P=O(\sqrt{T})$, then (14) is non-central χ^2, but is dominated by $\hat{\tau}_C$; it is divergent if $P>O(\sqrt{T})$ but is dominated by $\hat{\tau}_C$ if $P<O(T)$; both of the t-ratio and this term is O(T) if $P=O(T)$.

4. PROPERTIES OF THE TEST STATISITCS

Properties of the test statistics are characterized by the asymptotic distributions, in particular, if the asymptotic distribution simulates the small sample distribution accurately. Since there are two asymptotic results, it is necessary to examine the accuracy of the approximations to find which asymptotic result is more adequate than the other to characterize the test statistics.

Firstly, the accuracy of the asymptotic distribution derived by assuming P to be increasing with the sample size T. In this case, $\hat{\tau}_C/\sqrt{P}$ converges to a constant, say c, in the limit. The constant multiplied by $\sqrt{P}$ should be compared with the critical values of the test. If $c\sqrt{P}$ is less than the critical value, then the null hypothesis is rejected, and so on. It seems to us that this limit does not characterize the behavior of the test statistic at all since the $\hat{\tau}_C$ test values are not really large when the break points are misspecified by 10 %. As can be found from the Table 1, the critical values do not diverge away to infinity which is suggested by this asymptotic theory.

Secondly, the accuracy of the asymptotic distribution derived in Section 3 is examined. As it can be found from the figures in the Appendix 2, this second asymptotic results are more reliable to understand the small sample properties of the test statistics. At least, the asymptotic distributions do not diverge away to infinity even when the sample size is 1000. Our simulation results are not satisfactory yet since the ratio of the misspecified interval to the sample size is kept to 10% in all experiments. (The misspecified interval , not the ratio, had to be fixed in experiments.) It is expected that the asymptotic distributions will be found accurate if the misspecified interval is fixed to, for example, 10 in simulations. Then the small sample distribution and the asymptotic distributions may be very close.

APPENDIX 1: CRITICAL VALUES AND SIZE DISTORTION

Table 1: δ is -2, and the breaks are both in trend and intercept term

DGP BR	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
CHANGES IN 5% CRITICAL VALUES										
Coef-test										
correct spec	-20.8	-24.4	-27.1	-28.9	-30.1	-30.0	-29.5	-28.8	-27.2	-23.8
10% over-shoot	-24.4	-16.9	-10.0	-6.9	-5.0	-4.1	-3.3	-2.9	-2.4	-2.3
10% under-shoot	-23.8	-16.0	-18.2	-18.9	-19.4	-19.4	-19.3	-18.8	-18.0	-17.8
T-test										
correct spec	-3.5	-3.8	-4.1	-4.3	-4.3	-4.3	-4.3	-4.2	-4.0	-3.8
10% over-shoot	-3.8	-3.3	-2.2	-1.5	-1.1	-0.9	-0.8	-0.7	-0.6	-0.6
10% under-shoot	-3.8	-6.3	-6.3	-6.2	-6.1	-5.9	-5.6	-5.1	-4.4	-3.5
LR-test										
correct spec	6.7	5.7	6.3	6.8	7.0	7.0	6.8	6.7	6.3	5.7
10% over-shoot	5.7	13.1	12.0	11.2	11.0	10.8	10.5	10.4	10.1	14.1
10% under-shoot	5.7	29.5	20.1	20.3	20.3	20.1	19.3	18.5	16.8	13.2
SIZE DISTORTION (REAL SIZE - 5%)										
Coef-test										
10% over-shoot	0.0	-4.9	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0
10% under-shoot	0.0	-4.7	-4.9	-5.0	-5.0	-5.0	-5.0	-5.0	-4.9	-4.8
T-test										
10% over-shoot	0.0	-4.4	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0	-5.0
10% under-shoot	0.0	82.5	75.6	70.8	63.2	53.7	38.5	19.2	1.9	-4.1
LR-test										
10% over-shoot	0.0	58.3	36.8	24.6	19.8	19.0	18.0	20.2	24.5	29.6
10% under-shoot	0.0	90.4	87.1	87.0	87.2	87.2	86.5	85.1	78.4	58.4

R=10th 1-1 N=100

10% over-shoot: The critical values and the real size of the test are calculated when the test break-ratio (BR) is 10% greater than that of the DGP. (This is called over-shooting) For example, the model BR is 0.5 when the correct BR is 0.4. Real critical values are tabulated. Similarly, real sizes minus 5 are tabulated. The model BR is 0 when the correct BR is 0.9.

10% under-shoot: The critical values and the real size of the test are calculated when the test break-ratio (BR) is 10% smaller than that of the DGP. (This is called under-shooting) For example, the model BR is 0.4 when the correct BR is 0.5. Real critical are tabulated. Similarly, real sizes minus 5 are tabulated. The model BR is 0.9 when the correct BR is 0.

Table 2: δ is -1, and the breaks are both in the trend and intercept

DGP BR	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
SIZE DISTORTION (REAL SIZE - 5%)										
Coef-test										
10% over-shoot	0.0	-2.8	-3.8	-4.1	-4.4	-4.4	-4.4	-4.4	-4.4	-4.5
10% under-shoot	0.0	-3.2	-3.5	-3.7	-3.7	-4.1	-3.7	-3.7	-3.5	-2.8
T-test										
10% over-shoot	0.0	-2.4	-3.9	-4.2	-4.5	-4.5	-4.6	-4.5	-4.5	-4.5
10% under-shoot	0.0	24.8	18.0	13.3	10.3	7.4	4.3	1.7	-0.8	-2.3
LR-test										
10% over-shoot	0.0	9.6	1.2	-1.6	-3.0	-3.0	-3.5	-3.5	-2.7	-1.9
10% under-shoot	0.0	43.8	34.8	31.4	28.9	26.8	25.0	24.3	17.5	9.9

R=10th 1-1 N=100

Table 3: δ is -2, and the break is only in the intercept term.

DGP BR	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
SIZE DISTORTION (REAL SIZE - 5%)										
Coef-test										
10% over-shoot	0.0	0.1	-0.2	0.1	0.0	-0.2	-0.2	-0.4	-0.4	-0.4
10% under-shoot	0.0	-0.1	0.0	0.2	0.0	-0.3	-0.5	-0.5	-0.1	-0.4
T-test										
10% over-shoot	0.0	-0.4	-0.4	-0.1	0.0	0.1	-0.2	-0.2	-0.3	-0.3
10% under-shoot	0.0	-0.2	0.2	0.0	-0.4	-0.2	-0.3	-0.1	-0.3	-0.2
LR-test										
10% over-shoot	0.0	-0.4	-0.3	-0.4	0.0	-0.4	-0.3	-0.2	-0.6	-0.4
10% under-shoot	0.0	-0.1	0.1	0.1	-0.1	-0.1	-0.1	-0.3	-0.2	-0.1

R=10th 0-1 N=100

APPENDIX 2: FIGURES 1-4

Solid Line: Simulated Distribution, Dotted Line: Asymptotic Distribution

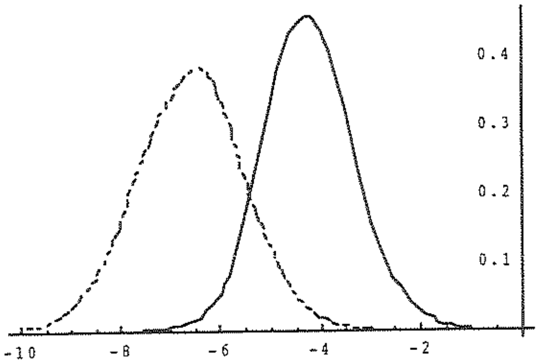

FIG 1: T=100 10% Undershoot Case:

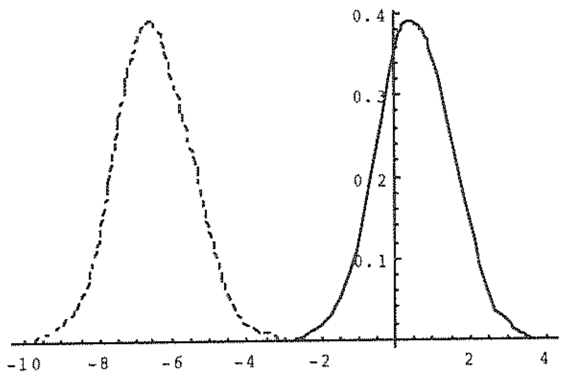

FIG2: T=100 10% Overshoot Case:

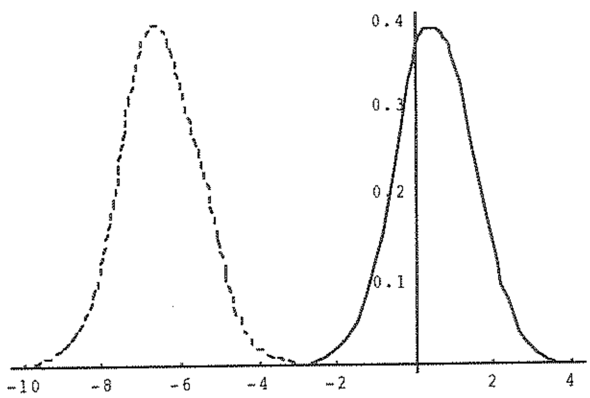

FIG 3: T=1000 10% Undershoot Case:

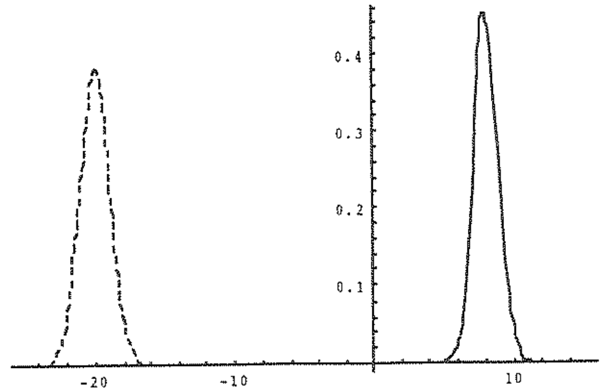

FIG 4: T=1000 10% Overshoot Case:

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