

The Matrix Method for Higher Education Modelling: A Temporal and Spatial Approach

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Abstract. — In this paper a matrix method for modelling higher education is proposed. The six basic higher education sets which represent the fundamental mathematical structure for higher education model building are introduced. Thus, the academic process is viewed as a six dimensional space, where elements of the basic sets interact among themselves. In order to overcome the problems of dimensionality and large scale, the method reduces the six dimensional description of higher education to a series of two-dimensional models. The method is used to develop a model for temporal and spatial analysis of academic processes.

1. INTRODUCTION

During the last decade important progress has been made in the research of different aspects of higher education. Researchers attempting to build models of the academic process have to take into account the existence, not only of dead objects, but also of live human beings. The human factor unavoidably introduces uncertainty into the decision making process based on the observed data. Such dual nature of the education process - dead objects and human beings - means that, depending on the nature of the process under analysis - physical and/or behavioural -science principles will be relevant.

In this paper, we propose a matrix method for analysis of higher education. We introduce basic higher education sets and define mathematical structures for operational and quantitative analysis of higher education. Some applications of this method has already been presented in Humo [1996], [1997].

2. BASIC HIGHER EDUCATION SETS

The matrix method to be developed is based on the following six sets:

The departments set D_U consists of all departments which are the members of a given university

$$D_U = \{D_1, D_2, \dots, D_j, \dots, D_n\} \quad (1)$$

where n is the total number of departments of the set.

The courses set C_U is made up of all the courses offered by particular university

$$C_U = \{c_1, c_2, \dots, c_i, \dots, c_m\} \quad (2)$$

where m is the total number of elements of the courses set.

The professors set P_U involves all the teachers of a given university

$$P_U = \{p_1, p_2, \dots, p_i, \dots, p_d\} \quad (3)$$

where d is the total number of teachers of the professors set.

The periods set H_U includes all the periods available for teaching on a weekly basis

$$H_U = \{h_1, h_2, \dots, h_i, \dots, h_b\} \quad (4)$$

where b is the total number of elements of the periods set.

The classrooms set R_U consists of all the available classrooms

$$R_U = \{r_1, r_2, \dots, r_i, \dots, r_k\} \quad (5)$$

where k is the total number of classrooms at university in question.

The students set S_U includes all the students population of a given university

$$S_U = \{s_1, s_2, \dots, s_i, \dots, s_h\} \quad (6)$$

where h is the total number of students enrolled at university under analysis.

The academic process may now be viewed as a six dimensional space, where the elements of the basic sets D_U , C_U , P_U , H_U , R_U and S_U interacting among themselves in accordance with the existing academic regulations, curricula and teachers, periods and classrooms schedules.

Mathematical modelling or even visualisation of the mutual interaction of the relevant elements in the six dimensional space of the basic sets is impossible. We must find an alternative approach, which would reduce multidimensionality and large scale of the analysis-synthesis problem of higher education.

Relational database theory, according to Korth [1987] is largely related with computer programming of the proposed matrix method.

When modelling interpersonal professors-students relations, Psychological Bulletin has published a series of articles in the general field of interdependent behaviours that are observed between individuals or groups of individuals, as viewed in Iacobucci [1990].

3. SOME PROPERTIES OF BASIC HIGHER EDUCATION SETS

Before beginning to formulate criteria for the formation of subsets of the basic sets, we must describe some properties of these sets relevant to modelling.

Property of Ordering. The order is an inherent property of set elements, which may be regarded as most important relation between them. The element of a given set can be ordered in many different ways according to the specific academic criteria chosen.

Property of Partition. The elements of a basic higher education set may be split down or grouped up in smaller or greater academic organisational entities. The only basic set whose elements possess this partition property is department set.

Property of Complexity. The department set is also the only basic set whose individual elements possess the property of complexity. This complexity is expressed by the fact that each element of the department set, through the specific academic organisation, is linked to a certain group of elements of another basic higher education set.

Basic structure. Applying the properties of partition and complexity to a department or to its smaller or larger academic entity, groups of other set elements, ordered by appropriate criteria may be associated. By introducing notations A, D, Co and U for the ordered set of elements which, by the property of complexity, respectively belong to a curricular area, department, college and university, it is possible to set up the following relation

$$A \subset D \subset Co \subset U \quad (7)$$

which describes this structure.

4. HIGHER EDUCATION AS OBJECT OF MODELLING

It is indisputable, that all the processes, operations and interactions pertinent to all the elements of the basic sets cannot be described in the form of a single model.

The complexity of higher education models is increased by the scale of the problem. The number of elements of some basic sets may be in the order of dozens, as in sets (1) and (5), or hundreds, as in sets (2) and (5), but may also run into thousands, as in sets (3) and (6).

In order to overcome the problem of dimensionality it is necessary to find an approach, which will enable us to start from a common basis, and then develop particular models of different aspects of the academic processes. The approach proposed here is to reduce the general six-dimensional description of higher education to a series of two-dimensional models each of which describe one particular aspect of functioning. The choice of the two concrete basic sets will depend on the particular problem to be considered.

The next step in reducing scale of the two dimensional models is to break up the basic higher education sets in accordance with the appropriate ordering criterion. A two dimensional model of two subsets of the basic sets considerably reduces the problem of the large scale.

The precedent description of academic processes leads us to postulate five main types of their contents: *operational, quantitative, dynamic, spatial and human.*

5. A MATRIX APPROACH TO HIGHER EDUCATION MODELLING

In order to present the general ideas of the method let us suppose that any two basic sets are given by

$$X = \{ x_1, x_2, \dots, x_j, \dots, x_n \} \quad (8)$$

$$Y = \{ y_1, y_2, \dots, y_i, \dots, y_m \} \quad (9)$$

where n and m are the total numbers of elements of sets (8) and (9), respectively. The order of elements in sets (8) and (9) is arbitrary. In the following considerations, it will be assumed that elements of the above sets are ordered.

Interaction among elements of basic sets (8) and (9) may be written in matrix form as follows

$$\begin{matrix} & y_1 & y_2 & \dots & y_i & \dots & y_m \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_j \\ \vdots \\ x_n \end{matrix} & \begin{bmatrix} f_{x_1 y_1} & f_{x_1 y_2} & \dots & f_{x_1 y_i} & \dots & f_{x_1 y_m} \\ f_{x_2 y_1} & f_{x_2 y_2} & \dots & f_{x_2 y_i} & \dots & f_{x_2 y_m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{x_j y_1} & f_{x_j y_2} & \dots & f_{x_j y_i} & \dots & f_{x_j y_m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{x_n y_1} & f_{x_n y_2} & \dots & f_{x_n y_i} & \dots & f_{x_n y_m} \end{bmatrix} \end{matrix} \quad (10)$$

where any element $f_{x_j y_i}$ of matrix (10) represents interaction of elements x_j and y_i of basic sets (8) and (9), respectively. The order of matrix (10) is $n \times m$.

It is known that interaction among elements of the basic sets may be functionally interdependent in a rather complicated way, which cannot be easily identified. In many situations, identification of these functions is not even necessary. The information as to whether there is interaction within a considered two-dimensional matrix model may often be sufficient.

Therefore, matrix (10) becomes *binary*. Element $f_{x_j y_i}$ is equal to 1 or 0, which depends on whether elements x_j and y_i of basic sets (8) and (9) interact when involved in the two-dimensional matrix. Thus we have

$$\begin{aligned} f_{x_j y_i} &= 1 \text{ if element } x_j \text{ "relates to" element } y_i \\ &= 0 \text{ otherwise} \end{aligned} \quad (11)$$

for every *ordered* pair of elements. Generally, $f_{x_j y_i}$ need not be equal to $f_{y_i x_j}$.

The inclusion of the quantitative content is accomplished by substitution of matrix elements $f_{x_j y_i}$, $j=1,2,\dots,n$; $i=1,2,\dots,m$; with their numerical values $qf_{x_j y_i}$, which correspond to the operations considered. So, the corresponding quantification matrix, which quantitatively describes its operation, is associated with matrix (10)

$$\begin{bmatrix} qf_{x_1 y_1} & qf_{x_1 y_2} & \dots & qf_{x_1 y_i} & \dots & qf_{x_1 y_m} \\ qf_{x_2 y_1} & qf_{x_2 y_2} & \dots & qf_{x_2 y_i} & \dots & qf_{x_2 y_m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ qf_{x_j y_1} & qf_{x_j y_2} & \dots & qf_{x_j y_i} & \dots & qf_{x_j y_m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ qf_{x_n y_1} & qf_{x_n y_2} & \dots & qf_{x_n y_i} & \dots & qf_{x_n y_m} \end{bmatrix} \quad (12)$$

Each element $qf_{x_j y_i}$ of quantification matrix (12) expresses the corresponding quantitative characteristic of operation $f_{x_j y_i}$ in matrix (10). Operation $f_{x_j y_i}$ obviously concerns the two-dimensional matrix mode academic process under consideration.

6. THE DECOMPOSITION OF THE BASIC MATRIX MODEL

General expressions (8) and (9) for basic sets may be written as a union of corresponding subsets X_1, X_2, \dots, X_N and Y_1, Y_2, \dots, Y_M , respectively

$$X = X_1 \cup X_2 \cup \dots \cup X_j \cup \dots \cup X_N \quad (13)$$

$$Y = Y_1 \cup Y_2 \cup \dots \cup Y_i \cup \dots \cup Y_M \quad (14)$$

Assuming that every basic set is union of corresponding subsets, relations (13) and (14), allows us to apply the previously developed principles of modelling to every reasonable two-dimensional submatrix of the interaction of elements of any two subsets

$$X_j = \{X_1^{Or}, X_2^{Or}, \dots, X_j^{Or}, \dots, X_{n_j}^{Or}\}, \quad (15)$$

$$Y_i = \{Y_1^{Or}, Y_2^{Or}, \dots, Y_i^{Or}, \dots, Y_{m_i}^{Or}\}, \quad (16)$$

of any two basic sets.

Here, $n_j < n$ and $m_i < m$ are total numbers of elements of sets X_j and Y_i , respectively. The superscript Or refers to an ordering criterion for elements of basic sets (8) and (9) with regard to finer grouping into subset (13) and (14). Thus, this approach enables *decomposition* of the general two-dimensional matrix form of the order $n \times m$ as in relation (10), into many particular two-dimensional matrices form of the order $n_j \times m_i$, where generally $(n_j \times m_i) \ll (n \times m)$. Hence, to create a particular submatrix, which represents the operational model of the academic process considered, it is necessary to order the elements of the two relevant basic sets according to the desired criterion of decomposition into particular subsets. When

such ordering is effected, basic matrix (10) may be decomposed into many particular two-dimensional submatrices. The same procedure applied to the basic matrix form may now be applied to any of its submatrices. Thus, basic matrix (10) may be rewritten in the form of matrix (17).

$$\begin{matrix} & & & & y_1^{Or} & y_2^{Or} & \dots & y_i^{Or} & \dots & y_m^{Or} \\ & y_1 & y_2 & \dots & & & & y_i & \dots & y_m \\ x_1 & \begin{bmatrix} f_{x_1 y_1} & f_{x_1 y_2} & \dots & & f_{x_1 y_i} & \dots & f_{x_1 y_m} \\ f_{x_2 y_1} & f_{x_2 y_2} & \dots & & f_{x_2 y_i} & \dots & f_{x_2 y_m} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \end{bmatrix} & & & & \\ x_2 & & & & & & & & & \\ \vdots & & & & & & & & & \\ x_j & & & & & & & & & \\ \vdots & & & & & & & & & \\ x_{n_j} & & & & & & & & & \\ \vdots & & & & & & & & & \\ x_n & \begin{bmatrix} f_{x_n y_1} & f_{x_n y_2} & \dots & & f_{x_n y_i} & \dots & f_{x_n y_m} \\ f_{x_{n_j} y_1} & f_{x_{n_j} y_2} & \dots & & f_{x_{n_j} y_i} & \dots & f_{x_{n_j} y_m} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \end{bmatrix} & & & & \end{matrix} \quad (17)$$

The decomposition of basic matrix (10) may be generally viewed through its form given by rel. (17). If all the elements of basic sets (8) and (9) are ordered to form subsets according to criterion Or and if number of the elements of subsets (13) and (14) are characterised by these equalities

$$n(X_1) + n(X_2) + \dots + n(X_j) + \dots + n(X_N) = n(X) \quad (18)$$

$$n(Y_1) + n(Y_2) + \dots + n(Y_i) + \dots + n(Y_M) = n(Y) \quad (19)$$

then basic matrix (17) may be decomposed into many submatrices of the form

$$\begin{matrix} & y_1^{Or} & y_2^{Or} & \dots & y_i^{Or} & \dots & y_{m_i}^{Or} \\ x_1^{Or} & \begin{bmatrix} f_{x_1^{Or} y_1^{Or}} & f_{x_1^{Or} y_2^{Or}} & \dots & f_{x_1^{Or} y_i^{Or}} & \dots & f_{x_1^{Or} y_{m_i}^{Or}} \\ f_{x_2^{Or} y_1^{Or}} & f_{x_2^{Or} y_2^{Or}} & \dots & f_{x_2^{Or} y_i^{Or}} & \dots & f_{x_2^{Or} y_{m_i}^{Or}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} & & & & & \\ x_2^{Or} & & & & & & \\ \vdots & & & & & & \\ x_j^{Or} & \begin{bmatrix} f_{x_j^{Or} y_1^{Or}} & f_{x_j^{Or} y_2^{Or}} & \dots & f_{x_j^{Or} y_i^{Or}} & \dots & f_{x_j^{Or} y_{m_i}^{Or}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} & & & & & \\ \vdots & & & & & & \\ x_{n_j}^{Or} & \begin{bmatrix} f_{x_{n_j}^{Or} y_1^{Or}} & f_{x_{n_j}^{Or} y_2^{Or}} & \dots & f_{x_{n_j}^{Or} y_i^{Or}} & \dots & f_{x_{n_j}^{Or} y_{m_i}^{Or}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} & & & & & \end{matrix} \quad (20)$$

The elements $f_{x_j^{Or} y_i^{Or}}$, $i=1, 2, \dots, m_i$; $j=1, 2, \dots, n_j$, of matrix (20) represent interaction among the corresponding elements x_j^{Or} and y_i^{Or} of sets (15) and (16), ordered according to criterion Or.

Thus, the proposed approach considerably reduces the scale of the problem by eliminating many obstacles immanent in the mathematical modelling of large-scale systems such as higher education.

By analogy with the previous, quantification matrix

associated with submatrix (20) is given by

$$\begin{bmatrix} qf_{x_1^{Or} y_1^{Or}} & qf_{x_1^{Or} y_2^{Or}} & \dots & qf_{x_1^{Or} y_i^{Or}} & \dots & qf_{x_1^{Or} y_m^{Or}} \\ qf_{x_2^{Or} y_1^{Or}} & qf_{x_2^{Or} y_2^{Or}} & \dots & qf_{x_2^{Or} y_i^{Or}} & \dots & qf_{x_2^{Or} y_m^{Or}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ qf_{x_j^{Or} y_1^{Or}} & qf_{x_j^{Or} y_2^{Or}} & \dots & qf_{x_j^{Or} y_i^{Or}} & \dots & qf_{x_j^{Or} y_m^{Or}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ qf_{x_n^{Or} y_1^{Or}} & qf_{x_n^{Or} y_2^{Or}} & \dots & qf_{x_n^{Or} y_i^{Or}} & \dots & qf_{x_n^{Or} y_m^{Or}} \end{bmatrix} \quad (21)$$

Every element $qf_{x_j^{Or} y_i^{Or}}$ of submatrix (21) represents the quantified value of interaction $f_{x_j^{Or} y_i^{Or}}$. The crucial question now is how to determine the function $f_{x_j y_i}$ representing interaction between elements of respective sets.

7. TEMPORAL AND SPATIAL DYNAMICS OF ACADEMIC PROCESSES

In connection with the proposed matrix approach for modelling higher education, the *dynamic content* of academic processes may be considered as the temporal coordination of different academic activities or as interaction among the elements of the periods set on one side, with the elements of the courses, professors, classrooms and students sets on the other side. Evidently, the classic concept of dynamic analysis, where the transient behaviour of elements of the basic sets is under consideration does not have any importance or even lose any sense. Therefore, every two-dimensional matrix model that, as one dimension, comprises elements of the periods set as in relation (4), provides the appropriate information about the behaviour in time of elements of another basic set which represents the second dimension of the matrix.

In this case, the "temporal matrix" may be written in the following way

$$\begin{matrix} & h_1 & h_2 & \dots & h_i & \dots & h_b \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_j \\ \vdots \\ x_n \end{matrix} & \begin{bmatrix} f_{x_1 h_1} & f_{x_1 h_2} & \dots & f_{x_1 h_i} & \dots & f_{x_1 h_b} \\ f_{x_2 h_1} & f_{x_2 h_2} & \dots & f_{x_2 h_i} & \dots & f_{x_2 h_b} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{x_j h_1} & f_{x_j h_2} & \dots & f_{x_j h_i} & \dots & f_{x_j h_b} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{x_n h_1} & f_{x_n h_2} & \dots & f_{x_n h_i} & \dots & f_{x_n h_b} \end{bmatrix} \end{matrix} \quad (22)$$

Besides rel.(11), for elements $f_{x_j h_i}$, $j = 1, 2, \dots, n$; $i = 1, 2, \dots, b$; of the matrix (22), the following relation

$$f_{x_j h_i} = f_{h_i x_j} \quad (23)$$

is valid for every ordered pair of set elements.

The *spatial content* of academic processes may be considered as, in a way analogous to the time content. Every two-dimensional matrix model (10) which, as one dimension, has elements of the classrooms set (5), may provide necessary information about the distribution of classrooms with respect to the elements of another basic set forming the second dimension of matrix (10). Thus, the resulting "spatial matrix" is this

$$\begin{matrix} & r_1 & r_2 & \dots & r_i & \dots & r_k \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_j \\ \vdots \\ x_n \end{matrix} & \begin{bmatrix} f_{x_1 r_1} & f_{x_1 r_2} & \dots & f_{x_1 r_i} & \dots & f_{x_1 r_k} \\ f_{x_2 r_1} & f_{x_2 r_2} & \dots & f_{x_2 r_i} & \dots & f_{x_2 r_k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{x_j r_1} & f_{x_j r_2} & \dots & f_{x_j r_i} & \dots & f_{x_j r_k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{x_n r_1} & f_{x_n r_2} & \dots & f_{x_n r_i} & \dots & f_{x_n r_k} \end{bmatrix} \end{matrix} \quad (24)$$

Here, besides relation (11), the following equality exists among elements $f_{x_j r_i}$, $j = 1, 2, \dots, n$; $r = 1, 2, \dots, k$; of matrix (24)

$$f_{x_j r_i} = f_{r_i x_j} \quad (25)$$

for every ordered pair of elements.

Binary interaction among elements of the periods and classrooms sets, matrix (26), traditionally

$$\begin{matrix} & r_1 & r_2 & \dots & r_i & \dots & r_k \\ \begin{matrix} h_1 \\ h_2 \\ \vdots \\ h_j \\ \vdots \\ h_b \end{matrix} & \begin{bmatrix} f_{h_1 r_1} & f_{h_1 r_2} & \dots & f_{h_1 r_i} & \dots & f_{h_1 r_k} \\ f_{h_2 r_1} & f_{h_2 r_2} & \dots & f_{h_2 r_i} & \dots & f_{h_2 r_k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{h_j r_1} & f_{h_j r_2} & \dots & f_{h_j r_i} & \dots & f_{h_j r_k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{h_b r_1} & f_{h_b r_2} & \dots & f_{h_b r_i} & \dots & f_{h_b r_k} \end{bmatrix} \end{matrix} \quad (26)$$

serves as a basis for scheduling lectures and classes. This matrix provides necessary information about the distribution of classrooms with respect to the daily available periods. Therefore, by means of matrix (26) we can quantify daily occupation of each classroom which disposes department under consideration.

Now, we may define *department daily occupation parameter* $Ocr_i^{D_i}$ as a quotient of number of periods

$q_{r_i^{D_1}}$ for which classroom $r_i^{D_1}$ is occupied by department D_1 and total number of daily periods b_d available for department D_1

$$Ocr_i^{D_1} = \frac{q_{r_i^{D_1}}}{b_d} \quad (27)$$

Parameter (27) is direct indicator of classroom occupation at the department level. It provides information for lectures and classes scheduling and other teaching activities.

8. CONCLUSION

This paper proposes the matrix method for modelling academic processes. The six basic sets represent fundamental mathematical structure. Thus, higher education is viewed as six dimensional space where elements of the basic sets interact among themselves. To overcome dimensionality and large scale problems, the method reduces six dimensional description of higher education to a series of two dimensional submatrix models.

The method is applied to develop the model for temporal and spatial analysis of academic processes. The temporal and spatial matrices are introduced. In addition, using periods-classrooms matrix, the department daily occupation parameter defined. It provides necessary information for classes scheduling as well as the other teaching activities.

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