H2U: A TRANSFER FUNCTION MODEL USING FRACTAL CHARACTERISTICS OF THE HYDROGRAPHIC NETWORK

I. GATOT SUMARJO
Centre for Soil and Agroclimate Research, Jl. Juanda 98 Bogor 16123, Indonesia.

J. DUCHESNE
Ecole Nationale Superieure Agronomique, 65 rue de St Brieuc, Rennes 35042, France.

P. PEREZ
Centre de Cooperation en Recherche Agronomique pour le Developpement, BP 5035, Montpellier 34032, France

ABSTRACT

Flood simulation using a conventional Unit Hydrograph Model has to be calibrated with an already existing data set. This is a major constraint for the extension of relevant flooding early warning systems. Many attempts have been carried out in order to link the hydrographic parameters to the characteristics of the watershed. The transfer function proposed hereafter, called *H2U model* (hydrogramme unitaire universel), is based on the description of the hydrographic network through a fractal approach. It is demonstrated that the parameters given by the fractal approach are related to the parameters used in the NASH's Instantaneous Unit Hydrograph Theory. The advantage of such a model is that it does not require any calibration: the two parameters have simply to be extracted from a precise map of the hydrographic network. After promising results in Europe and Africa, this model has been validated on the main Kali Garang watershed and on three sub watersheds in Central Jawa, Indonesia. The Kali Garang area is characterized by steep slopes with very short streams and by rapidly increasing impervious areas such as housing, factories, roads and other facilities. According to the existing rainfall and discharge data, flash-floods threaten the downstream provincial capital, Semarang, located on the coastal plain. The results of simulation show very closematching between the calculated hydrographs and the observed ones. This model needs a precise description of the rainfall pattern and of the land use characteristics.

1. INTRODUCTION

Usual flood simulation models using a conventional unit hydrograph model have to be calibrated with already existing rainfall /discharge data sets. This is the case for the NASH model (Nash, 1957), for which parameters n and k should be extracted from hydrograph records. This condition fails to solve some classical hydrologic problems, such as discharge prediction from ungaged basins (Gupta *et al*, 1980). That is of major constraint for countries such as Indonesia, which counts about thirty thousand islands and watersheds. Solution for this situation is how to find out a deterministic model without any calibration ?

Many efforts have been realized to establish a link between climate, drainage network parameters, and watershed response (Valdes *et al*, 1979; Rodriguez-Iturbe and Valdes, 1979; Gupta *et al* 1980; Rodriguez-Iturbo and Gonzales-Sanabria, 1982). These authors, using the Instantaneous Unit Hydrograph theory, found links between the response of a catchment and the geomorphologic parameters of the watershed. These parameters mostly consist of the mainstream length parameter (L_ω), where Ω corresponds to Strahler's order, of the streams number parameter (R_B), of the stream length parameter (R_L) and of the stream area (R_A). According to these parameters, the peak of discharge (q_p) and the time to peak (t_p) can be derived even to simulate IUH. Unfortunately, the authors didn't use the parameters directly in these equations but by intermediate parameters, which were calculated by regression. For example θ and k parameters (Rodriguez-Iturbe *et al.* 1982) were obtained by the following regression equations:

$$q_p = \theta . v$$

$$t_p = k/v$$

with: $\theta = 1.31 / L_\Omega . R_L^{0.43}$ (1)

and : $k = 0.44 . L_\Omega . R_B^{0.55} . R_A^{-0.55} . R_L^{-0.38}$ (2)

In the equation (1), linking θ, L_Ω and R_L, coefficient values 1.31 and 0.43 were obtained from statistical regressions. The same procedure was applied to the coefficients affecting L_Ω, R_B, R_A and R_L in the equation (2), that is to say 0.44, 0.55, -0.55 and -0.38.

So, according to Rodriguez-Iturbe *et al* (1982), it has to be noticed that :

(1) This model is not really deterministic because the coefficients of the Horton's geomorphologic laws are obtained from statistical regression.

(2) These relations are only valuable on basins for which they have been established. In order to extend the model to other watersheds, new coefficients have to be calculated.

Facing these on the two problems, we propose hereafter a deterministic model based on fractal characteristics called H2U, for Hydrogramme Unitaire Universel. The principle of this model is established on a quantitative description of the basin drainage network. The H2U model requires two parameters: n, the order of the basin according to Strahler's classification (Strahler, 1952) and $\overline{L}$, the average length of the drainage network. The advantage of H2U, beside being deterministic (rainfall-runoff transfer in the basin system can be physically explained), consists in the simple extraction of the two parameters from a precise map of the hydrographic network.

2. MATERIAL AND METHODS

2.1 H2U: a transfer function model

The H2U model has been developed by Duchesne and Cudennec (1997) from an initial analogy with the Maxwell's theory concerning the distribution of the gaz molecular velocities. The H2U model is based on the combination of two parameters: n and $\overline{L}$. These parameters are directly extracted from the drainage network representation. The geomorphologic response (IUH) links the parameters n, $\overline{L}$ and a so-called gamma function as following:

$$\rho(L) = \left(\frac{n}{2\overline{L}}\right)^{\frac{n}{2}} \cdot \frac{1}{\Gamma\left(\frac{n}{2}\right)} \cdot L^{\frac{n}{2}-1} . \exp\left(-\frac{nL}{2\overline{L}}\right) \quad (3)$$

with: n Strahler's basin order,

$\overline{L}$ mean hydraulic length

Γ gamma function.

With the H2U model we can answer the statement suggested by Rodriguez-Iturbe (1982) that the hydrological response of a basin depends only on some of the gross features, not on the details of the network geometry. In order to compare the H2U model and Nash model, we present the Nash model as following:

$$u(t) = \frac{1}{k\Gamma(n)}\left(\frac{t}{k}\right)^{n-1} . e^{-\frac{1}{k}}$$

k, n are parameters of basin extracted from discharge measurement and Γ is the gamma function.

Equation (3) and (4) show similarities in the gamma function and the other parameters. Getting the same expression (in t term) we replace $\overline{L}$ with $\overline{t}.v$ ($\overline{t}$ is average route time from the origin of water to the outlet and v represents the runoff velocity in the drainage network. We can write equation (3) as follow:

$$\rho(t) = \frac{n}{2\overline{t}\Gamma\left(\frac{n}{2}\right)}\left(\frac{n.t}{2.\overline{t}}\right).e^{-\frac{n.t}{2.\overline{t}}}$$

Finally the similarity between H2U model and Nash model can be expressed as.

$k_{NASH} = \frac{2.\overline{t}}{n}$ (H2U); $n_{NASH} = \frac{n}{2}$ (H2U)

2.2 Description of the watershed

After promising results in Europe and in Africa with several types of slopes, land uses, rainfalls (Cudennec, 1994; Corbierre, 1995), this model has been validated on the main Kali Garang watershed (195,05 km2) and on three sub watersheds, i.e. sub-basin Upper Kali Garang (82,86 km2), sub-basin Kali Kripik (16,59 km2) and sub-basin Kali Pancur (75,7 km2) in Central Java, Indonesia. The objectives of study are: (1) to validate the H2U as a deterministic model (2) to simulate rainfall-runoff transfer for predicting flood risk.

3. RESULT

3.1 H2U parameters of basin and sub-basins

The fractal parameters have been extracted from the hydrographic network which was designed from a 1:25000 scale topographic map. Results are shown in table 1.

Table 1. H2U parameters and the Γ function of the Kali Garang basin and its sub-basins.

Name of basin	n	$\overline{L}$ (km)	L (km)	$\Gamma\left(\frac{n}{2}\right)$
Kali Garang	6	19,22	35,49	2.00
Upper K. Garang	6	15,31	22,97	2.00
Kali Kripik	5	4,60	9,25	1,33
Kali Pancur	6	16,36	31,36	2.00

From the existing data we can establish the theoretical network density (Pdft) and compare to the observed network density (Pdf0). The figure 1 shows that Pdf0 and Pdft are well fitted in the case of the Kali Kripik sub-basin.

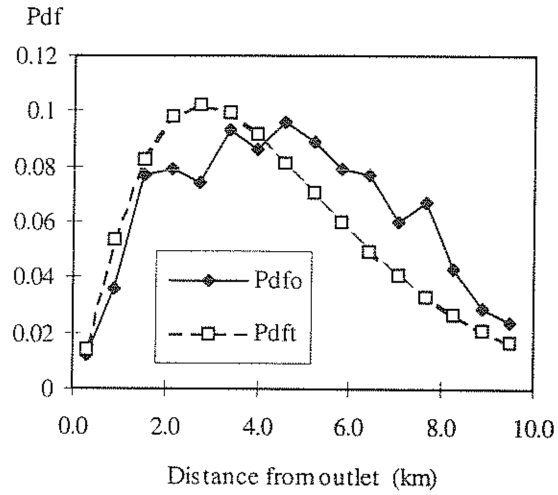

Figure 1. Probability density function of Kali Kripik sub-basin, theoretical (Pdft) and observed (Pdf0).

Results from the other sub-basins show also good fittings between theoretical and observed probability density functions. But the model overestimates the short stream length Pdf values from the Kali Garang basin. This is due, partly, to the fact that channels are more difficult to detect in the downstream areas of the catchment, characterized by weak slopes and urban settlements. Meanwhile, we cannot exclude that the fractal structure of the drainage network, relevant for homogeneous sub-basins, is probably slightly defective for the whole catchment.

3.2 Discharge simulation

The discharge by H2U model is calculated as following:

$$Q(t) = S\int_0^t \overline{i_{eff}}(\tau).\rho(t-\tau).d\tau$$

with: Q: Discharge (m3/s)
t : Time (s)
S: basin area (m2)
$\overline{i_{eff}}$: Average rainfall intensity (m/s)
ρ : Probability density function

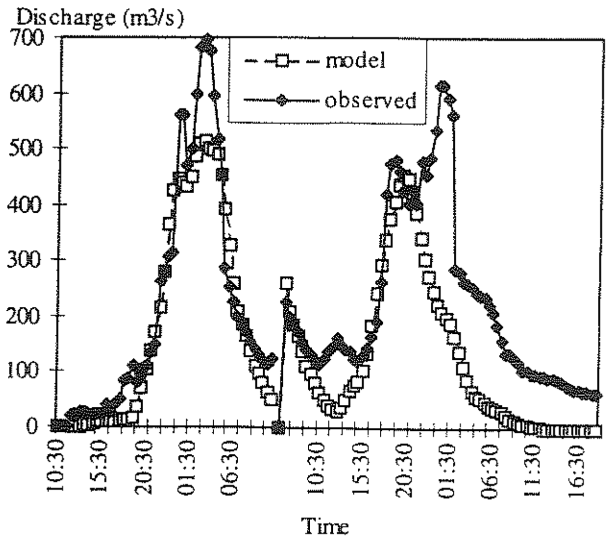

Figure 2. Comparison between observed and simulated discharge. Kali Garang basin, flood event from 25-26 December 1993.

Figure 2 shows a good fitting between observed and simulated discharge using H2U model. In this case, the simulated peak discharge is underestimated because only one rain recorder was available on the watershed at this time. Nevertheless, we have to suppose that rainfall intensity and duration are homogeneous in the Kali Garang basin for the selected event. The recorded maximum rainfall intensity was nearly 59,4 mm/hour and the duration was around 9 hours. Before this rainfall, the antecedent soil moisture condition could be considered as saturated.

4. DISCUSSION AND CONCLUSION

In the flood disasters of 1990 and 1993, about 75 casualties were claimed and infrastructure damaged such as roads, building and other facilities. In order to decrease the flood risk in the city of Semarang, capital of the province, an early warning system should be elaborated.

The changing from natural (forest and agricultural areas) to artificial land use occurs rapidly and increase impervious areas. This evolution modifies gradually the rainfall-runoff transfer rate and the average flow velocity within the watershed. But this characteristics are not homogeneous from one sub-basin to another.

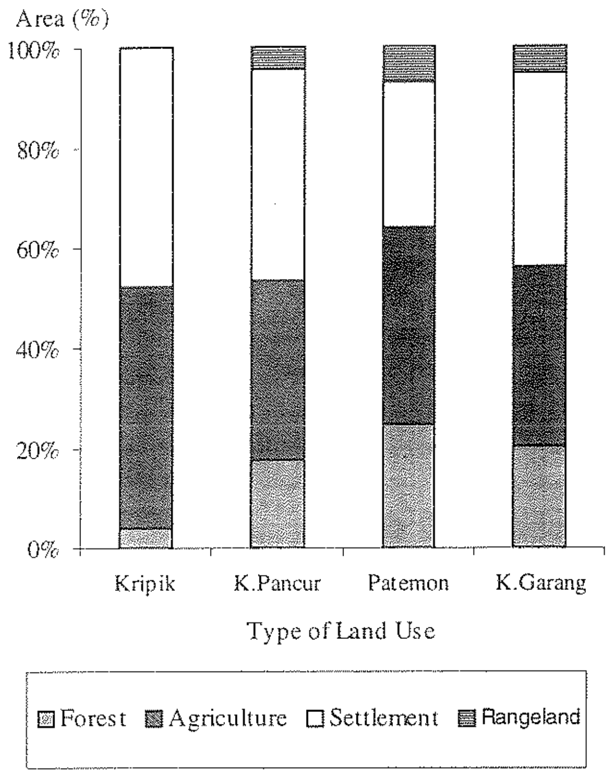

Figure 3. Different Land Use within the Kali Garang Basin and its main sub-basins.

The figure 3 shows that land use is dominated by settlement, with limited forest areas. As the problem of water management, including drought and floods, almost arise each year in the Kali Garang basin. There is a urgent need of reenforced land use regulation and of the extension of runoff harvesting methods such as small reservoirs or absorption wells.

The effect of different types of land use on the discharge characteristic are presented in figure 4.On one hand, the basin dominated by settlement (upper Kali Garang/Patemon) is characterized by a short time of response and by a high peak of discharge. This is due to the high velocity of surface runoff and low infiltration capacity. On the other hand, the basin characterized by agricultural land use (Kali Pancur) shows lower peak of discharge and time of response. So we have to consider that H2U model is already useful to simulate exceptional flooding and to built an early warning system; but for an hydrological survey land use description parameters have to be added.

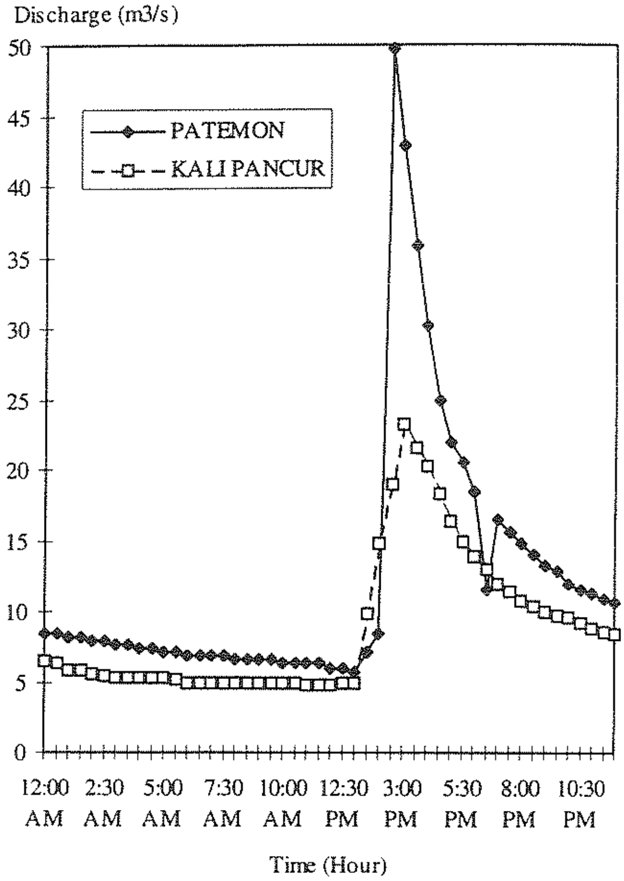

Figure 4. Comparison of two hydrographs corresponding to Patemon and Kali Pancur sub-basins. Flood events from 7 December 1996.

It is also possible to use H2U model as a tool for guiding raingauge installation. As a matter of fact, weighting methods used to compute average rainfall within a watershed, as Thiessen method or finite elements method, only take into account the distance between the different rainfall stations. Considering the fractal structure of the hydrological network, the probability density function (Pdf) points out the most frequent stream length observed in the hydrological network. This stream length value corresponds to the transfer function most influential stream areas. So, within the extreme limits of the watershed, raingauge installation should first favour these sensitive areas, without further information concerning the rainfall pattern or the soil hydrodynamics.

REFERENCE

Corbierre, V., Validation d'un nouveau model de transformation pluie-debit sur le resau pluvial d'Orly. *Memoire de fin d'etudes, Ecole Nationale Superieure Agronomique de Rennes*, 97 p, 1995.

Cudennec, C., Validation d'un nouveau modele physique de crues exceptionnelles. *Memoire de fin d'etudes, Ecole Nationale Superieure Agronomique de Rennes*, 83 p, 1994.

Duchesne, J and Cudennec, C., H2U: a universal unit hydrograph based both on the fractal structure of a catchment and on a mechanical statistical approach. *(submitted to Journal of hydrology)*, 1997.

Gupta, V.K., Waymire, E., Wang, C.T., A Representation of an Instantaneous Unit Hydrograph From Geomorphology *Water Resour. Res.,* 16 (5) 855-862, 1980.

Nash, J.E., The form of the instantaneous unit hydrograph. *AISH Publ., 42*, 114-118, 1957.

Rodriguez-Iturbe , I., Valdes, J.B., The Geomorphologic Structure of Hydrologic Response, *Water Resour. Res., 15 (6)* 1409-1420, 1979.

Rodriguez-Iturbe , I. And Gonzalez-Sanabria, M., A Geomorphoclimatic Theory of the Instantaneous Unit Hydrograph, *Water Resour. Res., 18 (4),* 877-886, 1982.

Strahler, A.N., Hypsometric analysis of erosional topography. *Bull. Geol. Soc. Am., 63*, 117-1142, 1952.

Valdes, J.B., Fiallo, Y., Rodriguez-Iturbe, I., A Rainfall-Runoff Analysis of the Geomorphologic IUH, *Water Resour. Res., 15 (6),* 1421-1434, 1979.