Stratified Flow past an Ideal Topography

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Abstract

A three-dimensional, non-hydrostatic, numerical turbulent model was used to study the flow pass a three-dimensional obstacle under a strong stratification condition. The numerical results clarify the stratification effect on the behavior of the flow at low Froude numbers. A vertical vorticity budget study shows that the tilting and friction term are important to the formation of the lee vortices while the baroclinicity term can be ignored.

1.INTRODUCTION

Flow structure over or around a three-dimensional obstacle changes drastically with stratification, which can be characterized by Froude number ($Fr=U/Nh_0$, where U is the flow speed, h_0 is the height of the obstacle, and N is the Brunt-Vaisalla frequency). At low Froude numbers, two phenomena are recognized: the onset of flow around the obstacle instead of that over the obstacle (i.e., flow splitting phenomenon) and onset of wave breaking above the obstacle (i.e., wave breaking phenomenon). Arguments from a linear theory[1], experiment[2] and numerical simulations[3-5] suggest the occurrence of the flow splitting and wave breaking phenomena at sufficiently low Froude numbers. Moreover, a pair of vortices (lee vortices) is formed on the lee side of the obstacle at lower Froude number[2-5]. In the atmosphere the air flows in approximately horizontal planes around topography as the stratification is strong enough. Despite their practical importance to aeronautics and air pollution dispersion, the distinctions among these phenomena are still little known, and the effects of stratification on flow splitting and wave breaking are far from clear. As for the lee vortices, there is still a debate on its formation mechanism. That is, interaction between the fluid and boundary is the only one mechanism of vortex formation under neutral stratification condition, while baroclinicity may be another candidate for vortex formation. There is an argument about the contribution of the later mechanism in a real atmosphere. The present work tries to investigate the importance of the stratification effect on the flow splitting and wave breaking phenomena by a numerical model. Vertical vorticity equation with its budget calculation is also provided to understand which terms in the equation are important for the formation of the lee vortices.

2. NUMERICAL METHOD

The numerical model used in this study was the same as in 6-7. Only a brief description of the model features will be given here.

The model equations are based on the atmospherical primitive equations simplified by adoption of the anelastic and Boussinesq approximations. A terrain-following coordinate system is used and the governing equations are of the three-dimensional, non-hydrostatic and dry numerical mode. In the turbulence modeling, the turbulent eddy diffusivity for momentum K_{vm} is evaluated from the turbulent kinetic energy E and the turbulent dissipation rate ε.

The prognostic equations are integrated forward explicitly in time with the time step chosen to satisfy the CFL-criterion. Centered difference approximations are used in space; the advective terms are evaluated by upstream difference with a spline technique. To increase the accuracy of the finite difference approximations, a staggered grid is used, both horizontally and vertically. In the horizontal directions, a grid interval of

$\Delta x=\Delta y=1000$ m is adopted, while in the vertical direction an expanding grid, with the greater resolution $\Delta z=20.2$ m near the ground, is used.

Numerical calculation is performed on a domain of which lateral boundaries are located at 100 km both in x and y directions, while the top boundary is located at z=8.6 km. The domain consists of 101*101*45 grid points. The mesoscale pressure is computed by solving a three-dimensional discrete Poisson equation with a Neumann boundary condition. The equation is solved directly by a Gaussian elimination method in the vertical direction, and by eigenfunction decomposition and fast Fourier transforms in the horizontal directions.

All experiments use the same mountain shape h(z) :

$$h(z)=h_0\left\{1+\left(\frac{x-x_0}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2\right\}^{-3/2}$$

where h_0 is the height of the mountain, and a and b are the half width of the mountain in the x, y directions centered at x_0, y_0.

A no-slip lower boundary condition u=v=0 is imposed. The surface temperature is constant. The Monin-Obukhov similarity law is used to determine the surface heat and momentum fluxes and u, v and θ at the nearest grids above the lower boundary. At the upper boundary all variables are fixed at their initial values. In the upper levels of the model, an absorbing layer is employed using Rayleigh damping to suppress downward reflection of energy. At the inflow lateral boundary, u,v,w are fixed at their initial values and an absorbing region similar to that in the upper level is installed upstream. This region serves to maintain the integrity of the upstream profiles throughout the integration. At the outflow lateral boundaries a radiation condition is used. Linear vertical profile of potential temperature and uniform profile of velocity are used to initialize the model by assuming the profiles horizontally homogeneous in the computational domain.

3. *RESULTS AND DISCUSSION*

Fig. 1 shows the wind (u, w), the potential temperature (θ) on the central y plane and the surface wind (u, v) for the case of Fr=0.44. The vertical wind field (u,w) and the potential temperature distribution show that the flow is in a wave breaking regime with a decline of the isotherm immediately after the mountain peak. The horizontal surface flow is characterized by flow splitting around the mountain and a defined vortex pair in the mountain wake.

Fig.2 is the same as Fig. 1 but for Fr=0.22. The overall patterns are quite similar to the ones in Fig.1, but the details are different. The vertical wind field (u,w) and the potential temperature distribution show an evidence of very reduced gravity wave activity in the cross sections. The horizontal surface flow is characterized by strong splitting around the mountain and a clearly defined vortex pair in the mountain wake. The vortices have axis in the vertical direction. After created above the lee side slope, they detach the slope and flow downstream. We have also done numerical experiment for the case of Fr=0.66.

These numerical results clarify the different behavior of the flow at low Froude numbers, showing the relative importance of the stratification effects on the flow splitting, wave breaking, and lee vortices phenomena. It is found that for Fr=0.22, the flow is characterized by strong streamline splitting with the formation of a very defined vortex pair (lee vortices). For Fr=0.44, the flow is in the wave breaking regime and the lee vortices still exist. For Fr=0.66, the flow is characterized by wave breaking with a strong hydraulic jump and no lee vortices can be found.

We further examine the vertical budget of vorticity equation to determine which terms are important to the formation of the lee vortices. After the stationary condition is attained, long-term averaging was made for each term of the vortivity equation. The time-averaged value of the local change of the vertical vorticity is assumed to be zero. Figure 3a-f are the surface distributions of the vertical vorticity itself, and terms of advection, stretching, tilting, baroclinicity, and friction for the case of Fr=0.44. Fig.3a shows respectively the lee vortices on the lee side of the slope. The advection and stretching terms (Fig.3b-c) are one order smaller than the tilting term (Fig.3d) or friction term (Fig.3f), but they are responsible for the increase or decrease of the vertical vorticity. Fig.3d and Fig.3f show that the tilting and friction term are most important for maintaining the vertical vorticity. It is also found in Fig.3e that the baroclinicity term is much more smaller than the other terms and can be ignored. Therefore, it can be concluded that tilting and friction are responsible for the maintenance of the vertical vorticity, and the contribution of baroclinicity can be ignored for it.

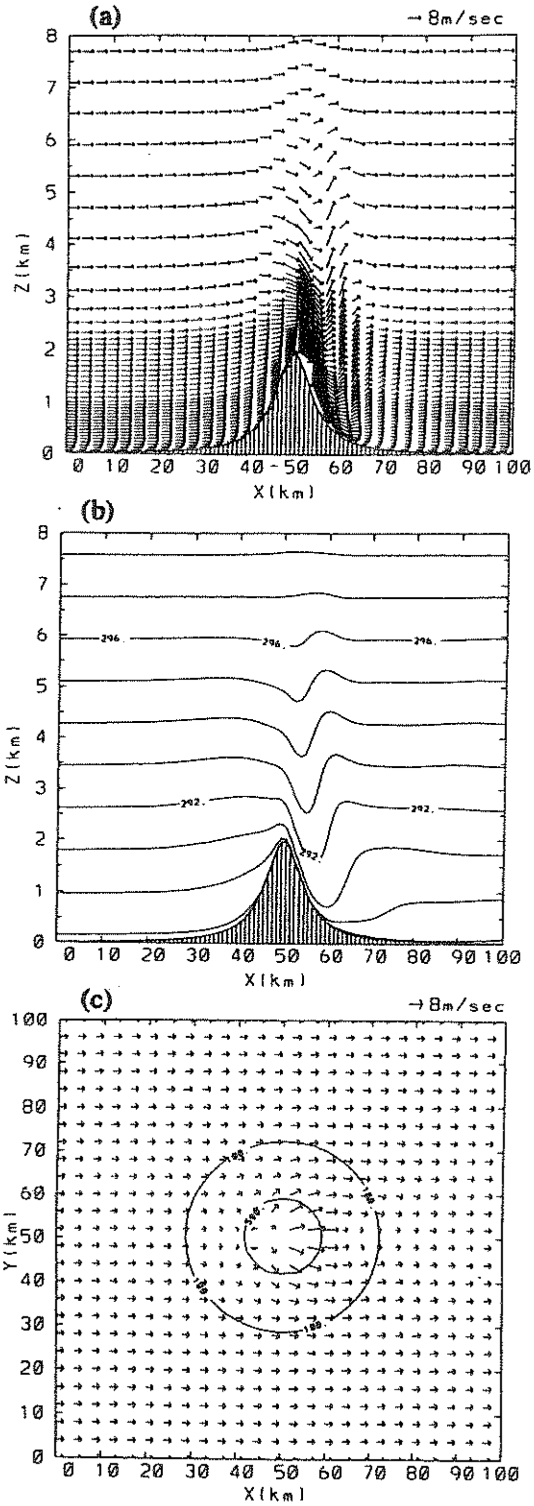


Figure 1. For a case of Fr=0.44, (a) vertical cross section wind field (u,w), (b) vertical cross section potential temperature distribution on the central y plane, (c)horizontal cross section wind field (u,v) on the lowest level (z=20.2m) with contours of the mountain.

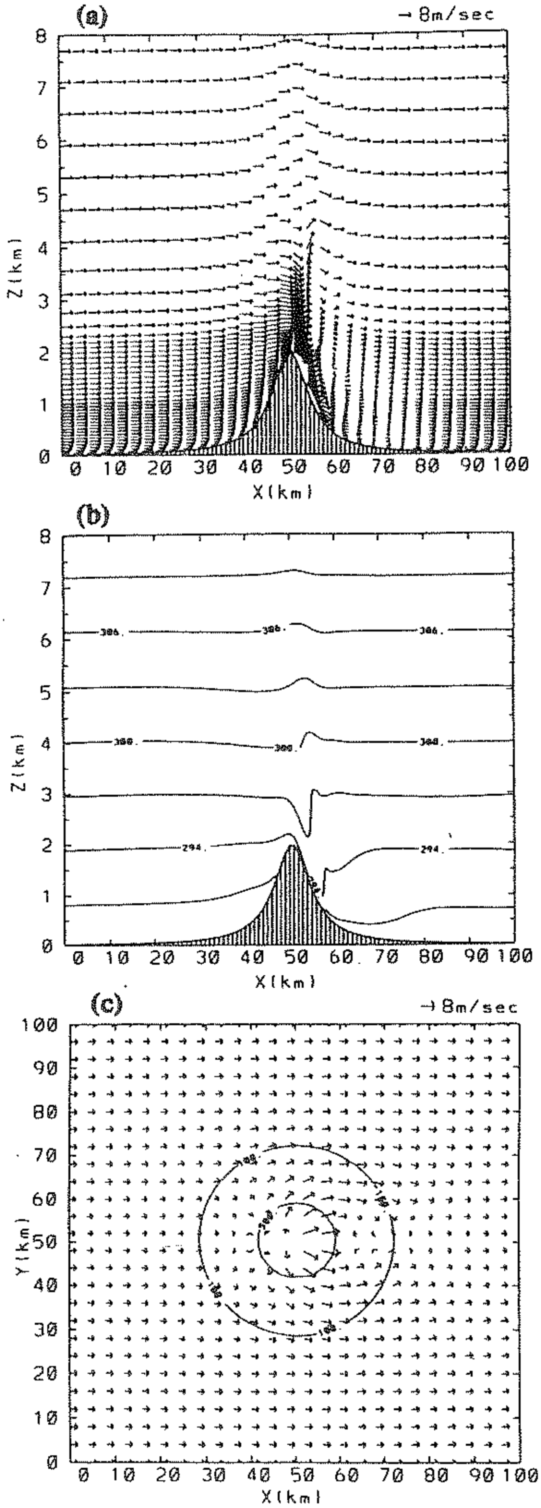
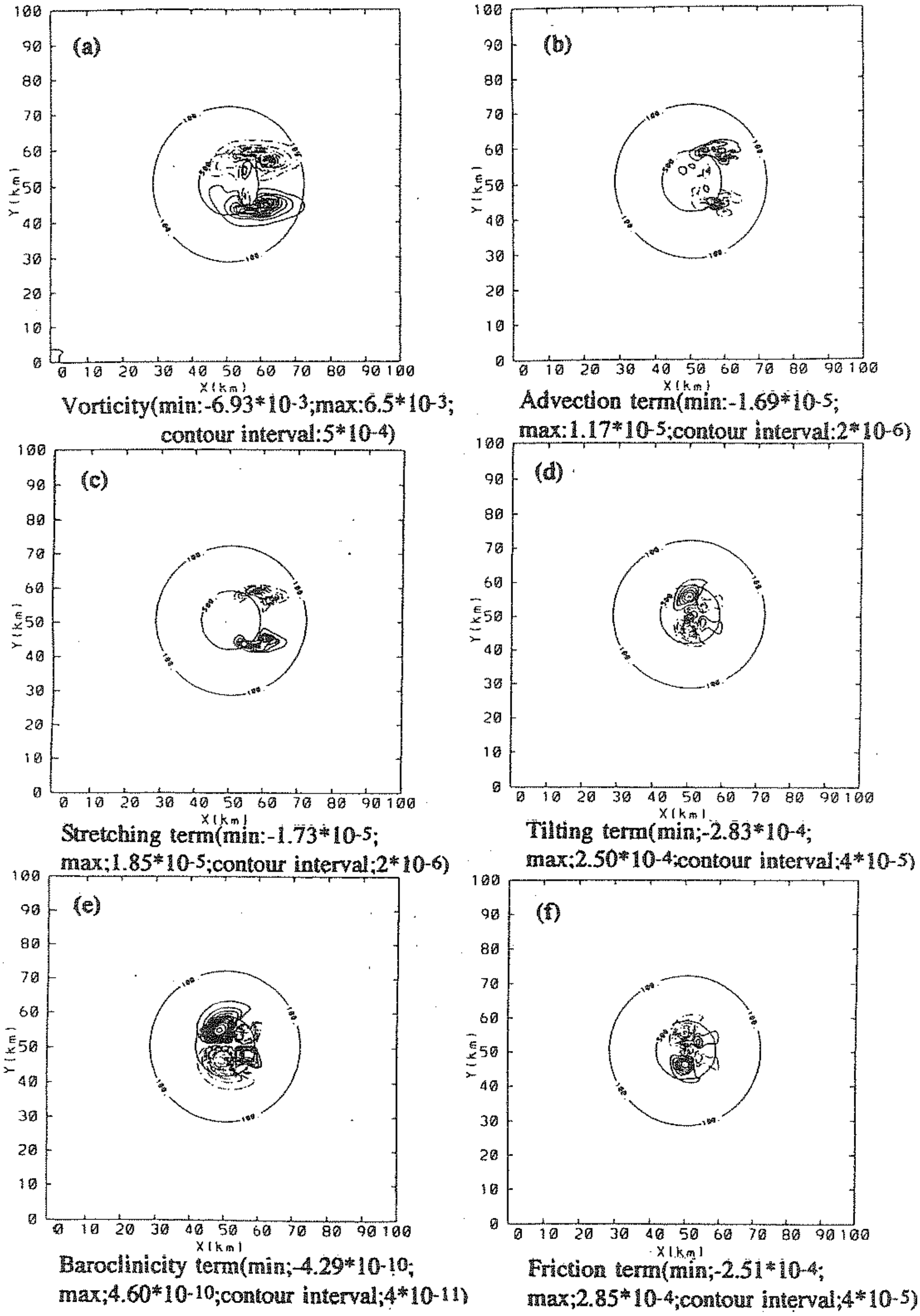


Figure 2. As same as Fig. 1 but for Fr=0.22.



(a) Vorticity(min:$-6.93*10^{-3}$;max:$6.5*10^{-3}$; contour interval:$5*10^{-4}$)

(b) Advection term(min:$-1.69*10^{-5}$; max:$1.17*10^{-5}$;contour interval:$2*10^{-6}$)

(c) Stretching term(min:$-1.73*10^{-5}$; max;$1.85*10^{-5}$;contour interval;$2*10^{-6}$)

(d) Tilting term(min;$-2.83*10^{-4}$; max;$2.50*10^{-4}$;contour interval;$4*10^{-5}$)

(e) Baroclinicity term(min;$-4.29*10^{-10}$; max;$4.60*10^{-10}$;contour interval;$4*10^{-11}$)

(f) Friction term(min;$-2.51*10^{-4}$; max;$2.85*10^{-4}$;contour interval;$4*10^{-5}$)

Figure 3. Surface distribution of (a) vertical vorticity, (b) advection term, (c) stretching term, (d) tilting term, (e) baroclinicity term, and (f) friction term on the lowest level (z=20.2m) for case of Fr=0.44.

4.CONCLUSIONS

A three-dimensional, non-hydrostatic, numerical turbulent model was used to study the flow pass a three-dimensional obstacle under a strong stratification condition. The numerical results clarify the behavior of the flow at low Froude numbers, showing the relative importance of the stratification effects on the flow splitting, wave breaking, and lee vortices phenomena.

A vertical vorticity budget study shows that the tilting and friction term are important to the formation of the lee vortices. On the other hand, the advection and stretching terms are responsible for carrying the vorticity to the lee side. The baroclinicity term can be ignored.

Acknowledgments

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References

[1]Smith, R.B., 1980: Linear theory of stratified hydrostatic flow past an isolated mountain. Tellus, 32, 348-364.

[2]Hunt, C.R. And W.H.Snyder, 1980: Experiments on stably and neutrally stratified flow over a model three-dimensional hill. J. Fluid Mech., 96, 671-704.

[3]Smolarkiewicz, P.K. And R.Rotunno, 1989: Low Froude number flow pass three-dimensional obstacle. Part 1: Baroclinically generated lee vortices. J. Atmos. Sci., 46, 1154-1164.

[4]Crook, N.A., T.L.Clark and M.W.Moncrieff, 1990: The Denver Cyclone. Part 1: Generation in low Froude number flow. J. Atmos. Sci., 47, 2725-2742.

[5] Miranda, M.A. And I.N.James,1992: Non-linear three-dimensional effects on gravity-wave drag: Splitting flow and breaking waves. Q. J. R. Meteorol. Soc., 118, 1057-1081.

[6]Sha, W., T.Kawamura and H.Ueda, 1991: A numerical study on sea/land breezes as a gravity current: Kelvin-Helmholtz billows and inland penetration of the sea-breeze front. J. Atmos. Sci., 48, 1649-1665.

[7] Sha, W., W.Grace and W.Physick, 1996: A numerical experiment on the Adelaide gully wind of South Australia. Aust. Met. Mag., 45, 19-40.