A 3-dimensional numerical simulation of salt-fingering convection

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Abstract We have studied the 3-dimensional development of salt finger convection by using recently developed software named α-FLOW. The domain of numerical calculation is a box of 5cm high with a rectangular section of 1.25 cm x 1.25 cm. The upper (lower) half of the box is occupied by hot-salty (cold-fresh) water, and initial random disturbances are given on the interface of these two fluids. The results are as follows: (1) Salt finger convection is well reproduced. (2) Horizontal scale of the salt finger is proportional to the power of -1/4 of the temperature difference between upper and lower layers. (3) Spatial patterns of fingers in the horizontal plane are highly irregular. However, (4) vertical flux of salt is similar to those of previous laboratory experiments and proportional to the power of 4/3 of the initial salt difference between both layers.

1. Introduction

Salt finger convection is one of the important processes of vertical heat and salt transports in the ocean, especially below the hot-salty water. In the upper layer of the world ocean, the vertical temperature and salinity distribution shows that about two thirds of the water mass have stratification favoring development of salt finger (Kulikov and Karlin [1995]). Thus, many researcher have intensively studied generation and development of salt fingers theoretically and experimentally. Numerical modeling of the finger was first carried out by Piacsek and Toomre[1980] (We call [PT] here after). Their numerical calculation agreed fairly well with previous theory and laboratory experiments for growth rate and horizontal scale of developing fingers. Whitfield et al.[1989] and Shen [1989,1991] also conducted similar numerical experiments. However, their calculations were restricted in 2-dimensional structure of salt fingers. Recent observation of asymmetric "salt fountains" by Osborn[1991] gave motivation for studying the 3-dimensionsl structure of fingers. By using a 3-dimensional linear theory, Schmitt [1994] found that there was rich variety of salt finger plan forms having the same growth rate, and one of these, an asymmetric finger, has a plan form consistent with Osborn's observation.

In this study, we investigate numerically 3-dimensional structure and development of fingers together with flux of heat and salt and the scale of fingers.

2. Numerical calculation

We used a recently developed software named α-FLOW, where the linearlized equation of state is used together with the ordinal fundamental momentum, heat and salt equations. As shown in Fig. 1, the domain of numerical calculation is a box of 5cm high with a rectangular section of 1.25cm x 1.25cm. As the grid points in the box are 64 x 64 x 64 , the grid sizes of horizontal and vertical directions are 0.2 mm and 0.8 mm, respectively. The upper (lower) half of the box is occupied by hot-salty (cold-fresh) water, and initial random disturbances are given just below the interface of these two fluids at first. Numerical calculations are carried out for three cases as shown in Table 1.

The case -1 has the initial temperature and salt differences, ΔT and ΔS, almost same as those of [PT]. The cases 2 and 3 have the values 8 and 16 times larger than that of the case -1. To elucidate the dependence of scale of finger and salt flux on the temperature (salt) difference of the two layers, these unrealistic large initial difference such as 70.4° C (4.8PSU) is used in case -3.

	TH	TL	SH	SL
CASE 1	14.4	10.0	34.3	34.0
CASE 2	45.2	10.0	36.4	34.0
CASE 3	80.4	10.0	38.8	34.0

Table 1 Initial value of temperature(T) and Salinity(S) in the upper(H) and lower layer(L)

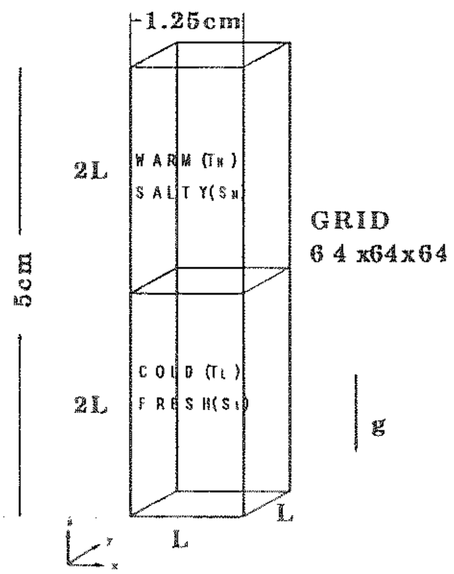


Fig. 1 Schematic view of the calculating domain

3. Results

Fig.2 shows the initial stage of salt concentration in the case-1 in which ΔS and ΔT are almost the same to those in [PT]. Small scale undulations are clearly found in and near the interface due to the given random initial disturbances.

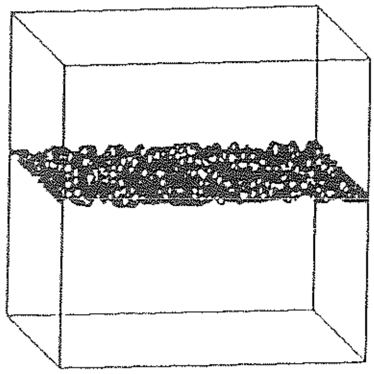


Fig. 2 Initial disturbance

After a while, however, salt fingers with preferable horizontal scale develop as shown in Fig. 3, and the top of fingers shows mushroom-like structure (Fig. 4). A pattern of salt concentration in the horizontal plane near the initial boundary of two fluids in Fig. 5(a) shows irregular form of salt fingers, which supports the possibility of rich variety of finger forms presented by Shmitt mentioned previously. Although, the preferable horizontal scale estimated by 2-D Fourier analysis is 3.72 mm, which corresponds to 1/3.4 of the width of domain. This result is almost consistent with that in the 2-D model by [PT], where the scale was nearly 1/4 of the width of the computational domain. Salt flux (Fs) increases with time and becomes almost constant at 60 sec after the beginning of convection as shown in Fig. 6.

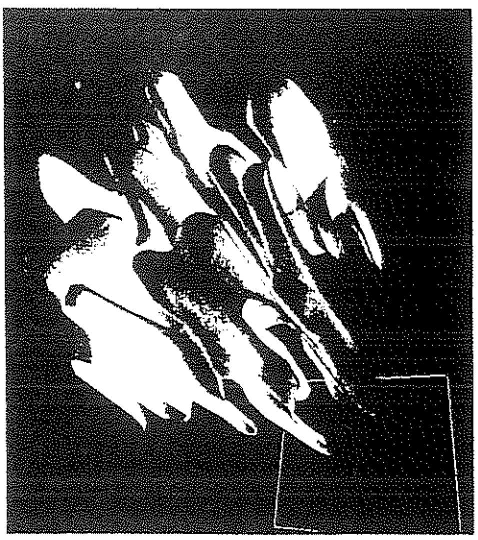


Fig. 3 Development of fingers

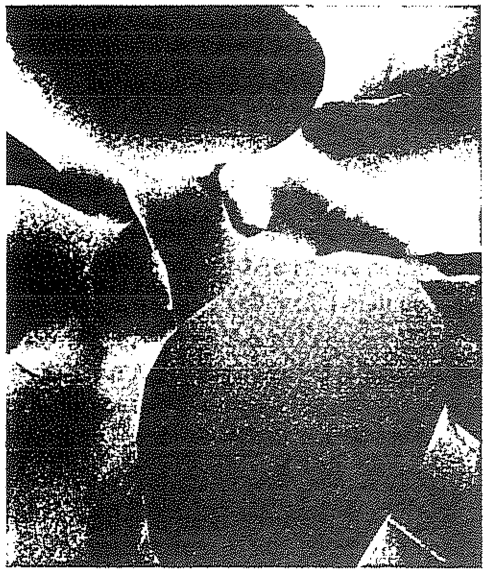
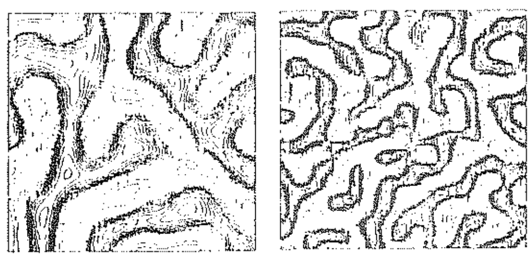


Fig. 4 Mushroom structure of top of fingers



a) b)

Fig. 5 Plan view of salt finger convection
a):case-1
b):case-3

In case-1, we succeeded to reproduce a salt finger convection, whose characteristics are similar to those by 2-D model except irregular horizontal pattern of fingers. Next, we investigate dependence of the horizontal scale of fingers on the initial temperature difference between two fluids.

Other two cases listed in Table 1 started numerical calculation for the initial temperature differences 8 times (case-2) and 16 times (case-3) larger than that of the case -1, respectively. Fig. 5(b) shows horizontal pattern of salt concentration, which scale seems to be smaller than that in case -1 in Fig. 5(a). The two dimensional Fourier analysis of this pattern gives the prevailing scale of 1.86 mm, which is a half of that in case-1. In Fig. 7, the horizontal scale, wave length of fingers in other words, is plotted against the initial temperature difference. The result shows that the wave length is proportional to the -1/4 power of the initial temperature difference ΔT, which is consistent with linear theory and laboratory experiments.

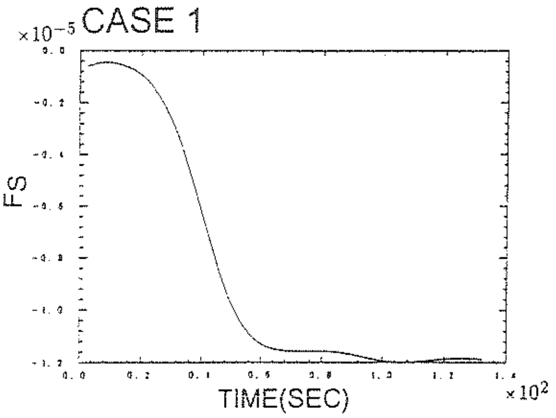


Fig. 6 Downward salt flux (MKS unit) increases with time and becomes almost constant at 60 sec after onset of fingers.

The vertical flux of salt and heat due to fingering convection has been investigated theoretically and experimentally. Stern (1969) derived the so-called 4/3 power law where salt flux (Fs) is proportional to the initial salt difference (DS). This flux law is elucidated by Linden 1973) in his laboratory experiment. The salt flux (Fs) estimated in the present study is listed in Table 2 together with the recent empirical formula. The results of the present 3-dimensional numerical experiment coincide with such an empirical formula.

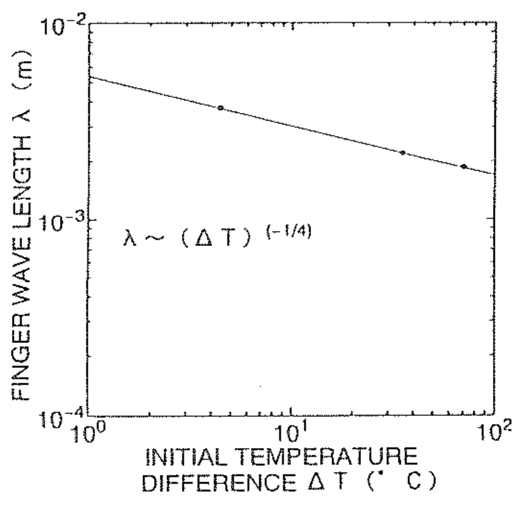


Fig. 7 Dependence of salt finger width on the initial temperature difference.

	Numerical Experiment	Laboratory Experimen'	Δ S
CASE 1	1.2×10^{-3}(1)	1.14×10^{-3}(1)	0.3
CASE 2	1.9×10^{-2}(15.8)	1.83×10^{-2}(16.0)	2.4
CASE 3	4.4×10^{-2}(36.7)	4.61×10^{-2}(40.4)	4.8

$$g\beta Fs = C_1 K_T^{1/3}(g\beta \Delta S)^{4/3}$$

$$(C_1=0.04 + 0.327R\rho^{-1.91} ; R\rho=\alpha \Delta T/\beta \Delta S)$$

Table 2 Vertical flux of salt (MKS unit)

4. Conclusion

We have studied the 3-dimensional development of salt finger convection by using recently developed software named α-FLOW. The results are as follows:(1) Salt finger convection is well reproduced. (2) Horizontal scale of the salt finger is proportional to the power of -1/4 of the temperature difference between upper and lower layers. (3) Spatial patterns of fingers in the horizontal plane are highly irregular, which supports the possibility of rich variety of plan forms presented by Shumitt. (4) Vertical flux of salt is consistent with those of previous laboratory experiments and proportional to the power of 4/3 of the initial salt difference between both layers.

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