

# A Comparison of Algorithms for Streamflow Recession and Baseflow Separation

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**Abstract** Vertical plane models for outflow of groundwater into a stream suggest that the form of the storage-discharge relationship changes from linear for a confined aquifer to quadratic for an unconfined flow. Tests of the form of streamflow recessions in 11 streams, during periods of no recharge, show that for most catchments the storage-discharge relationship is much more strongly nonlinear. For recessions of duration up to about 10 days, the linear model remains a very good approximation, using a biased value of the groundwater turnover time. However, estimates from the stream hydrograph of recharge during a storm event are very sensitive to the form of the storage-discharge relationship. The results of this study also show great variability in the parameters of the recession algorithm from one recession to another, attributable to spatial variability in groundwater recharge. An extension of the linear model to 'leaky' catchments, where the recession reaches zero flow, has been tested on two data sets. The second part of the paper deals with algorithms for base flow during surface runoff events - the problem of hydrograph separation. Algorithms with one, two and three parameters have been compared, using data for the same 11 streams, and the results show significant differences in the baseflow index (BFI) predicted for some catchments. The two-parameter algorithm, which is fitted subjectively, is more consistent in providing plausible results than either the one- and three-parameter algorithms, both of which can be fitted objectively.

## 1. INTRODUCTION

The characteristics of flow in perennial streams during extended dry spells have long been recognised as different from those experienced during and following storm rainfall. Since the development of modern hydrology, these differences have been studied in contrasting ways by engineering hydrologists, systems analysts and scientific hydrologists.

The engineering hydrology approach has resulted from the need to develop models for the flood runoff from storm events. Here the underlying dry weather runoff (or 'base flow') is subtracted from the total stream flow, and the difference (called 'direct runoff' or 'storm runoff') is related to the causative rainfall. The base flow is generally regarded as due to groundwater discharging into the stream, while the direct runoff is considered to result from overland or near surface flow.

In the systems analysis approach, an attempt is made to develop a rainfall-runoff model from the time series of rainfall and stream flow. Jakeman and Hornberger [1993] have shown that, after applying a nonlinear loss function to the rainfall data, the response of a wide range of catchments is well represented by a linear model with two components, interpreted as defining a 'quick flow' and 'slow flow' response to the filtered rainfall.

Scientific hydrologists, interested in understanding the processes by which rainfall becomes stream flow, have used tracers to determine the time-varying proportion of 'old flow' and 'new flow' in the stream hydrograph. Here the old flow is identified as being water which was already in the catchment before the start of rainfall, while the new flow has the same quality characteristics as the incoming rainfall.

Thus it would be expected that the old flow would result from subsurface pathways, while the new flow would be identified with surface runoff. However, a result of this work has been the recognition that the old flow has many of the characteristics of quick flow, and although it can be modelled by algorithms used for base flow separation, selection of parameter values requires experimental data from tracer experiments [Chapman and Maxwell, 1996].

Linear models underlie both the usual engineering equations for base flow recession during non-recharge periods and the systems analysis of slow flow. The first part of this paper explores the extent to which the assumption of linearity is valid. An extension of the linear model is then applied to the case of a leaky catchment, where stream base flow may be zero in extended dry periods.

The second part of the paper deals with the behaviour of base flow during a storm runoff event, and attempts to find some common ground between the separation algorithms used by engineering hydrologists and the hydrographs of slow flow obtained by the systems analysis approach.

## 2. MODEL DEVELOPMENT

### 2.1 Recession Curves

The most used equation for base flow during a non-recharge period is

$$Q_t = Q_0 e^{-t/\tau} = Q_0 k^t \quad (1)$$

where  $Q_t$ ,  $Q_0$  are the flows at times 0 and  $t$ ,  $\tau$  is the turnover time of the groundwater storage, and  $k$  is the recession constant for the selected time units. The first, and physically more meaningful form of (1) was first put forward by Maillet [1905], and the second form, more popular with engineering hydrologists, by Barnes [1939].

Eqn (1) is readily shown to be the result of a linear storage, in which the groundwater storage  $S$  is related to the stream flow  $Q$  by

$$Q = S / \tau = a S \quad (2a)$$

where  $a = 1/\tau$ . Linear behaviour of groundwater would be expected *a priori* from the Darcy equation, and Werner and Sundquist [1951] showed that (1) can be derived from the equation for one-dimensional flow in a confined aquifer.

However, the depth of groundwater flow varies in the more usual case of unconfined groundwater flow to a stream, and the vertical plane analysis by Werner and Sundquist [1951], for the case where the stream bed intersects impermeable bedrock, was shown by Chapman [1963] to infer that the flow would be proportional to the square of the volume of groundwater storage. These results can be generalised into the nonlinear relationship [Coutagne, 1948]

$$Q = a S^n \quad (2b)$$

which results in the recession equation

$$Q_t = Q_0 \{1 + (n-1)t/\tau_0\}^{-n/(n-1)} \quad (3)$$

where  $\tau_0 = S_0/Q_0$  is now the turnover time at time 0. Eqn (1) is the special case of (3) when  $n=1$ . Boughton [1986] obtained a value of  $n = 1.78$  for a small catchment in Queensland, while Padilla et al [1994] documented three karst springs with  $n$  equal to 1.5 or 2.

## 2.2 Extension to Leaky Catchments

The above equations are based on the assumption that all the groundwater storage in the catchment is released as stream flow recorded at the gauging station. This is not necessarily the case, and it is proposed that the appropriate model for a leaky catchment could be as shown in Figure 1, where a linear groundwater storage feeding the stream overlies a storage which has its outflow outside the catchment boundaries.

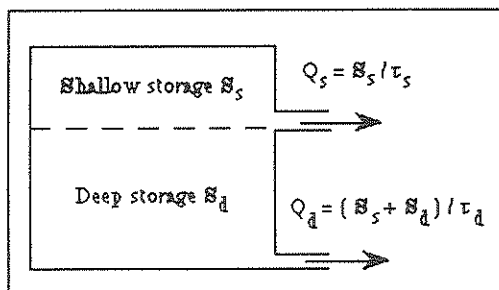


Figure 1. Linear storages model for a leaky catchment

While there is base flow,  $S_s > 0$ ; and  $S_d$  is constant. Under these conditions, it is readily shown that the recession equation takes the form

$$Q_t + q = (Q_0 + q) e^{-t/\tau^*} \quad (4)$$

where  $q = S_d / (\tau_s + \tau_d)$  and  $\tau^* = \tau_s \tau_d / (\tau_s + \tau_d)$ . The flow becomes zero at time

$$t = \tau^* \ln(1 + Q_0/q) \quad (5)$$

Figure 2 shows that there is little obvious departure from linearity of the graph of  $\log Q_t$  against  $t$ , for short recession curves of the form of Eqns (2) and (4).

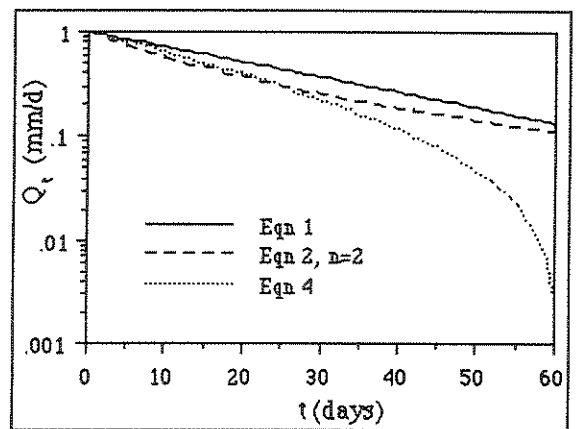


Figure 2. Comparison of Eqns (1), (2) and (4), for  $\tau = \tau_0 = \tau_s = 30$  d,  $\tau_d = 100$  d,  $S_d = 10$  mm.

## 2.3 Baseflow separation algorithms

Separation of base flow from a stream hydrograph starts with identifying the points at which the direct runoff starts and ends. The start point is readily identified as the time when the flow starts to increase, while the end point is usually taken as the time when a plot of  $\log Q$  against time becomes a straight line.

Having established the end points for the separation, a wide range of graphical techniques are available for defining the base flow between these points [Dickinson et al., 1967]. These techniques are inconvenient when separations are to be undertaken on a long continuous record of stream flow, rather than just a few storm period hydrographs, and this has led to the development of numerical algorithms, as described below.

### 2.3.1 The 1-parameter algorithm

Lyne and Hollick [1979] appear to have been the first to suggest the use of a digital filter, which has been used in Australia [Chapman, 1987; Nathan and McMahon, 1990; O'Loughlin et al., 1982] and overseas [Arnold et al., 1995]. Chapman [1990; 1991] showed that the form of the filter implied that the base flow would be constant when there was no direct runoff, and proposed a reformulation, which was subsequently simplified [Chapman and Maxwell, 1996] to a form which is based on the base flow being a simple

weighted average of the direct runoff and the base flow at the previous time interval, i.e.

$$Q_b(i) = k Q_b(i-1) + (1-k) Q_d(i) \quad (6)$$

where the algorithm parameter  $k$  is now seen as the recession constant during periods of no direct runoff. As the total stream flow  $Q$  is the sum of the base flow  $Q_b$  and direct runoff  $Q_d$ , the latter can be eliminated to give the recession algorithm

$$Q_b(i) = \frac{k}{2-k} Q_b(i-1) + \frac{1-k}{2-k} Q(i) \quad (7)$$

### 2.3.2 The Boughton 2-parameter algorithm

More flexibility is provided by introducing a second parameter  $C$  in place of  $(1-k)$  in (6). The separation algorithm then becomes

$$Q_b(i) = \frac{k}{1+C} Q_b(i-1) + \frac{C}{1+C} Q(i) \quad (8)$$

which is the algorithm used in the AWBM model developed by Boughton [1993].

### 2.3.3 The IHACRES 3-parameter algorithm

In the usually chosen form of the linear module of the IHACRES model [Jakeman and Hornberger, 1993], the rainfall excess  $u$  is partitioned into slow and quick components, here called  $Q_b$  and  $Q_d$ , by the equations

$$Q_b(i) = \beta_s u(i) - \alpha_s Q_b(i-1) \quad (9a)$$

$$Q_d(i) = \beta_q u(i) - \alpha_q Q_d(i-1) \quad (9b)$$

where the  $\alpha$ 's and  $\beta$ 's are parameters, and the suffixes  $q$  and  $s$  refer to quick and slow flow respectively.

Eliminating  $u$  from these equations, and expressing the direct runoff as the difference between the base flow and the total stream flow results in the following equation for base flow:

$$Q_b(i) = -\frac{\alpha_s \beta_q + \alpha_q \beta_s}{\beta_q + \beta_s} Q_b(i-1) + \frac{\beta_s}{\beta_q + \beta_s} \{Q(i) + \alpha_q Q(i-1)\}$$

It should be noted that the  $Q$  in this equation is strictly the modelled flow rather than the observed flow used in separation algorithms.

By putting  $C = \beta_s/\beta_q$  and  $k = -\alpha_s - \alpha_q \beta_s/\beta_q$  in the above equation, we obtain

$$Q_b(i) = \frac{k}{1+C} Q_b(i-1) + \frac{C}{1+C} \{Q(i) + \alpha_q Q(i-1)\} \quad (10)$$

which can be seen as an extension of (8), with an additional parameter. Given that  $\alpha_q < 0$ , (10) implies that the rate of change of base flow is positively linked to the rate of change of the stream flow.

### 2.3.4 The Base Flow Index

The base flow index (BFI) is defined as the long-term ratio of base flow to total stream flow. For the IHACRES model, it can be obtained by summation of the terms in (10) over a sufficiently long period that the difference between the initial and final flows is negligible relative to the other terms in the final expression. This gives

$$BFI = C(1 + \alpha_q)/(1 + C - k) \quad (11)$$

Putting  $\alpha_q = 0$  gives the corresponding result for the 2-parameter algorithm, and then putting  $C = 1-k$  gives the BFI for the one-parameter algorithm as 0.5. This clearly shows that the one-parameter algorithm can only work successfully when operated with the constraint that the modelled base flow must not exceed the observed stream flow. It is the extent of operation of this constraint which causes differences in the BFI obtained by this algorithm.

## 3. DATA

The data used in this study were stream flow records for the Queensland and NSW catchments in the data set prepared by Chiew and McMahon [1993]. The locations of the gauging stations are shown in Figure 3, and details of the catchments are given in Table 1.

The record length varied from 8 to 16 years, with very few missing data. Flows for the 24 h period up to 9 a.m. were used for the NSW catchments, and up to midnight for the Queensland sites. Daily flows in ML were converted to an equivalent depth in mm over the catchment. No base flows were identified for the Mitchell grass at Naradhan sites, and these were not examined further.

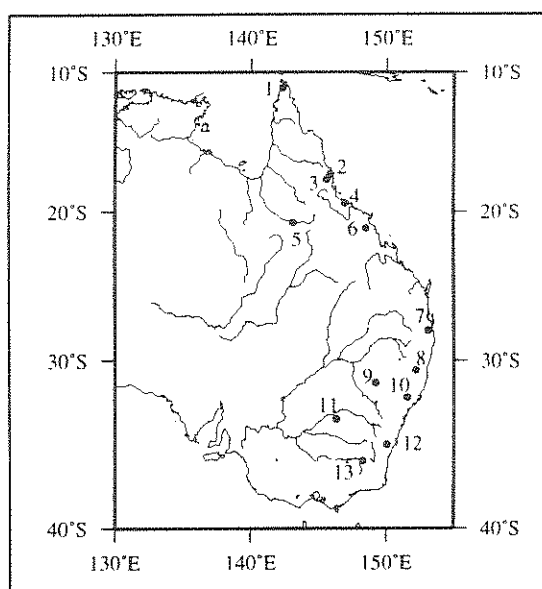


Figure 3. Location of study catchments

## 4. METHODOLOGY AND RESULTS

Computer programs were written to display plots of observed flow and modelled base flow (usually on a log scale) against time for any selected period.

Table 1. Some details of catchments used in study

Map Ref.	National No.	Catchment	Area (km <sup>2</sup> )	Mean rain (mm)
1	927001	Jardine R. at Telegraph Line	2500	1700
2	111105	Babinda Ck at the Boulders	39	5400
3	113004	Cochable Ck at Powerline	93	2400
4	118106	Alligator Ck at Allendale	69	1100
5	915001	Mitchell Grass at Richmond	3	450
6	120204	Broken R. at Crediton	41	2100
7	145103	Cainable Ck at Good Dam Site	41	900
8	206001	Styx R. at Jeogla	163	1300
9	420003	Belar Ck at Warkton	133	1100
10	210022	Allyn R. at Halton	215	1200
11	412093	Naradhan Ck at Naradhan	44	450
12	215004	Corang R. at Hockeys	166	800
13	401554	Tooma R. above Tooma Res.	114	1700

4.1 Baseflow recessions

Periods of base flow were identified as sections of the hydrograph, of at least 4 days' duration, which were apparently linear on the semilog plot. Each period was then fitted to Eqn (3), using as objective function the mean sum of squares of differences between the logs of the observed and modelled flows. This objective function gives equal weight to a given proportional error in the modelled flows. The value of  $Q_0$  in Eqn (3) was taken as a parameter to be optimised, in addition to  $\tau_0$  and  $n$ .

The response surface is very flat, resulting in a range of values of  $n$  which have little effect on the objective function. The results of these tests, summarised in Table 2, show values on  $n$  significantly greater than 1, and with one exception greater than 2. There appeared to be no relation between the value of  $n$  and the flow  $Q_0$  at the start of the recession.

Table 2. Summary of base flow recession characteristics

Catchment	Recessions		Parameter $n$		Other parameters		
	No.	Mean length (d)	Mean	SD	$Q_0$ (mm/d)	$\tau_0$ (d)	$S_0$ (mm)
Jardine	25	72.2	1.62	0.34	1.66	148.7	225.9
Babinda	119	9.1	2.61	0.48	10.48	45.6	383.2
Cochable	98	8.1	2.57	0.43	4.55	62.3	210.5
Alligator	52	8.0	2.46	0.63	1.69	30.7	37.0
Broken	41	9.3	2.91	0.58	3.70	51.6	134.2
Cainable	20	9.7	2.68	0.86	0.29	33.7	7.1
Styx	95	9.2	3.16	0.68	0.96	66.5	48.0
Allyn	65	7.5	3.07	0.84	0.37	46.6	15.6
Belar	57	8.5	2.87	0.82	0.28	37.9	6.1
Corang	86	9.1	3.22	0.75	0.32	41.6	10.7
Tooma	95	7.1	3.24	0.74	3.43	74.8	167.9

The exponential recession, Eqn (1), was also fitted to the data, using the same objective function (OF). The Student's t-test showed that the mean of this OF was significantly greater than that for fitting Eqn (3), at better than 90% level for 4 catchments, 95% for 2, and 99% for 5.

The mean value of the storage  $S_0$  at the beginning of each recession (Table 2) is shown in Figure 4 to be strongly related to the mean catchment rainfall for the period of

record. With one exception (Styx), values of  $S_0$  below 150 mm were associated with catchments that occasionally experienced zero flow.

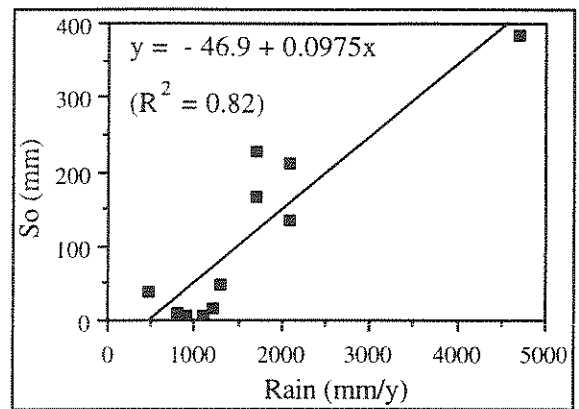


Figure 4. Relation between mean catchment storage at beginning of recession and mean annual rainfall

4.2 Tests of the leaky catchment model

Of the six catchments which had zero flow at some time, only two (Alligator and Corang) had recessions of sufficient duration for analysis by the leaky catchment model. Four recessions were identified for each catchment, and these were fitted to Eqn (4) by determining the value of  $q$  which gave the best fit to a straight line when  $Q_t + q$  was plotted against  $t$ . The value of  $\tau_s$  was obtained from the previously fitted exponential recession. The results (Table 3) suggest that the Alligator has a relatively small deep storage with a turnover time about the same as that of the storage feeding the baseflow recession, while the Corang has a very small deep storage with a relatively low turnover time. G. Pocock [1997, pers. comm.] states that the bed of the Alligator Creek at Allendale is of a sandy to light gravelly material, which would be in agreement with the calculated turnover time; the depth to bedrock is unknown, so that the plausibility of the estimated storage depth cannot be checked. I. Milroy [1997, pers. comm.] states that the gauging station on the Corang at Hockeys is on a rock bar, with a downstream pool; the results may therefore indicate leakage through fissures in the rock bar, but with negligible storage due to the underlying rock.

Table 3. Average parameter values for leaky catchment model

Catchment	$\tau_d$ (d)		$\tau_s$ (d)		$S_d$ (mm)	
	Mean	SD	Mean	SD	Mean	SD
Alligator	16.5	14.9	17.2	4.6	2.26	0.96
Corang	4.6	2.5	58.3	8.0	0.10	0.07

4.3 Baseflow separation

The one-parameter algorithm (Eqn 7) was fitted to the data by minimising the differences between the modelled and recorded flows during the periods of recession previously identified, using the same objective function as before. The fitting was generally done on three years of data, and the average values of  $k$  then applied to the whole data set. This had a minimal effect on the calculated value of the BFI.

This optimisation technique cannot be used for the two-parameter algorithm (Eqn 8), as the optimisation simply

increases the value of the parameter C until the modelled data coincide with the observed data. The fitting is therefore subjective, and Boughton [1994, pers. comm.] recommends basing the choice "mainly on the endings of surface runoff, particularly in the big runoff events". The values of the parameters and the BFI shown in Table 4 have been kindly provided by Dr Boughton.

For the same reasons, the three-parameter separation also cannot be optimised in the form of Eqn (10), but the IHACRES parameters and the BFI have been determined by Ye [1996], and the parameters converted to the form of Eqn (10) for display in Table 4.

Table 4. Baseflow separation parameters and baseflow index (BFI) for three algorithms

Catchment	Eqn (7)		Eqn (8)			Eqn (10)			
	k	BFI	k	C	BFI	k	C	$\alpha_q$	BFI
Jardine	0.993	0.48	0.990	0.019	0.63	1.145	0.197	-0.81	0.72
Babinda	0.971	0.46	0.975	0.069	0.62	0.975	0.033	-0.26	0.43
Cochable	0.987	0.45	0.975	0.047	0.60	*			
Alligator	0.953	0.38	0.965	0.021	0.30	*			
Broken	0.977	0.39	0.985	0.042	0.44	0.974	0.030	-0.01	0.53
Cainable	0.956	0.29	0.970	0.018	0.24	0.984	0.012	-0.02	0.43
Styx	0.981	0.43	0.980	0.066	0.60	0.976	0.026	-0.52	0.24
Allyn	0.976	0.30	0.950	0.033	0.28	0.989	0.011	-0.43	0.28
Belar	0.949	0.39	0.960	0.050	0.40	0.907	0.105	-0.17	0.44
Corang	0.972	0.32				**			
Tooma	0.971	0.49	0.970	0.085	0.70				

\* IHACRES model could not be fitted satisfactorily

\*\* No slow flow identified by the IHACRES model

The results show some large discrepancies between the estimated values of the BFI for the three algorithms, with no particular pattern emerging. Figure 5 illustrates the case where the one-parameter model reaches the total flow hydrograph well down the recession curve, resulting in an unrealistically low BFI for the Jardine catchment. In contrast, the slow flow from the IHACRES model is seen to have implausibly sharp peaks and rates of rise, and meets the total flow hydrograph before what would be usually regarded as the start of recession; the BFI is consequently high.

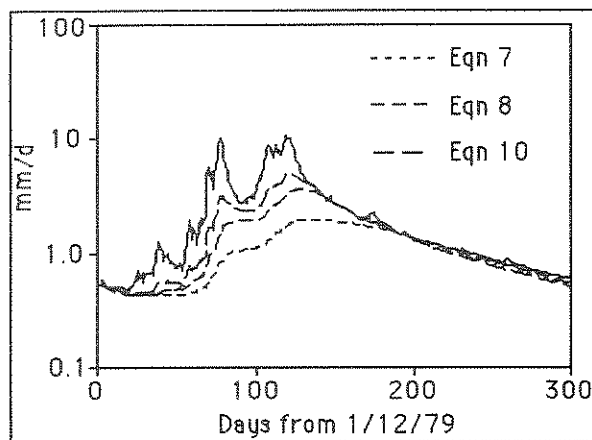


Figure 5. Comparison of separation algorithms for Jardine River at Telegraph Line

For the period on the Styx shown in Figure 6, the peaky nature of the IHACRES slow flow is again seen, and the predicted recession is well below the observed hydrograph, resulting in a low BFI for this catchment. Again, the

flexibility of the two-parameter algorithm appears to provide plausible results, particularly during the first peak event.

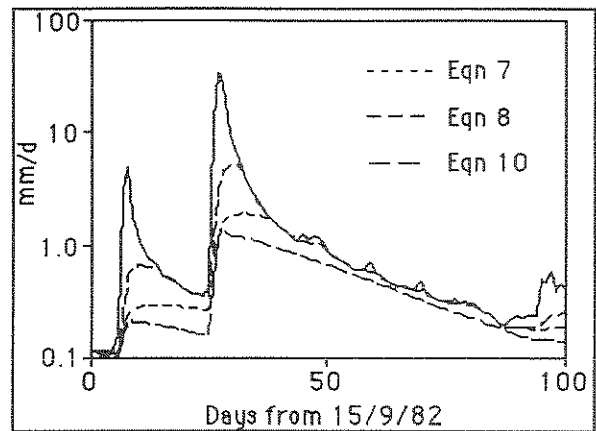


Figure 6. Comparison of separation algorithms for Styx River at Jeogla

## 5. DISCUSSION AND CONCLUSIONS

### 5.1 Baseflow recessions

With hindsight, the high values identified for the index  $n$  in the nonlinear recession model (Eqn 3) can be explained in terms of flow convergence in the groundwater system. When using the vertical plane model of Werner and Sundquist [1951], the value of  $n$  changes from 1 for confined flow with uniform flow depth, to 2 for unconfined flow in which the flow depth decreases downstream. It is reasonable then to expect  $n$  to increase further in the three-dimensional situation, due to the horizontal convergence of flow lines.

Even when the value of  $n$  is high, the linear storage model can still be used as a very good approximation in most cases. Noting that, with one exception, the mean duration of recessions in the study catchments is about 7-10 days, reference to Figure 3 shows that the recession can be fitted very well by Eqn (1), with the value of  $\tau$  slightly decreased. Only in the very long recessions of the Jardine catchment does the nonlinearity have significance in modelling, as can be seen in Figure 5 from the divergence between the modelled and observed stream flows after about day 200.

However, estimates of the recharge during a runoff event are very sensitive to the nonlinearity of the storage. Such estimates are based on the water balance, so that the recharge  $R$  is given by

$$R = S_2 - S_1 + \int_{t_1}^{t_2} Q_b dt \quad (12)$$

where  $S_1$  and  $S_2$  are the groundwater storages at times  $t_1$  and  $t_2$  respectively, before and after the period of surface runoff. For the event shown in Figure 7, the recharge calculated using the linear storage model was 9.5 mm, while the nonlinear storage gave a value of 19.8 mm. Such differences far outweigh any effects due to selecting a different separation algorithm (for the separation used in Figure 7, the base flow output during the period shown was 7.5 mm).

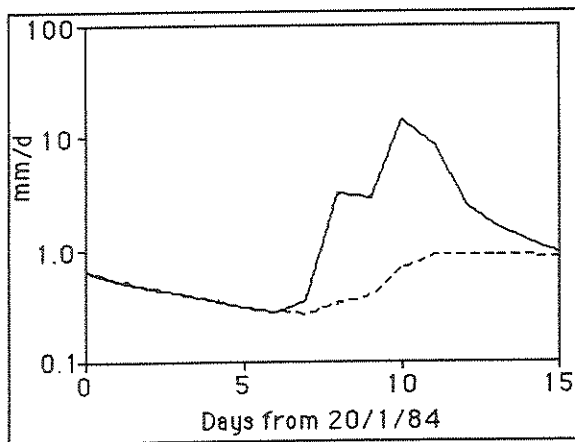


Figure 7. An event on the Allyn River at Halton, used for recharge estimation

Another difficulty in modelling baseflow recessions, irrespective of the linearity or otherwise of the model, is the variation in values of turnover time  $\tau$  from one event to the next, as evidenced by an average coefficient of variation of 0.61 for this parameter. It can be hypothesised that this is due to spatial variations in groundwater storage, occurring as a result of spatial variations in rainfall and subsequent recharge. There is a trend for  $\tau_0$  to decrease with an increase in the base flow  $Q_0$  at the start of a recession, but this explains only a small part of the variance in  $\tau_0$ .

The leaky catchment model requires testing and comparison with site conditions on more catchments, before its merits can be evaluated.

## 5.2 Baseflow separation

Of the algorithms tested, the two-parameter Boughton algorithm appears overall to be the most satisfactory, although suffering from the disadvantage that the parameter selection is subjective. However, the variations from one operator to another seem likely to have less effect on the BFI than the use of the objectively fitted one- and three-parameter algorithms, both of which can result in implausible separations on some catchments, with consequent high or low BFI values.

While one would expect a three-parameter algorithm to have considerable flexibility, the form of the algorithm derived from the IHACRES model results in sharp peaks in the slow flow hydrograph. As shown by Young and Jakeman [1996, pers. comm.], the unit hydrograph for the two-parameter model has a smooth peak, while that for the IHACRES model is structured to have a sharp peak at one time unit.

These algorithms are all based on the underlying concept of an exponential baseflow recession, and it would seem worthwhile to investigate whether algorithms based on a nonlinear storage-discharge relationship could prove useful.

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