Climate Change and the Regional Allocation of CO2 Emission Reductions

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Abstract We provide a comparison of two independent methodologies for the allocation of world greenhouse gas emission reductions on a regional level. Both methodologies make use of the solution of a 'Stage 1' world emission reduction problem. This exploits techniques of mathematical programming to produce an optimal emission reduction strategy for the world by maximizing an economic utility function. We accomplish the regional allocation of the optimal world emission reduction strategy, firstly, via the solution of an auxiliary optimization problem minimizing disruption from the so-called '*business as usual*' emission strategy. Secondly, regional allocations are made via an application of a game theoretic concept called the Shapley value. Here we view regions as players in a 'cooperative game' in which we try to answer the question: Given the characteristic function, what is an equitable share of the benefits/costs of cooperation to a particular player? The analysis illustrates that allocations of regional CO_2 emission reductions are not only methodology dependent but also sensitive to targets placed on global CO_2 concentrations. It is hoped that such analyses will help the international community narrow the range of environmental targets to one that might contain a realistically achievable emission reduction strategy.

1. Introduction

There is a growing concern that emissions of carbon dioxide (CO_2) and other radiatively active trace gases or greenhouse gases from human activities might cause an increase in the Earth's surface temperature, and change the Earth's climate through the enhanced greenhouse effect. This global change will not only manifest itself in altered patterns of surface temperature and precipitation, but might also cause impacts, such as droughts, sea level rise, shift in the growing zones for vegetation (both natural and agricultural), and changes in the supply of freshwater, etc. These diverse impacts could certainly induce great economic, social and political changes. Sea level rise may threaten densely populated low-lying countries as well as damage coastal areas and ecosystems. On the other hand climate change can also provide benefits to some countries, for example, increased rainfall or longer growing seasons in the present marginal areas for crop production will only be welcomed by the affected regions.

At first sight, it might appear that global environmental problems such as the greenhouse effect have no relation to the subjects of mathematical programming and/or game theory. However, the widely anticipated international agreements on the reductions in emissions of greenhouse gases will, undoubtedly, involve complex tradoffs among the signatories of such agreements. Prior to the signing of such a treaty, the participating countries will, presumably, have an understanding of a scheme that apportions shares of the cost of these reductions. This paper adopts the point of view that these agreements and subsequent cost allocations can, and perhaps, should, be regarded as a two-stage problem of the form:

Stage 1: Find $\mathbf{U}^0$, a world CO_2 emission strategy, that

$$\begin{array}{llll} & & \text{maximizes} & C(\mathbf{U}) \\ \text{s.t.} & & & \\ & \text{(i)} & e_i(\mathbf{U})=0 & i\in I \\ & \text{(ii)} & p_j(\mathbf{U})=0 & j\in J \\ & \text{(iii)} & g_l(\mathbf{U})=0 & l\in L \end{array}$$

where $C(\mathbf{U})$ is a world economic utility function (Nordhaus, 1994), $e_i(\mathbf{U})$, $p_j(\mathbf{U})$ and $g_l(\mathbf{U})$ represent economic dynamics, climate dynamics and the environmental target constraints respectively. Once an optimal $\mathbf{U}^0$ is obtained from above:

Stage 2: Find u_r, a regional CO_2 emission strategy (for each world region r) satisfying $\sum_r u_r = \mathbf{U}^0$.

We accomplish the regionalization via two independent methodologies based on mathematical programming and game theory respectively. Firstly, in the mathematical programming approach we find the solution of an auxiliary optimization problem:

$$\begin{array}{lll} & & \text{minimizes } D(u_1,u_2,\ldots,u_\rho) \\ \text{s.t.} & & \\ & \text{(i)} & \sum_r u_r = \mathbf{U}^0 \\ & \text{(ii)} & m_r u_r^A \le u_r \le M_r u_r^A, \\ & & \text{where } r=1,2,\ldots,\rho. \end{array}$$

where D is a function that captures the level of '*disruption*' caused by deviations from the so called *business as usual* (BaU) strategy $(u_1^A, u_2^A, \ldots, u_\rho^A)$. Constraint (i) provides consistency with international agreements and (ii) ensure that no world region is required to reduce its CO_2 emissions by an unrealistically large, or small amount.

Secondly, regionalization of emission reductions are accomplished via an application of a game theoretic concept known as the Shapley value[1]. In implementing the Shapley value we regard world regions as players who join an initially empty coalition one by one until the grand coalition I (a coalition containing all the players) is formed. Each coalition K (a subset of I) must know its *value* $\nu(K)$, representing the optimal benefit/cost which players in K can guarantee for themselves no matter what the remaining players do. The Shapley value assigns to each player a share in the benefit/cost reflecting this accounting. More precisely,

$$S_r = \sum_{K \ni r} \frac{(N-|K|)!(|K|-1)!}{N!} \left[\nu(K) - \nu(K \backslash r)\right] \tag{1}$$

which defines a vector

$$S = (S_1, S_2, \ldots, S_r).$$

However, this allocation need not be stable, in the sense that certain coalitions of players could, perhaps, do better for themselves by breaking away from the grand coalition I. If that is the case, then for the Shapley value to be implemented there either must be a general acceptance of the rationale behind it, or a central authority capable of enforcing that allocation.

2. The Model

2.1 OMEGA+ Environmental Optimizer

In Filar et al (1995), a methodology involving a two stage optimization process is developed using the coupled economic/global climate model OMEGA+ (Optimization Model for Economic and Greenhouse Assessment, Filar et al, 1995 and Janssen, 1995). The OMEGA+ environmental optimizer (Figure 1) couples the prominent DICE (Dynamic Integrated model of Climate and Economy) climate and economy model (Nordhaus, 1994) with that of a mathematical system (Braddock et al, 1994 and Zapert, 1994) extracting the core of the global climate model IMAGE 1.0 (Integrated Model to Assess the Greenhouse Effect, Rotmans 1990). DICE is primarily an economic optimization model with highly simplified climate dynamics. The model calculates optimal trajectories for both *capital accumulation* and *greenhouse gas emission reductions* by maximizing a discounted value of 'utility' or satisfaction from consumption subject to economic and geophysical constraints. The integrated modular system of IMAGE 1.0 incorporates relatively simple modules of the main components of the global climate system.

The OMEGA+ two stage environmental optimizer solves, in Stage 1, a *world emission reduction problem* producing an optimal emission reduction strategy for the world by maximizing an economic utility function. Stage 2, addresses a *regional emission reduction allocation problem* via the solution of an auxiliary optimization problem minimizing disruption from the so-called business as usual emission scenario[2]. Regional allocations are made on a four region basis: OECD, former USSR and Eastern Europe, China and centrally planned Asia, and the Rest of the World (RoW). The 145 countries comprising these regions are those with a population of at least one million or a GNP of at least US $1 bn (Economist Books, 1990). To simplify equations we will denote the four regions as region 1, 2, 3 and 4 respectively.

[1] A Shapley value solution determines a unique disbursement of the benefits/costs solely by the use of a characteristic function of the game (eg., see Owen, 1968).

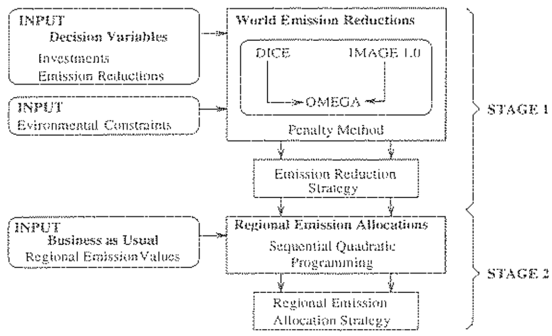

Figure 1: The OMEGA+ Environmental Optimizer

2.2 Cooperative Game Approach

In Filar and Gaertner 1995, the solution to the above Stage 2 problem is considered as a '*cooperative game*' (Jones, 1980) whose solution is based on the Shapley value concept. In a cooperative game we are particularly interested in the formation of coalitions between players. This is because economic costs are assumed to be in monetary form and that coalitions will act to minimize their joint cost. Hence, given a *characteristic function*[3], we try to answer the question: What is an equitable share of the benefits/costs of cooperation to any particular player? The outcome of a cooperative game is viewed as a distribution of the total available benefits/costs amongst the various players. A possible distribution of available benefits/costs is called an *imputation*. An imputation must satisfy:

$$\mathbf{x} = (x_1, x_2, \ldots, x_N) \tag{2}$$

s.t.

(i)	$x_i \leq \nu(i)$ for all i	individual rationality
(ii)	$\sum_{i=1}^{N} x_i = \nu(I)$	collective rationality.

Ideally, the final outcome of a cooperative game would be a unique imputation arrived at via a bargaining procedure conducted by players who are motivated by maximizing their individual utilities. The Shapley value solution suggests one rational way of uniquely dividing the benefit/cost among the players.

[2] In the business as usual emission scenario economic growth is based on an increase in fossil fuel consumption, especially coal because of the higher prices of decreasing oil and gas resources. In this scenario environmental concerns neither alter ways of life nor lead to any substantial effort to reduce greenhouse gas emissions.

[3] A characteristic function is simply a function defining the worth, $\nu(K)$ of any particular coalition K, as defined earlier.

3. World Emissions Reduction Problem

Stage 1 of the OMEGA+ model poses the following world emission reduction optimization problem with decision variables $\mathbf{z}(t) = (z_1(t), z_2(t))$, where $z_1(t)$ defines the *rate of CO_2 emission reductions* at time t and $z_2(t)$ defines *rate of investment in tangible capital* at time t. Following Nordhaus (1994) we assume that the levels of decision variables change linearly over fixed time intervals of ten years.

$$\begin{aligned} &\max f(\mathbf{z}) \\ \text{s.t.} \quad & \\ &\Phi_t(\mathbf{z}) \leq \rho\mathrm{CO}_2 \qquad t = 1990, 2010, \ldots 2100 \\ &0 \leq \mathbf{z} \leq 1 \end{aligned}$$

where $f(\mathbf{z})$ defines a world economic utility function (Filar et al, 1995), $\Phi_t(\mathbf{z})$ define environmental target constraints at which we fix the limits on atmospheric CO_2 concentration, ρCO_2.

This stage is solved utilizing the penalty method and the Powell direction set method (Press et al, 1988 and Brent, 1973) for a time horizon that extends to the year 2100. However, we generally do not present model results for time periods beyond the year 2080 so as to avoid reporting results that may be unduly distorted by the existence of an artificial model termination date.

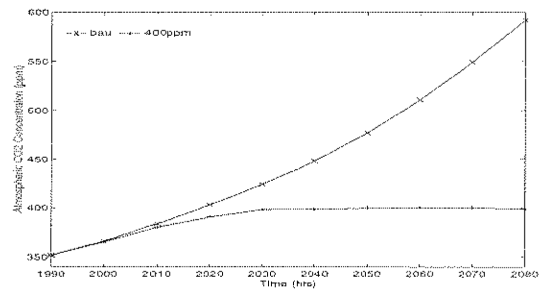

Figure 2: Atmosphere CO_2 Concentration Targets

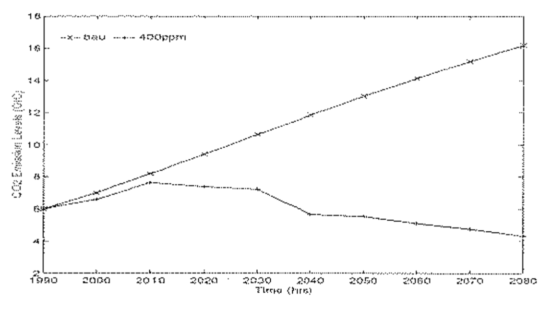

Figure 3: Atmosphere CO_2 Emission Targets

Figure (3) shows world CO_2 emission levels that need to be adhered to in order to maximize economic growth while still achieving our environmental target of 400 ppm (parts per million), see Figure (2). It is worth mentioning that the results are sensitive to this target. In Filar et al (1995), we show that considerably smaller emission reductions need to be made when the target is relaxed to 500 ppm.

4. Regional Allocations via Global Optimization

The optimization approach to the Stage 2 regional allocation problem is based on the solution of an auxiliary optimization problem, minimizing reductions from regional business as usual emission levels subject to these reductions being consistent with the strategy $\mathbf{U}^0(\cdot)$ found in Stage 1, and regional practical constraints. Decomposing Stage 1 results into regional values $u_r(t)$ yields a minimization problem:

$$\min \sum_t \sum_r \omega_r (u_r^A(t) - u_r(t))^2$$

s.t.

$$\begin{aligned} &\text{(i)} \ \sum_r u_r = \mathbf{U}^0 && \text{for all } t \qquad (3) \\ &\text{(ii)} \ m_r u_r^A \leq u_r \leq M_r u_r^A && \text{for all } r, t \end{aligned}$$

where (ii) consists of 'practical' constraints such as: permitting a region to emit at least $m_r\%$, but no more than $M_r\%$[4] of its business as usual scenario emissions. We define ω_r to be the weight associated with region r, and $u_r^A(t)$ to be the regional emissions due to the business as usual scenario (IPCC, 1992) in year t. Note that (3) will be a convex program in the decision variables $u_r(t)$ and hence a global minimum should be computable, and provide '*minimum disturbance*' emissions for each region, subject to the total being the '*optimal*' emissions from Stage 1. Weights on time periods are also possible. Because of the availability of data the decision times in the Stage 2 problem will be taken at ten year intervals.

The constrained minimization of Stage 2 was performed using the constrained optimization toolbox within MATLAB 4.2 (MATLAB, 1992). The solution methodology is based on the solution of the Kuhn-Tucker (KT) equations, which are necessary and sufficient conditions for optimality in a convex program. For a comprehensive overview of global optimization see Horst and Pardalos, 1995. The results of this analysis are shown in Figure (4) below.

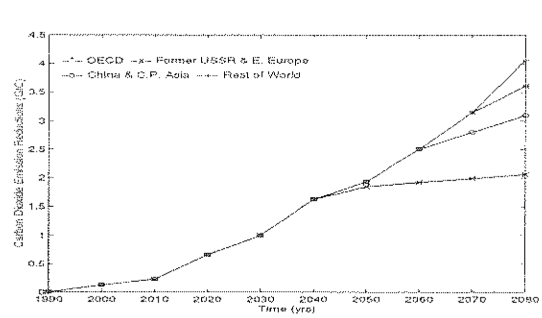

Figure 4: Optimization Experiments

In the above experiments the upper limit on the practical constraint (ii) of the formulation in (3) was set at $M_r = 1$, for the business as usual (BaU) scenario. Thus no region was allowed to emit levels of

[4] It is possible that M_r could be more than 100% thus allowing region r's CO_2 emissions to exceed those of the business as usual scenario.

CO_2 above their business as usual scenario. The lower limit m_r was set to the highest value below M_r that provided a feasible solution. Consequently, the solution will provide the minimum disruption from the business as usual scenario for each region and satisfy the environmental target conditions. The regions were all given equal importance, thus regional weights were $\omega_r = 1$ for $r = 1, \ldots, 4$. Notice, that until 2040 all four regions have nearly the same emission reductions. Of course, this does not mean emission reductions are uniform over all regions, since a reduction of 1 GtC (gigatons of carbon) of CO_2 in region 1 (OECD) would not mean the same as a 1 GtC reduction in region 4 (Rest of the World). Note also that by the year 2080 the reductions demanded from OECD and the Rest of the World are almost twice those required from the former USSR & Eastern Europe.

5. Regional Allocations via Shapley Value

The Shapley value solution to the allocation problem, besides determining a unique disbursement of the benefits/costs solely by the characteristic function of the game, has built into it a certain equity principle. This solution might therefore be a strong contender for the status of a 'normative' solution, ie, one which 'rational players' ought to accept.

We begin constructing the characteristic function by introducing the notion of a *flow* $f_{i,j}$ from region i to region j which is intended to capture the degree of relative dependence (or 'fear') of i on j. Thus we want $f_{i,j} \in [0,1]$ and $\sum_{j \neq i} f_{i,j} = 1$ implying that the sum of these dependencies over all regions excluding i itself is one. While there may be many ways of deriving such dependencies, for illustration purposes we propose to use:

$$f_{i,j} = \frac{GNP_j}{\sum_{k \neq i} GNP_k} \quad \text{for } i = 1, \ldots, 4. \tag{4}$$

Thus, rather crudely, the above suggests that the dependence of i on j is proportional to j's GNP, however, the importance of that dependence is properly reflected by the ratio of j's GNP to the sum of the GNP's of all the regions other than i. This is because if, hypothetically, j refused to trade with i, that loss could be compensated by trade with other regions.

In our four region network (Figure 5), the flows are computed using recent GNP data (Table 1) obtained from the World Bank, (1990).

World Region	Total Exports (US \$ bn)	GNP (US \$ bn)
OECD (1)	8651.8552	16367.3994
USSR & E.E. (2)	187.4203	748.3501
China & C.P. Asia (3)	392.3726	700.7378
ROW (4)	1789.5002	2446.5769
Total	11021.1484	20263.0643

Table 1: Regional Exports and GNP

We wish to combine the interdependency notion modelled with $f_{i,j}$'s with the notion of 'weights' or

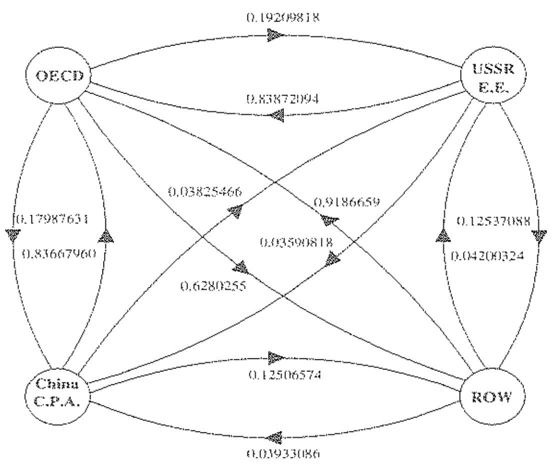

Figure 5: Four Region World Network

'trade strengths' E so that the network takes on a more realistic appearance[5]. In this paper we take these strengths to be the total exports (in US \$ bn) for each of the four regions, thus

$$E = \begin{bmatrix} (1) & 8651.8552 \\ (2) & 187.4203 \\ (3) & 392.3726 \\ (4) & 1789.5002 \end{bmatrix}$$

Of course these weights are only used here as a demonstration. The exact mechanism for the calculation of regional weights or trade strengths as well as how they can be accurately applied can undoubtedly be the subject of some discussion.

The worth of a coalition is calculated by creating a cut in the network. A *cut* is defined to be the set of directed arcs containing their 'tail' in K and 'head' in K^c (complement of K). Incorporating the notion of weights or trade strengths, we define the value or worth of a coalition by:

$$\nu(K) := \frac{\sum_{i \in K} E_i \left(\sum_{j \in K^c} f_{i,j} \right)}{\sum_{i \in K} GNP_i}. \tag{5}$$

Thus $\nu(K)$ is a measure of the amount of export that coalition K might lose if K^c were against them. We view the value of a coalition as a 'cost' as it represents a measure of the amount the coalition stands to lose. The value of the grand coalition I (the whole world cooperating) must be zero, since $I^c = \{\}$ and hence poses no threat. The value of the null coalition $\nu(\phi)$ must also be zero, since the players are not obligated to 'throw away' any portion of their benefits/costs, hence

$$\nu(I) = \nu(\phi) = 0. \tag{6}$$

The value of all other coalitions is given in Table (2).

[5] The precise data for the amount of export from each of the four regions to each of the other regions were unavailable at the time of writing the paper.

Single Player	Double Player	Triple Player
$\nu(1) = 0.5286$	$\nu(1,2) = 0.4102$	$\nu(1,2,3) = 0.3090$
$\nu(2) = 0.2504$	$\nu(1,3) = 0.4195$	$\nu(1,2,4) = 0.0835$
$\nu(3) = 0.5599$	$\nu(1,4) = 0.1788$	$\nu(1,3,4) = 0.0898$
$\nu(4) = 0.7314$	$\nu(2,3) = 0.3851$	$\nu(2,3,4) = 0.5466$
	$\nu(2,4) = 0.5879$	
	$\nu(3,4) = 0.6553$	

Table 2: Values of Regional Coalitions

Applying equation (1) we obtain the following Shapley values for each of the four world regions in the equivalent '0 – 1 transformed' game (see Filar and Gaertner, 1995 and Jones, 1980):

$$\hat{S}_1 = 0.185544 \quad \hat{S}_3 = 0.316629$$
$$\hat{S}_2 = 0.110126 \quad \hat{S}_4 = 0.387702.$$

These values can now be used to allocate the emission reductions required by the solution of the Stage 1 problem.

If we define $\delta(t)$ to be the difference between the business as usual emission scenario $U^A(t)$ and the Stage 1 emissions $U^0(t)$, then $\delta(t)$ is the sum of all the regional adjustments, therefore[6]

$$\delta(t) = U^A(t) - U^0(t). \tag{7}$$

The higher the transformed Shapley value the greater the reduction and thus penalty to that region. Note that $\delta(t)$ represent the total reduction in CO_2 emissions required at time t to achieve the environmental target of 400 ppm. Then $\delta_r(t)$, the regional reductions, can be found by portioning $\delta(t)$ among the regions according to the transform Shapley values

$$\delta_r(t) = \hat{S}_r\delta(t). \tag{8}$$

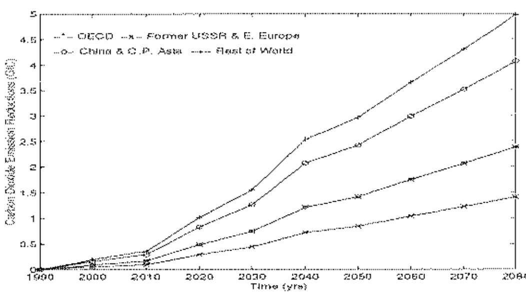

Figure 6: Shapley Value Experiments

Figure (6) displays regional levels of CO_2 emission reductions (according to the Shapley allocation) that must not be exceeded in order to achieve the environmental target of 400ppm on CO_2 concentration. From Figure (6) we see that the former USSR and Eastern Europe and the OECD (regions which arguably have contributed most to the greenhouse problem) are required to reduce their emission levels the least.

[6]It may be possible for $\delta_r(t)$ to be negative. This would mean that the region under consideration is allowed to increase their emissions for the indicated period. This occurrence may happen when looking at third world countries.

Of course, this fact may seem unreasonable, as it indicates that the two regions largely responsible for the increase in atmospheric CO_2 concentration, have much lower reductions placed on them than other regions. However, we have based our characteristic function on regional GNPs, exports and the ratio of exports to GNP, consequently, regions that are largely self-sufficient (regions having a low export to GNP ratio, eg. regions 1 & 2) would also be under less threat from others if they chose not to abide by the emission reductions agreements. This explains the low share of the burden assigned to the former USSR & Eastern Europe. In the case of OECD, its low share of the burden is due to the economic threat it can exert on others.

6. Comparison of Methodologies

Figures (7) – (10) show comparisons between the optimization and Shapley value approaches to the regional emission reduction problem.

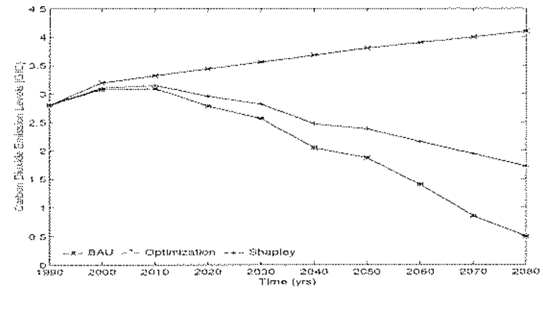

Figure 7: OECD

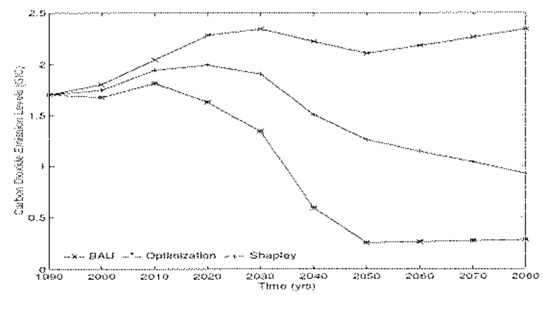

Figure 8: Former USSR & Eastern Europe

Perhaps, the most significant finding of the preliminary analysis is the demonstration of how sensi-

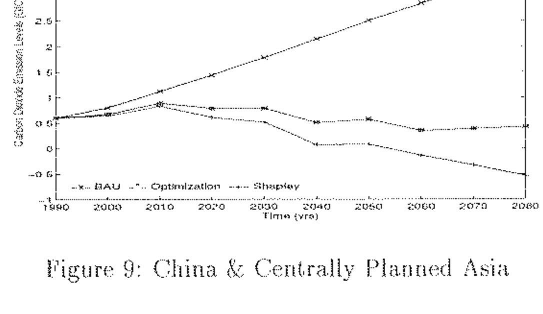

Figure 9: China & Centrally Planned Asia

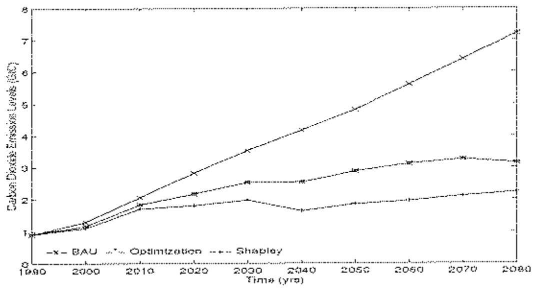

Figure 10: Rest of the World

tive the optimal emission reduction strategies to the philosophy underlying the allocation scheme. For instance, the optimization allocations with the target of ρCO_2 = 400 ppm require emission reductions in the year 2020 of 19.0% for OECD, 28.7% for the former USSR and Eastern Europe, 45.5% for China and centrally planned Asia and 23.2% for the Rest of the World. The Shapley value allocations require emission reductions in the year 2020 of 14.1% for the OECD, 12.7% for the former USSR and Eastern Europe, 57.6% for China and centrally planned Asia and 36.0% for the Rest of the World. Thus the OECD and the former USSR and Eastern Europe are better off under a Shapley allocation then under an optimization scheme.

7. Conclusions

Ultimately, complex tradeoffs will have to be made concerning the selection and extent of industries to be regulated in order to achieve (internationally agreed upon) emission reductions. We do not advocate either of the allocation schemes discussed here, we merely point out how a philosophical rationale underlying a scheme can be converted to the required emission reductions. Optimal control models, or models based on game theory could be extremely useful in determining the 'best' reductions under a number of different rationales and thus can provide useful tools for the environmental policy analyst. Equally important, is that we may have demonstrated that solution methodologies play an important part in determining allocations of emission reductions.

8. Acknowledgments

The research undertaken in this paper was supported, in part, by the World Resources Institute (WRI), Washington D.C. However, the views expressed in this paper are those of the authors and do not, necessarily, represent those of the World Resources Institute. The authors would also like to thank Radoslaw Zapert and Marco Janssen for their useful comments and help in obtaining data.

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