Simulation of Chaotic Behavior in Lotka-Volterra Model

Choi, Chang-Hyeon
Dept. of Public Administration
College of Law & Political Science
Kwandong University
522, Naegok-dong
Kangnung City, Korea
ZIP: 210-701
TEL: (0391) 41-0671
FAX: (0391) 41-1010

Abstract The idea of applying the natural scientific evolutionary, non-equilibrium, self-organizing, or chaos theory initiated by Prigogine and others to social systems at large is nowadays widely spreading. Newtonian paradigm tends to focus on linear relationships, causal relationships, and equilibrium, or order and stability of the system, while chaos theory diverts attention to nonlinear relationships, feedback loops, and non-equilibrium, or chaos and unstability of the system. In terms of nonequilibrium chaos theory, an organization is destined to disorder or death, since a thermodynamic equilibrium, by definition, means a disorder. All evolving systems has the capability to reconfigure the existing structure into a dynamic new order through fluctuations. This implies that reordering of the constituent components (in the case of an organization, reengineering, restructuring, technological innovation, or holographic design, etc.) must be encouraged to create a dynamic new order, if an organization is in a state of bifurcation point, far-from-equilibrium, or chaos. System dynamic approach using "Dynamo" was used to simualte L-V model. The implications of using simulation in the analysis of chaotic behavior are presented.

1. INTRODUCTION: WHAT IS A CHAOS THEORY?

The idea of applying the natural scientific evolutionary, non-equilibrium, self-organizing chaos theory initiated by Prigogine and others to social systems at large is nowadays widely spreading [Loye &Riane,1987]. While research into chaos theory in such areas as chemical, physical, and biological sciences has made a significant progress during the last decade, scientific study of chaos is relatively new in the social sciences [Gleick,1987].

Social scientists have attepmted in vain to explain and predict the social phenomenon and particularly the behavior of the social system, with the unsatisfactory result that they were not so successful in terms of the accuracy of the prediction that they started to look into chaos theory. There might be several reasons why their predictions are not so accurate. Even if a social system such as individuals, groups, or organizations are faced with the same initial internal state and the same environment, and are governed by the same causal relations, the system has the potential to exibit a totally different behaviors. This use of the word 'chaos' denotes something quite distinct from other causes of error in empirical studies, such as randomness, exogenous variables, and measurement error. Addtionally, as used here, chaos does not imply antisocial or psychopathic meanings of the word [Gregersen & Sailer,1993]. In chaos theory, chaos means a deterministic chaos.

The Newtonian mechanical paradigm states that the behavior of a system can be predicted by identifying its parts and the cause-effect relationships among them (and this is the very principle of atomism and mechanism). This crude assumption leads to a set of differential equations that rules the behavior of the system. However, real systems evolve, that is to say, they interact in the feedback loops over time, and deterministic model does not reflect this. Thus evolution must result from what has been removed in the reduction process [Allen,1988]. This might be inevitable if we stick to a methodological reductionism. In this paper, a system dynamic approach using "Dynamo" was used to simualte the chaotic behavior of Lotka-Volterra model. L-V model in population ecology theory was simulated in that it is a nonlinear model, and has feedback loops. The implications of using simulation in the analysis of chaotic behavior are presented. [Andersen,1988].

2. THE CHARACTERISTICS OF CHAOS

Newtonian paradigm tends to focus on linear relationships, causal relationships, and equilibrium, or order and stability of the system, while chaos theory diverts attention to nonlinear relationships, feedback loops, and non-equilibrium, or chaos and unstability of the system [Prigogine & Stengers,1984; Davies,1988; Pagels,1988; Nicolis & Prigogine,1977]. It should be noted that a stable equilibrium state or an unstable chaotic state is just temporal states in the evolutionary system.

In this section, such characteristics of chaos theory as nonlinearity, feedback loop, sensitive dependency on intial conditions, and the resulting nonequilbrium chaotic system is contrasted with Newtonian paradigm, and simulated using Dynamo.

The best way to understand the dynamics of NDS is to compare the behavior of such systems with those of linear dynamic systems (LDS). In linear systems, the relationships among relevant variables remain stable over time, which means that the dynamics of the linear systems will typically show smooth and regular behavior. Linear systems respond to the changes in the parameters, or to external shocks, in a proportionate and consistent manner. On the other hand, NDS is typified by the dynamic relationships among variables. As these relationships change, the temporal behavior of the system might change from smooth and regular to unstable and irregular and even up to the point of seemingly random, referred to as chaotic state [Kiel,1993]. While it's true that some balance should be struck between mathematical tractability and reality of the system, it must be recognized that the presence of nonlinearity is often the reason for the chaotic behavior of the system [Beaumont,1982].

Theory-testing not only in natural sciences but also in social sciences is focused on oneway causality between predictor variables and predicted ones. Rare exception is the case of LISREL which tests reciprocal causality [Choi,1992]. Nevertheless LISREL still is unable to test feedback loops model. In the field of organization theory, only a few feedback models have been proposed. Natural selection theories are dynamic. One explains the pattern of variations observable at one point in time through reference to a theory which considers the time path of some set of variables... The dynamic quality of natural selection theories focuses attention on the speed with which various processes occur and the lag structures which result [Morgan,1986].

Ecological studies abandoned the assumption of micro economic assumption of equilibrium, and this has methodological implications. Longitudinal data and dynamic models such as time-series model or rate model are required instead of cross-sectional data and static models. In this paper competition theory of population ecology model is simulated by means of system dynamics methodology which is appropriate for dynamic models and longitudinal data.

In NDS, when nonlinearity is coupled with deviation- amplifying feedback loop, a trivial difference in the initial conditions can generate a chaotic behavior of the system. This is so called a sensitive dependency on initial conditions [Stewart,1989]. In atmospheric science, it's called Butterfly Effect. To fully understand the sensitive dependency on initial conditions, the famous equations of Lorenz [Lorenz,1963]. would be helpful. Lorenz found that three simplified atmospheric nonlinear differential equations could reveal chaotic behaviors extremely sensitive to initial conditions. Deterministic chaos is always associated with the presence of a basic instability that allows small random fluctuations (noise) to be amplified by the deviation-amplifying feedback loops and finally influence the overall behavior of the system. In system dynamics models, this instability is associated with the negative feedback [Moskilde, et. al.,1988].

When nonlinear interactions coupled with feedback loops dominate, the system may extend beyond its stability boundary and pushed to the critical point of instability, referred to as the bifurcation point. Nicolis & Prigogine (1977) refers to systems in this far-from-equilibrium condition as a dissipative structure.

3. EVOLUTIONARY SYSTEM BASED ON CHAOS: L-V MODEL

Most of organization theory is based on equilibrium system theory, and thus has primarily focused on the function of pattern maintenance. In other words, an organization is regarded rational if an organization adapts to the changing environment well, and maintain the fit between the organization and the environment, which implies the state of equilibrium. This makes sense when we are only talking about equilibrium theory.

L-V model in population ecology theory was simulated in that it is a nonlinear model, and has feedback loops. Even though system structural view sets organization in terms of environment, it neglects the relation of organization in terms of other organizations and external selection of environment. Population eclogy theory (PET) shifts focus from a Lamarckistic adaptation (how organizations adapt to environment) to Darwinistic selection (how environments selects in or out a certain organization for survival). This view is an organizational application of Social Darwinism.

The assumptions of PET are: [Hannan & Freeman,1988].

1) Organizational forms are selected in or out of niches based on the principle of isomorphism that there is a one to one relation between an organizational form and matching environment (niche).

2) The concept of structural inertia which limits adaptive capacity provides the rationale for the replacement of adaptation perspective with selection view.

Lotka and Volterra suggested models of population dynamics incorporating interpopulation competition. Starting with isolated population whose growth rate is sigmoid curve, the average change rate of the number of population of organizations, N from the time a to a+dt is

$dN/dt=\rho_N N$ ....(1)

where $\rho_N=\lambda_N-\mu_N$ ....(2)

N= current size of population of organization

$\lambda_N=a_0-a_1N \quad a_1>0$ .....(3)

$\mu_N=b_0+b_1N \quad b_1>0$......(4)

$dN/dt=\{a_0-a_1N-(b_0+b_1N)\}N$

$=(a_0-b_0)N-(a_1+b_1)N^2$......(5)

Expressing logistic population growth model in terms of environmental carrying capacity K,

$dN/dt=rN[K-N/K]$.........(6)

where r=intrinsic (or natural) growth rate

=structural adaptive capacity to environmental change

If $N<K$, $r>0$; If $N=K$, $r=0$; If $N>K$, $r<0$

This carrying capacity, K can be more elaborated as a function of such ecological exogenous variables as human population (Carroll & Delacroix, 1982), labour force (Hannan & Freeman, 1987), and industrial consumption (Brittain & Sterns, 1985). For the purpose of making system dynamics model as simple as possible, in this paper, growth rate r and carrying capacity k are not elaborated.

Let us introduce the second population in equation (6). If one population lowers the carrying capacity of another, it can be said that the two populations are in competition. If the two populations compete each other, another term, competition coefficient should be introduced in equation (6).

$dN_1/dt=r_1N_1[k_1-N_1-\alpha_{12}N_2/k_1]$..(7a)

$dN_2/dt = r_2 N_2 [K_2 - N_2 - \alpha_{21} N_1 / K_2]$..(7b)

where α_{12} , α_{21} =competition coefficient

Let equation 7a and 7b equal to zero

$1/\alpha_{21} < K_2/k_1 < \alpha_{12}$

Therefore two popualtions can coexist only in k2/k1 ratio . This is so called principle of competitive exclusion (Hannan & Freeman, 1982).

If we generalize equatious 7a & 7b

$dN_i/dt = r_i N_i (k_i - \sum \alpha_{ij} N_j / k_i)$

$\alpha_{ii} = -1$

N_i = number of firms in population i ;
K_i = environmental carrying capacity for population i;
r_i = instantaneous growth parameter for population i;
α_{ij} = interaction coefficient, the size of effect of density of jth population to growth rate of ith population.

The ideas and results of the population dynamics can easily be applied to the social systems, such as the evolving technology, the diffusion of technology, or economic activities, in cases where the origin of differnt strategies and the adoption or the rejection by the surrounding populations are analyzed.

4. SYSTEM DYNAMICS (SD) AS A RESEARCH METHODOLOGY

System Dynamics based on a general systems approach helps consturct a causal-loop theory in terms of feedback loops, simualtaneously dealing with the dynamic processes of a complex system. SD analyzes a system in terms of feedback loop structure with level variable representing system state and rate variable representing system activity or decision.

SD has been widely used in such areas as urban problems, environmental problems, educational policy, cultural policy, political science, economics, business administration [Sohn & Surkis,1985; Andersen & Sturis,1988] However, the value of this methodology has not been fully recognized in organizational studies. The areas in which SD can be used has a certain characteristics. First it has variables changing over time. Secondly the feedback loops among variables play an important role in system behavior [Richardson & Pugh III,1986].

If we compare SD with econometric model such as time-series model, SD is subjective and prediction-oriented, while econometric model is objective and forecasting-oriented.

5. SIMULATION OF CHAOTIC BEHAVIOR IN L-V Model

In this section the before-mentioned characteristics of chaos theory, that is, nonlinearity, feedback loop, sensitive dependency on intial conditions, and the resulting nonequilbrium chaotic system is simulated.

Going back to the before-mentioned L-V Model, let's express the types of interactions possible between population I and J in terms of interaction coefficients (α_{ij},α_{ji}). If (α_{ij},α_{ji}) is (-,-), then full competition in which case presence of I or J suppresses other's growth. If (α_{ij},α_{ji}) is (-,0) then partial competition in which case I's growth rate is decreased, J's is unaffected. If (α_{ij},α_{ji}) is (+,-) then predatory competition in which case I expands at expense of J. If (α_{ij}, α_{ji}) is (0,0) neutrality in which case I and J do not affect one another. If (α_{ij},α_{ji}) is (+,0) then commensalism in which case I benefits from presence of J, but J is unaffected by I. If (α_{ij},α_{ji}) is (+,+) then symbiosis where both I and J benefit from presence of other.

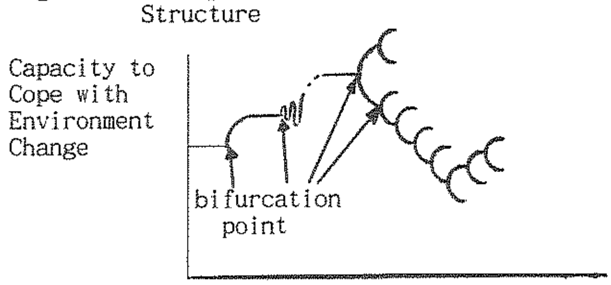
Table 1: Population Size Trends [Choi,1993]

(α_{ij}, α_{ji})	Interaction	α_{12}	α_{21}
(-,-)	Full compeition	.5	.5
(-,0)	Partial competition	.5	0
(+,-)	Predatory competition	-.5	.5
(0,0)	Neutrality	0	0
(+,0)	Commensalism	-.5	0
(+,+)	Symbiosis	-.5	-.5
(-,-)	**	1	1

N1=10, N2=15, K1=K2=30, r1=r2=.1, α_{ii}=0
Carrying capacity manipulated into K1=40 and K2=80

Leifer (1989) proposed a dissipative structure model of an organizational change. As environenmental crisis increases, the analyzability of the environment decreases, with the result that it becomes difficult to maintain homeostasis or stability. (see Figure 1) [Leifer,1989].

Figure 1: Dissipative Structure



Source: Leifer, R. (1989)

In order to verify the conditions of the dissipative structure shown in Figure 1, the before-mentioned L-V model is simulated, and the simulation results are shown in Figure 2 and Figure 3. Figure 2 is a simulation result of L-V model in equilibrium state, and Figure 3 is a simulation result of L-V model in nonequilibrium state. (for a detailed discussion on the simulation of L-V model in equilibrium state, refer to Choi [1993])

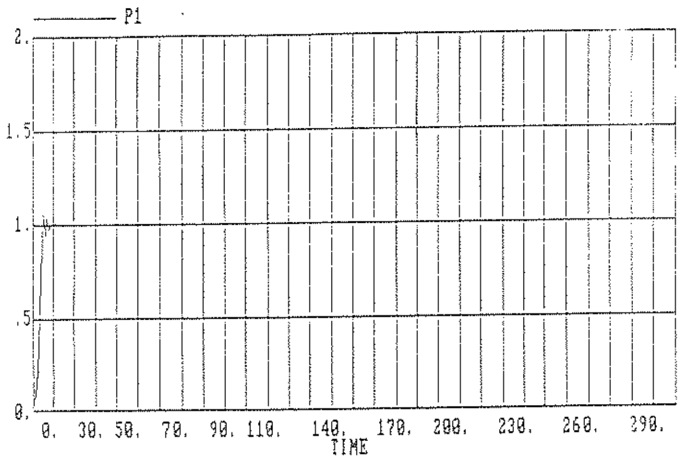


Figure 2: L-V Model in Equilibrium State

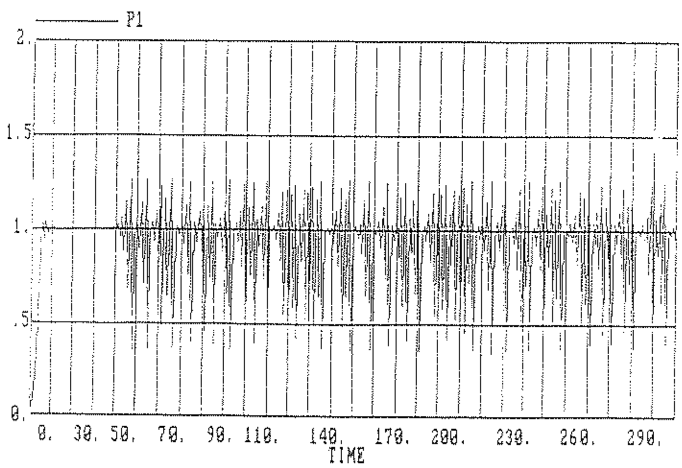


Figure 3: L-V Model in Chaotic State

VI. CONCLUSIONS

In System Dynamics (SD), it is considered more important to conceptualize feedback model rater than to estimate parameter values precisely. SD predicts future by showing feedback mechanisms among system components rather than forecasting future by time-series data. Therefore, SD can test and simulate feedback model without longitudinal data. Feedback loop analysis in organizational science can enrich theoretical framework. For example, in terms of PET, all the possible interaction patterns such as competition and cooperation among populations can be analyzed in feedback loop model.

The implications of the simulation results are 1) it's important to grasp the dynamics of the chaotic systems in long term perspective. So long as social scientists continue to rely on cross-sectional studies, it's unlikely that they will discover and model the chaotic nature of social system [Gregersen & Sailer, 1993]. 2) Poor analytical results (e.g., low r^2 values and lack of statistical significance) are to be expected when analyzing chaotic systems with standard statistical method [Kiel, 1989; Oh & Lee, 1994] Thus, simulation is one way of dealing with this problem.

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