Modelling Australian Interest Rate Swap Spreads by Mixture Autoregressive Conditional Heteroscedastic processes

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EXTENDED ABSTRACT

An interest rate swap is a contract between two parties to exchange periodically fixed rate payments for floating rate payments based on an agreed-upon notional principal and maturity. The fixed rate is known as the *swap rate* and a *swap curve* can be constructed using swap rates of different maturities. The swap curve is widely used by financial market participants as the benchmark for the pricing of investment grade corporate bonds. The floating rate is usually the Bank Bill Swap Reference Rate (BBSW) in the Australian market.

The Australian interest rate swap market is the most important over-the-counter (OTC) derivative market in Australia. The outstanding notional amount at the end of June 2006 was US$815.8 billion, which was much greater than other derivative instruments such as the forward rate agreements and interest rate options. The swap market size is comparable to the stock market in Australia, which had a market capitalisation of US$893.3 billion at the end of June 2006.

The observed difference between the swap rate and the government bond yield of corresponding maturity is known as the *swap spread*. The swap spread reflects the risk premium that is involved in a swap transaction instead of holding risk-free government bonds. It is primarily composed of the liquidity risk premium and the credit risk premium. In recent years there has been growing interest in modelling swap spreads because the swap spread is the key pricing variable for the swap rate.

In this paper we apply the class of mixture autoregressive conditional heteroscedastic (MARCH) models to three (3-year, 5-year and 10-year) swap spread series in Australia. The MARCH model is able to capture both of the stylised characteristics of the observed changes of the swap spread series: volatility persistence and the dependence of volatility on the level of the data. The proposed MARCH model also allows for regime switches in the swap spreads.

A MARCH (2; 3,0; 1,0) model is consistently identified for the three observed series. The fitted MARCH models can be interpreted as AR(3)–ARCH(1) processes mixed with small portions (5% to 10%) of independent shocks/breaks. In addition, we use the ex ante conditional probabilities as a tool for detecting possible shocks in the swap spread data. Around 50 observations of the 5-year swap spread series are identified as likely to come from the shock component. These detected shocks are mainly from the fourth quarter of 2001 (after terrorist attacks in the United States on 11 September 2001) and the summer of 2003 (retreat of mortgage-backed securities convexity hedging in the United States).

1. INTRODUCTION

An interest rate swap is a contract between two parties to exchange periodically fixed rate payments for floating rate payments based on an agreed-upon notional principal and maturity. The fixed rate is known as the *swap rate* and a *swap curve* can be constructed using swap rates of different maturities. The swap curve is widely used by financial market participants as the benchmark for the pricing of investment grade corporate bonds (e.g., see Schumacher, 1998). The floating rate is usually the three- or six-month London Interbank Offer Rate (LIBOR) or the Bank Bill Swap Reference Rate (BBSW) in the Australian market.

The observed difference between the swap rate and the government bond yield of corresponding maturity is known as the *swap spread*. The swap spread reflects the risk premium that is involved in a swap transaction instead of holding risk-free government bonds. It is primarily composed of the liquidity risk premium and the credit risk premium. In recent years there has been growing interest in modelling swap spreads because the swap spread is the key pricing variable for the swap rate. See, for example, Duffie and Singleton (1997), Grinblatt (2001), Liu *et al.* (2006) and Johannes and Sundaresan (2007).

The Australian interest rate swap market is the most important over-the-counter (OTC) derivative market in Australia. The outstanding notional amount at the end of June 2006 was US\$815.8 billion (Bank for International Settlements, 2007), which was much greater than other derivative instruments such as the forward rate agreements and interest rate options. The swap market size is comparable to the stock market in Australia, which had a market capitalisation of US\$893.3 billion at the end of June 2006 (Australian Securities Exchange, 2007).

Previous studies of the Australian swap market focus on searching for the determinants of swap spreads and the linkage between the US dollar and Australian dollar swap markets. See, for example, Brown *et al.* (2002), Fang and Muljono (2003) and In *et al.* (2004). Observed swap spreads commonly vary over time (i.e., they are volatile) and the Australian market is not an exception. Given that the swap spread is in effect the current swap price, changes in the swap spread can significantly affect the value of an on-going swap position for both the market maker and the corporate end-users of the agreement. In this paper we apply the class of mixture autoregressive conditional heteroscedastic (MARCH) models to three (3-year, 5-year and 10-year) swap spread series in Australia. The MARCH model is able to capture both of the stylised characteristics of the observed changes of the swap spread series: volatility persistence and the dependence of volatility on the level of the data. Lekkos and Milas (2004) find that the dynamics of the US and UK swap spreads are best described by a regime-switching model. Our proposed MARCH model also allows for regime switches in the Australian swap spread data.

The paper proceeds as follows. Section 2 provides a brief review of MARCH modelling. Section 3 presents the data and empirical results. The discussion and conclusion follow in the final section.

2. THE MARCH MODEL

2.1 Model Specification

Wong and Li (2001) introduce the class of mixture autoregressive conditional heteroscedastic (MARCH) models. A time series Y_t is said to follow a MARCH $(K; p_1, p_2, \ldots, p_K; q_1, q_2, \ldots, q_K)$ model if

$$F(y_t|\mathcal{F}_{t-1}) = \sum_{k=1}^{K} \alpha_k G\left(\frac{e_{k,t}}{\sqrt{h_{k,t}}}\right), \qquad (1)$$

$$e_{k,t} = y_t - \mu_{k,t},$$

$$\mu_{k,t} = \phi_{k0} + \phi_{k1} y_{t-1} + \ldots + \phi_{kp_k} y_{t-p_k},$$

$$h_{k,t} = \beta_{k0} + \beta_{k1} e_{k,t-1}^2 + \ldots + \beta_{kq_k} e_{k,t-q_k}^2.$$

Here, $F(y_t|\mathcal{F}_{t-1})$ is the conditional cumulative distribution function of Y_t given the past information, evaluated at y_t; $\mathcal{F}_t$ is the information set up to time t; $G(\cdot)$ is the cumulative distribution function of the standard normal distribution and mixing proportions $\alpha_1 + \cdots + \alpha_K = 1$ with $\alpha_k > 0$, for $k = 1, \ldots, K$. This model consists of a mixture of K autoregressive components with autoregressive conditional heteroscedasticity, that is, the conditional mean of y_t follows an AR process while the conditional variance of y_t follows an ARCH process (Engle, 1982). To avoid the possibility of zero or negative conditional variance, the following conditions for β_{ki}s must be imposed: $\beta_{k0} > 0$ $(k = 1, \ldots, K), \beta_{ki} \geq 0$ $(i = 1, \ldots, q_k; k = 1, \ldots, K)$.

One important feature of the MARCH model is its flexibility in the modelling of changing conditional variance. The conditional variance of y_t is given by

$$\text{Var}(y_t|\mathcal{F}_{t-1}) = \sum_{k=1}^{K} \alpha_k h_{k,t} + \sum_{k=1}^{K} \alpha_k \mu_{k,t}^2 - \left(\sum_{k=1}^{K} \alpha_k \mu_{k,t}\right)^2. \qquad (2)$$

The first term allows the modelling of the dependence of the conditional variance on the past "errors". The second and third terms model the change of the conditional variance due to the difference in the conditional means of the components.

The squared autocorrelations of the time series that are generated by a MARCH model are similar to those that are generated by an ARCH model. As an example, for a MARCH (K; 0,...,0; 1,...,1) model with $\phi_{k0} = 0$ for all $k = 1, \ldots, K$, the autocorrelations of the squared time series are given by

$$\text{corr}(Y_t^2, Y_{t-l}^2) = \left(\sum \alpha_k \beta_{k1}\right)^l.$$

Note that the squared autocorrelation function is similar to that of an ARCH(1) model with the lag 1 coefficient replaced by the coefficient $\sum \alpha_k \beta_{k1}$. As a generalisation of the ARCH model, the range of possible squared autocorrelations should be as great as that of the corresponding standard ARCH process.

2.2 Model Estimation

The estimation of the parameters of the MARCH model can be performed by the maximum (conditional) likelihood method. For $k = 1, \ldots, K$, define

$$\begin{aligned} \boldsymbol{\alpha} &= (\alpha_1, \ldots, \alpha_{K-1})'; \\ \boldsymbol{\Phi}_k &= (\phi_{k0}, \phi_{k1}, \ldots, \phi_{kp_k})' \\ \boldsymbol{\beta}_k &= (\beta_{k0}, \beta_{k1}, \ldots, \beta_{kq_k})'. \end{aligned}$$

The parameters in the MARCH model (1) can be grouped into

$$\boldsymbol{\Theta} = (\boldsymbol{\alpha}', \boldsymbol{\Phi}_1', \boldsymbol{\beta}_1', \ldots, \boldsymbol{\Phi}_K', \boldsymbol{\beta}_K')'. \quad (3)$$

Suppose that the observation $\mathbf{Y} = (y_1, \ldots, y_n)'$ is generated from the MARCH model (1). Let $\mathbf{Z} = (\mathbf{Z}_1, \ldots, \mathbf{Z}_n)'$, where $\mathbf{Z}_t$ is a K-dimensional unobservable random vector with its kth component equal to one if y_t comes from the kth component of the conditional distribution function, and to zero otherwise. Denote the kth element of $\mathbf{Z}_t$ as $Z_{k,t}$. The observation y_t will have the following contribution to the (conditional) log-likelihood:

$$\mathcal{L}_t = \sum_{k=1}^K Z_{k,t} \ln \alpha_k - \sum_{k=1}^K \frac{Z_{k,t}}{2} \ln h_{k,t} - \sum_{k=1}^K \frac{Z_{k,t} e_{k,t}^2}{2h_{k,t}},$$

where $e_{k,t}$ and $h_{k,t}$ are parts of the MARCH model defined in (1). The normalised log-likelihood function for the MARCH model is given by

$$\mathcal{L} = \frac{1}{N} \sum_{t=p+q+1}^n \mathcal{L}_t, \quad (4)$$

where $N = n - p - q$ with $p = \max(p_1, \ldots, p_K)$ and $q = \max(q_1, \ldots, q_K)$.

Many numerical methods can be used to maximise the log-likelihood function in (4) and obtain the maximum likelihood estimate of the MARCH parameter $\boldsymbol{\Theta}$ in (3). In this paper we employ the EM algorithm (Dempster *et al.*, 1977), which is the most readily available procedure in estimating mixture type models. One advantage of the EM algorithm is that it ensures that the likelihood values increase monotonically. See McLachlan and Basford (1988) and McLachlan and Krishnan (1997) for a discussion of the EM algorithm and other alternatives. The standard errors of the parameter estimates can be computed by Louis' method (1982), after the EM estimation. The details of the EM estimation algorithm for estimating the class of MARCH models are given in Wong and Li (2001).

2.3 Model Selection

There are two aspects of model selection in the MARCH models, namely, the number of components (K) and the orders of each AR-ARCH component (i.e., p_k and q_k, respectively). Here, we do not discuss the selection problem for the number of components, K, as it is difficult to handle even in the special case of the homogeneous MAR model (Wong and Li, 2000). The use of the Bayesian information criterion (BIC) that is proposed by Schwarz (1978) to choose K is somewhat non-standard as it corresponds to testing problems with nuisance parameters that do not exist under the null hypothesis (Davis, 1987). However, a two-component MARCH model should be sufficient in most applications. In this paper we consider only MARCH models with $K = 2$.

After the number of components K has been decided, the BIC can be used for the selection of the orders, p_k and q_k, of each AR-ARCH component. Wong and Li (2001) illustrate the performance of the minimum BIC procedure with simulation studies. They find that the minimum BIC procedure performs well. They also find that the minimum AIC procedure (Akaike, 1973) is not appropriate for the model selection problem of the class of MARCH models.

3. DATA AND EMPIRICAL RESULTS

The observed difference between the swap rate and the government bond yield of corresponding maturity is known as the swap spread. In this section, we consider MARCH modelling of daily Australian swap spread rates. The series under study are 3-year ($SS3_t$), 5-year ($SS5_t$) and 10-year ($SS10_t$) swap rates. The analysis is based on the first-order differenced series, which are $DSS3_t = (SS3_t - SS3_{t-1})$, $DSS5_t = (SS5_t - SS5_{t-1})$ and $DSS10_t = (SS10_t - SS10_{t-1})$. The time frame of the study is 3 January 2000 to 29 December 2006, with 1821 observations for each series. Figure 1 plots the $DSS5$ series and Table 1 provides the summary statistics for the data.

Table 1. Descriptive statistics

	$DSS3$	$DSS5$	$DSS10$
Mean	−0.0001	−0.0001	0.0000
Median	0.0000	0.0000	0.0000
Maximum	0.1650	0.1800	0.1495
Minimum	−0.1950	−0.2000	−0.1700
Std. Dev.	0.0294	0.0296	0.0267
Skewness	−0.0753	0.0994	−0.0477
Kurtosis	7.8368	6.7792	4.6743
Correlation			
$DSS3$	1.0000		
$DSS5$	0.7159	1.0000	
$DSS10$	0.5532	0.6318	1.0000

Table 1 shows that the series display means and medians around zero with similar values of standard deviation. The series are quite symmetrically distributed but highly leptokurtic. Given the large kurtosis of the distributions, the frequency of extreme values is likely to be underestimated using ordinary statistical models (e.g., regression analysis) under the Gaussian assumption. Therefore, it might be preferable to model these DSS series by means of mixture Gaussian time series processes.

In this paper we only entertain two-component ($K = 2$) MARCH models. For each DSS series, MARCH (2; $p_1, p_2; q_1, q_2$) models with different combinations of orders ($p_k \leq 5$ and $q_k \leq 5$ for $k = 1, 2$) are estimated, and their corresponding BIC values are computed. We find that a MARCH (2; 3,0; 1,0) model, without intercept (i.e., $\phi_{k0} \equiv 0$), is consistently identified by the BIC as the best model for the observed $DSS5$ and $DSS10$ series; while a similar model, MARCH (2; 2,0; 1,0), is selected for the $DSS3$ series. It should be noted that a MARCH (2; 2,0; 1,0) model can be regarded as a special case of a MARCH (2; 3,0; 1,0) process with $\phi_{13} = 0$. Table 2 summarises the MARCH model estimation results. The standard errors of the estimates are reported in parentheses.

The conditional volatility as implied by the fitted MARCH model can be computed as the square root of the conditional variance equation (2), with all the parameters replaced by their corresponding estimates in Table 2. Figure 2 plots the time series of the conditional volatility for the $DSS5$ series. The calculated conditional volatilities in Figure 2 match reasonably the fluctuation patterns of the $DSS5$ series in Figure 1.

4. DISCUSSION

Given that the swap spread is in effect the current swap price, changes in the swap spread can significantly affect the value of an on-going swap position for both the market maker and the corporate end-users of the agreement. In this paper we apply the class of MARCH models to three (3-year, 5-year and 10-year) swap spread series in Australia. A MARCH (2; 3,0; 1,0) model is consistently identified for the three observed DSS series.

Table 2. Fitted MARCH model parameters for DSS data

	$DSS3$		$DSS5$		$DSS10$	
Parameter	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$
α_k	0.9379 (0.0123)	0.0621 (0.0123)	0.9033 (0.0173)	0.0967 (0.0173)	0.9493 (0.0124)	0.0507 (0.0124)
ϕ_{k1}	−0.4040 (0.0243)		−0.4278 (0.0251)		−0.4523 (0.0258)	
ϕ_{k2}	−0.1690 (0.0199)		−0.2226 (0.0227)		−0.2463 (0.0229)	
ϕ_{k3}			−0.0975 (0.0201)		−0.1391 (0.0207)	
β_{k0}	0.0003 (0.0000)	0.0052 (0.0010)	0.0003 (0.0000)	0.0036 (0.0006)	0.0003 (0.0000)	0.0037 (0.0008)
β_{k1}	0.1911 (0.0292)		0.1878 (0.0288)		0.2299 (0.0367)	

Figure 1. First difference of daily 5-year swap spreads (2000 - 2006).

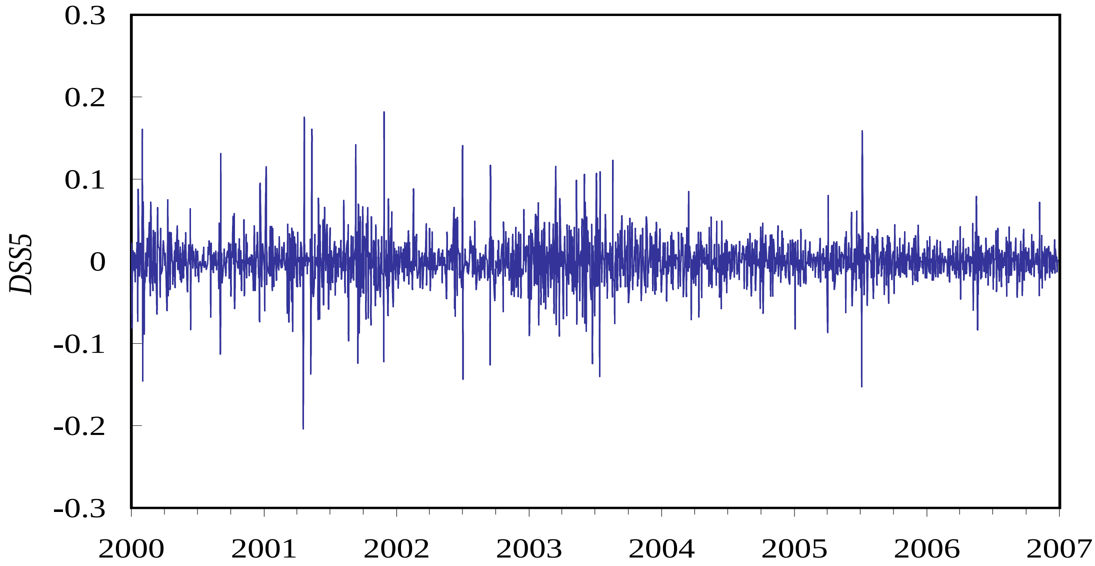
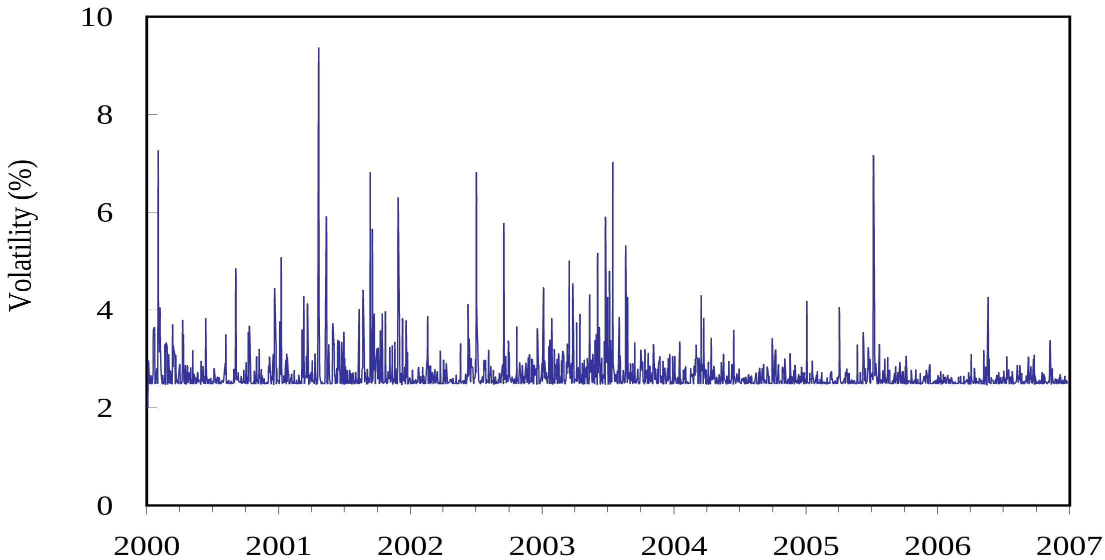


Figure 2. Conditional volatility for the $DSS5$ series as implied by the estimated MARCH model.



There is an interesting interpretation of the structure of the identified MARCH (2; 3,0; 1,0) model. The first component of the model is an AR(3)–ARCH(1) process. The second part, $p_2 = 0$ and $q_2 = 0$, is simply an independent random normal variate with a zero mean and variance of β_{20}. From Table 2, we find that $\hat{\alpha}_2$s range from 5% to 10% for the three DSS series and their corresponding $\hat{\beta}_{20}$s are much larger than $\hat{\beta}_{10}$s. Therefore, the fitted MARCH models can be interpreted as AR(3)–ARCH(1) processes that are mixed with small portions (5% to 10%) of independent shocks/breaks.

The empirical evidence for the time-varying conditional volatility of high-frequency financial time series is overwhelming in the literature. However, even if conditional heteroscedasticity is a stylised fact in observed financial time series data, outliers/shocks may still occur. Sakata and White (1998) propose an outlier-robust estimation procedure for conditional heteroscedasticity models. Franses and Ghijsels (1999) apply the outlier detection and adjustment method of Chen and Liu (1993) to ARCH processes. The fitted MARCH (2; 3,0; 1,0) models in this paper are able to accommodate possible outliers/shocks as a component of the model.

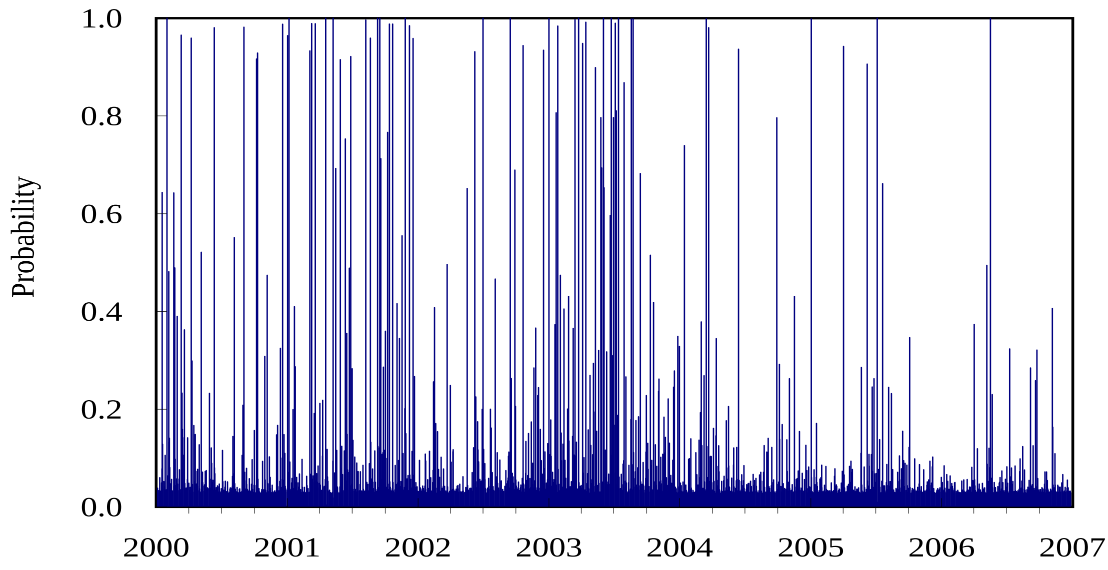
Furthermore, we can use the ex ante conditional probabilities as a tool to detect possible shocks in the DSS data. The ex ante conditional probability of a MARCH model is

$$\pi_{k,t} = \mathrm{E}[Z_{k,t}|\mathcal{F}_{t-1}],$$

and $Z_{k,t}$ is the indicator variable that is defined in Section 2.2. The $\pi_{k,t}$ probabilities can be estimated during the EM algorithm (Wong and Li, 2001). Figure 3 plots the $\hat{\pi}_{k,t}$ of the second component ($k = 2$) implied by the fitted MARCH (2; 3,0; 1,0) model for the $DSS5$ series. Around 50 observations are identified as likely ($\hat{\pi}_{2,t} > 0.90$) to come from the second component (i.e., the shock component). These detected shocks are mainly from the fourth quarter of 2001 and the summer of 2003.

Kobor, Shi and Zelenko (2005) mention two events that might be responsible for the major aberrant jumps that are observed in the US swap market from 2000 – 2004. The first event is the terrorist attacks in the United States on 11 September 2001. The second event is the retreat of mortgage-backed securities (MBS) convexity hedging in 2003. From June to August in 2003, the surge in long-term US Treasury bond yields forced MBS investors to unwind their convexity hedges in swaps. The sharp rises in the swap rate relative to the treasury yield caused the 10-year US$ swap spread to widen drastically and this created waves of large volatility in the swap spreads of all maturities. The results in In, Fang and Brown (2004, p.55) demonstrate that shocks in the US swap market have an impact on the Australian swap market but not vice versa. In this paper we also detect shocks in the Australian swap market in the fourth quarter of 2001 and the summer of 2003. These detected shocks are likely the result of "impulse" transmission from the US market.

Figure 3. The ex ante conditional probabilities of the second regime implied by the MARCH (2; 3,0; 1,0) model for the $DSS5$ data.



5. CONCLUSION & FURTHER RESEARCH

In this paper we apply the class of mixture autoregressive conditional heteroscedastic (MARCH) models to three (3-year, 5-year and 10-year) swap spread series in Australia. The MARCH model is able to capture both of the stylised characteristics of the observed changes of the swap spread series: volatility persistence and the dependence of volatility on the level of the data. The proposed MARCH model also allows for regime switches in the swap spreads.

In Table 1 it is clear that the three DSS series are not independent. It would be worthwhile to extend the problem of modelling the swap rate spreads in a multivariate context. Even though Fong *et al.* (2007) successfully derive a set of statistical procedures for modelling mixture vector autoregressive processes, method for model building of multivariate MARCH models has not been developed.

The selection of the number of components in the MARCH mixture, which is denoted by K in this paper, is another possible topic for further research.

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7. REFERENCES

Akaike, H. (1973) Information theory and an extension of the maximum likelihood principle. In *The Second International Symposium on Information Theory* (Eds B.N.Petrov and F. Csake), pp. 267-281. Hungary: Akademiai Kiado.

Australian Securities Exchange (2007) Historical market statistics. Available: www.asx.com.au/research/market_info /historical_equity_data.htm. Accessed 4 May 2007.

Bank for International Settlements (2007) Interest rate derivatives by instrument, counterparty and currency. Available: www.bis.org/statistics/otcder/dt07.pdf. Accessed 3 May 2007.

Brown, R., In, F., Fang, V. (2002) Modeling the determinants of swap spreads. *Journal of Fixed Income* **12**, 29–40.

Chen, C., Liu, L.M. (1993) Joint estimation of model parameters and outlier effects in time series. *Journal of the American Statistical Association* **88**, 284–297.

Davies, R.B. (1987) Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* **74**, 33–43.

Dempster, A.P., Laird, N.M., Rubin, D.B. (1977) Maximum likelihood from incomplete data via the EM algorithm (with discussion). *Journal of Royal Statistical Society* **B39**, 1–38.

Duffie, D., Singleton, K.J. (1997) An econometric model of the term structure of interest-rate swap yields. *Journal of Finance* **52**, 1287–1321.

Engle, R.F. (1982) Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* **50**, 987-1007.

Fang, V., Muljono, R. (2003) An empirical analysis of the Australian dollar swap spreads. *Pacific-Basin Finance Journal* **11**, 153–173.

Fong, P.W., Li, W.K., Yau, C.W., Wong, C.S. (2007) On a mixture vector autoregressive model. *The Canadian Journal of Statistics* **35**, 135–150.

Franses, P.H., Ghijsels, H. (1999) Additive outliers, GARCH and forecasting volatility. *International Journal of Forecasting* **15**, 1–9.

Grinblatt, M. (2001) An analytic solution for interest rate swap spreads. *International Review of Finance* **2**, 113–149.

In, F., Fang, V., Brown, R. (2004) Australia and US Interest rate swap markets: Comparison and linkages. *Accounting and Finance* **44**, 45–56.

Johannes, M., Sundaresan, S. (2007) The impact of collateralization on swap rates. *Journal of Finance* **62**, 383–410.

Kobor, A., Shi, L., Zelenko, I. (2005) What determines US swap spreads? *The World Bank Working Paper No. 62*, The World Bank.

Lekkos, I., Milas, C. (2004) Common risk factors in the US and UK interest rate swap markets: Evidence from a nonlinear vector autoregression approach. *Journal of Futures Market* **24**, 221–250.

Liu, J., Longstaff, F.A., Mandell, R.E. (2006) The market price of risk in interest rate swaps: The roles of default and liquidity risks. *Journal of Business* **79**, 2337–2359.

Louis, T.A. (1982) Finding the observed information matrix when using the EM algorithm. *Journal of Royal Statistical Society* **B44**, 226–233.

McLachlan, G.J., Basford, K.E. (1988) *Mixture Models: Inference and Applications to Clustering*. New York: Marcel Dekker.

McLachlan, G.J., Krishnan, T. (1997) *The EM Algorithm and Extensions*. New York: Wiley.

Sakata, S., White, H. (1998) High breakdown point conditional dispersion estimation with application to S&P500 daily returns volatility. *Econometrica* **66**, 529–567.

Schumacher, M. (1998) Swap spreads do matter. *Journal of Fixed Income* **8**, 59–64.

Schwarz, G. (1978) Estimating the Dimension of a Model. *The Annals of Statistics* **6**, 461–464.

Wong, C.S., Li, W.K. (2000) On a mixture autoregressive model. *Journal of the Royal Statistical Society* **B62**, 95–115.

Wong, C.S., Li, W.K. (2001) On a mixture autoregressive conditional heteroscedastic model. *Journal of American Statistical Association* **96**, 982–995.