

Sensitivity Analysis for Evaluating Importance of Variables Used in an Urban Water Supply Planning Model

King, D. M.¹ and B. J. C. Perera¹

¹School of Architectural, Civil and Mechanical Engineering, Victoria University, Victoria
Email: david.king@vu.edu.au

Keywords: Yield estimation, sensitivity analysis, Morris, FAST, Sobol'

EXTENDED ABSTRACT

The yield of an urban water supply system is defined as the average annual volume of water that can be supplied from the water supply system over a given planning period, subject to streamflow variability, operating rules and demand pattern, without violating the adopted level of service. Since yield plays a key role in the management of urban water supply systems, it is important for water authorities to accurately estimate it with minimal inherent uncertainty. Sensitivity analysis can identify key variables used in yield estimation, allowing water authorities to improve the knowledge of those variables (or input factors) and thus to improve the confidence and reliability of the system yield.

The increase of computational power that has become available over the past decades has meant the variance based sensitivity analysis techniques, FAST (Fourier Amplitude Sensitivity Test) and Sobol', have become favourable. Additionally, the Morris method, a computationally inexpensive screening technique, is commonly used to economically identify non-influential input variables so they can be disregarded from the more computationally expensive variance based techniques.

The case study considered in this study was a simple urban water supply system consisting of two storages and a single urban demand centre simulated using the REALM simulation package. Historic monthly data was used for streamflow, rainfall and evaporation. Twenty-eight input variables were identified within the model consisting of historic data, empirical values and model parameters.

A screening experiment using the Morris method, the results were confirmed by the Extended FAST method (EFAST, a derivative of FAST), was used to identify non-influential input variables. These variables were then set at their nominal values and

disregarded from the subsequent experiments using the FAST, EFAST and Sobol' techniques.

The ranked results from the Morris and EFAST screening techniques showed close similarity with only some differences occurring in the lower influential variables. The sensitivity indices were dominated primarily by the streamflow, with the supply reliability, upper restriction rule curve and the maximum number of consecutive restriction months variables showing some significance.

Detailed sensitivity analyses on the simple model were then performed using FAST, EFAST and Sobol' considering the ten highest ranked variables from the screening experiments. The remaining 18 variables were kept at their nominal value. Experiments of increasing model simulations, and hence accuracy, were performed until a convergence was met or the required number of simulation was impracticable.

Comparison of the results of the detailed sensitivity analyses again indicated the dominance of the streamflow, and the minor significance of the supply reliability, upper restriction rule curve and the maximum number of consecutive restriction months variables. The results obtained from FAST and EFAST were reasonably similar, however the Sobol' experiments gave erroneous results, where the total-order sensitivity is less than the first-order effect. Nevertheless, these errors were found only for lower influential variables.

1. INTRODUCTION

The reliable supply of clean potable water is increasingly viewed as an essential commodity throughout the world. Government authorities continually confront various issues, problems and limitations in their attempt to provide the community's needs of a clean and reliable water supply. Lack of rainfall, water quality, suitability of source, cost, infrastructure and storage, and the community's acceptable security of water supply

are issues which need to be addressed to ameliorate urban water supply.

The main issue for urban water supply systems is to continually supply a demand that does not outstrip the volume of water entering a system over a long period. The recent drought experienced in Australia has meant that many water supply systems are required to supply a demand that exceeds a sustainable volume. This shortfall can be reduced by; decreasing the demand via water saving measures and schemes, and education; and/or increasing the yield of the system by; optimising system management, or augmentation with additional water sources.

The definition of yield used in this study and commonly used by many water authorities throughout Australia is: the average annual volume of water that can be supplied from the water supply system over a given planning period, subject to streamflow variability, operating rules and demand pattern without violating the adopted level of service. Yield is commonly estimated by increasing or decreasing the average annual demand until the accepted level of service is just violated, by using a computational model that simulates the specific water supply system that incorporates streamflow variability, operating rules and demand pattern. REALM (REsource ALlocation Model), a water supply simulation software tool, is commonly used in Australia for modelling water supply systems (Perera and James 2003, Perera et al. 2005).

REALM is a generalised computer simulation software package that models the harvesting and bulk distribution of water resources within a water supply system. It uses a fast network linear programming algorithm to optimise the water allocation within the network during each simulation time step, in accordance with user-defined operating rules including Target Rule Curves (TRC) and Restriction Rule Curves (RRC) (Perera et al. 2005).

The yield of a water supply system is dependant on numerous variables including climate dependant data (e.g. streamflow and demand), empirical inputs (e.g. operating rules), and model parameters (e.g. transmission losses). As these inputs are determined through measurement, optimisation or modeller experience, they inherently contain unquantified errors which are conveyed through the model structure to the output. Minimising these errors will increase the confidence in the yield estimate. However, input variables may have different significance in terms of their influence on the estimation of yield. Investigation into some of

the variables may result in little improvement in the confidence of the yield estimate. Therefore it is more efficient to identify, investigate and improve only those variables that provide a significant effect on the output. The identification of influential, or important, variables is a primary goal of Sensitivity Analysis (SA).

In this case the principle aim was to determine the variables that have the greatest influence on the estimation of yield, whilst evaluating the appropriateness of various sensitivity analysis techniques on a water allocation model.

This paper briefly discusses the principles of SA, and introduces the Morris method, and two variance based techniques: the Fourier Amplitude Sensitivity Test (FAST) and the Sobol' Method. This is followed by a discussion of the urban water system case study and appropriate handling of the model's input variables. Finally, the results of the case study are presented along with some discussion, recommendations and conclusions.

2. SENSITIVITY ANALYSIS

Sensitivity analysis can be defined as the study of how the variation in the output of a model can be apportioned, (qualitatively or quantitatively) to different sources of input variation. It can provide valuable information regarding the structure of the model, and its reliance upon the input variables, or lack thereof (Saltelli 2000). The sensitivity of an input variable or parameter is an indication of the effect that a variation of that input will have on the output; an input variable of higher sensitivity will result in a greater variation of the output and vice-versa. The sensitivity of a variable illustrates the care that modellers must take to obtain and employ an appropriate value for the variable, but can also signify its importance in relation to its dependency by the model structure (Saltelli *et al.* 1999).

The successful application of sensitivity analysis largely depends upon the model structure and the selection of an appropriate technique(s) to accurately investigate the nature of the variables and model. For example, a purely linear model (i.e. a model where the input-output relationship is linear) can be easily investigated with the use of first-order, differential or one at a time (OAT) techniques. However, for a model that is non-linear, first-order differential analyses are ineffective as they cannot identify or handle non-linearity, interactions, or correlations between variables. Computational advancements have allowed the use of variance based techniques that can accommodate non-linearity and interactions within a model and its variables.

Two such variance based methods, the Fourier Amplitude Sensitivity Technique (FAST) (Cukier *et al.* 1973, Saltelli *et al.* 1999) and the method of Sobol' (Sobol' 1993), are commonly used for detailed analyses after using a screening analysis employing the computationally efficient screening technique: the Morris method (Morris 1991). The Morris method is first applied to quickly determine which variables are non-influential. These can then be held at constant values and eliminated from subsequent analyses, so as to reduce the number of model simulations that the computationally expensive variance based methods require.

These methods were selected as the most appropriate as they are regarded as the best for non-linear, complex, computationally demanding simulation models such as the REALM model used in this study.

2.1. The Morris Method

The Morris method is a specialised randomised OAT design that has proved to be an efficient and reliable technique to identify and rank important variables (Morris 1991, Campolongo *et al.* 2007). It gives a modeller insight into the nature of the influence of input variables on a model's output with a limited number of model simulations. The method is based on the OAT assumption that if all variables are changed by the same percentage, the variable that exhibits the largest variation in the output is the most sensitive.

To perform this, a multiple number of trajectories through the parameter space are generated to provide an efficient and systematic method to explore the model output. Each trajectory provides a single estimation of the Elementary Effect (EE) for each model input, as defined by:

$$EE_i(\mathbf{x}) = \frac{[y(x_1, \dots, x_{i-1}, x_i + \Delta, x_{i+1}, \dots, x_k) - y(\mathbf{x})]}{\Delta} \quad (1)$$

where Δ is a value in $\{1/(p-1), \dots, 1-1/(p-1)\}$, p is the number of levels that divide the parameter space, \mathbf{x} is the set of input values, and y is the model output.

Morris (1991) proposed two measures, namely the mean (μ) and standard deviation (σ) of the set of EEs for each variable. The sensitivity index, μ , assesses the overall influence of a variable on the model output, including higher-order and interaction effects. When μ is high, the variable said to be highly sensitive as a unit change causes a large deviation of output and vice versa. The spread, or standard deviation, denoted as σ ,

provides a measure that indicates possible interaction of a variable with other variables and/or the variable has a non-linear effect on the output (Campolongo and Braddock 1999).

Campolongo *et al.* (2007) propose a third output index, μ^* , the mean of the absolute EE's, which addresses the possible misrepresentation of the magnitude of sensitivity of the variables in a non-monotonic model given by μ . Such variables would produce positive and negative elementary effects, from which the mean value of the EE's, μ , would indicate a lower overall sensitivity measure for a variable that is highly sensitive. The benefit of μ^* is that only the magnitudes of the changes are considered, avoiding some effects that may cancel out each other (Campolongo *et al.* 2007), hence providing a more accurate measure of overall sensitivity compared to μ .

2.2. Variance Based Techniques

Owing to the vast increase of computing power over the past recent decades, more powerful methods of SA have become feasible. The primary developments have been made on Variance Based Sensitivity Analysis methods. These methods can identify and quantify interactions between variables, and can be applied to a single or group of variables. They are also model independent so they can be used on a model whose algorithms are unknown or complex. The main drawback of variance based measures is their computational cost since they involve the estimation of k-dimensional integrals.

The FAST and Sobol' methods determine the same first order sensitivity index: S_i , the estimate of the ratio of the variance due to the i -th variable to the variance due to all variables. Therefore, if the model is purely additive the sum of S_i equals 1, while for non-uniform, non-additive models the sum of S_i is less than 1.

The second sensitivity measure that can be computed using variance based methods is the total sensitivity index S_{Ti} . This is defined as the sum of all effects involving the i -th variable. It can be computed using Sobol' and Extended FAST (EFAST), a derivative proposed by Saltelli *et al.* (1999) of the original FAST. Further higher order indices can be calculated by both variance based methods, but generally only determined using Sobol'. This is due to the relatively small number of extra model simulations that are required, compared to the number required for FAST.

Method of Sobol'

The key principle behind Sobol's approach is to decompose the total output variance $V(Y)$ in the form:

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \dots + V_{12\dots k} \quad (2)$$

where:

$$V_i = V(E(Y | X_i)), \quad (2a)$$

$$V_{ij} = V(E(Y | X_i, X_j)) - V_i - V_j, \text{ etc.} \quad (2b)$$

Corresponding sensitivity indices are given by $S_i = V_i/V$, $S_{ij} = V_{ij}/V$ etc., where S_{ij} , indicates the two-factor interaction effect.

The Sobol' decomposition also allows for an estimate of the total sensitivity index, S_{Ti} , a measure of the sum of all order sensitivity effects involving the i -th input variable.

Fourier Amplitude Sensitivity Test Method

The Fourier Amplitude Sensitivity Test (FAST) (Cukier *et al.* 1973), and related Extended FAST (EFAST) (Saltelli *et al.* 1999), use the Fourier principles of frequency analysis to determine the same indices as Sobol's method. Each input variable is assigned a certain angular frequency ω_i to transform the input variables into an approximately space filling curve from which the samples are selected. The Fourier coefficients A and B at all frequencies of the resultant model output are determined. S_i is then estimated by considering the ratio of their magnitude at the i -th variable's angular frequency (and harmonics) to the total magnitude of all frequencies.

Total indices S_{Ti} can also be calculated by using EFAST. The basic idea is to consider the frequencies that are not harmonics of the frequencies $\{\omega_1, \omega_2, \dots, \omega_k\}$ (Saltelli *et al.* 1999).

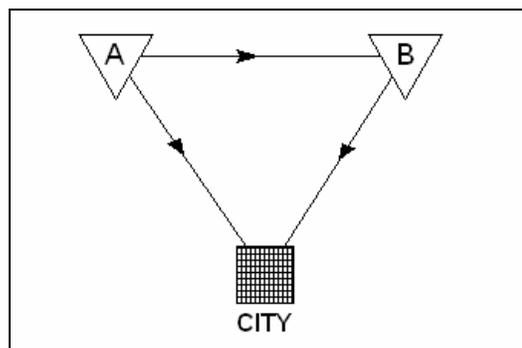


Figure 1. Case Study Water Supply System.

These frequencies contain information about the residual variance that is not accounted for by the first order indices.

Accuracy of the sensitivity indices depends on the selection of space filling curve and set of angular frequencies. The set of frequencies should be incommensurate and selected so that common Fourier transform issues, such as aliasing and interference, are prevented (Cukier *et al.* 1973).

3. CASE STUDY

3.1. System Description and Data

A hypothetical example of a two-reservoir system (VU and DSE 2005) was considered as the case study for this paper. A schematic diagram of the system is shown in Figure 1, while basic system, streamflow and demand data are as given in VU and DSE (2005).

Reservoirs A and B supply water to a demand centre. Both reservoirs receive streamflow from their own catchments and are both subjected to evaporation losses and rainfall gains; modelled using storage volume - surface area relationship, rainfall and evaporation climatic data and empirical factors A and B. (Note: rainfall and evaporation data are used for storage gains and losses only.) The mean annual flow at the two reservoirs is approximately 104,000 MI (1 MI = 10^3 m³). Reservoir A, which has a capacity of 100,000 MI, can transfer water to the 60,000 MI capacity reservoir B, according to the defined TRCs. Both reservoirs have a minimum capacity of zero MI. The monthly demand disaggregation factors, which reflect typical high demands during summer months and low demands during winter months, were used to disaggregate annual demand into monthly demands. These monthly demands were further adjusted by a 'climatic index variable' (CLINX) to account for climatic variability. The streamflow data at the reservoirs, climatic data (i.e. rainfall and evaporation) for modelling reservoir evaporation and climatic index data for disaggregating annual demand data into monthly data were available for a 28 year period.

A four-level demand restriction policy, consisting of upper and lower rule curves, including four intermediate restriction zones (with definitions of relative positions and percentage restrictable levels), and a base demand curve, was used to restrict the demand during low storage volume periods. Storage TRCs were defined by a single set of five-point curves for all months of the year, indicating the preferred individual storage volumes for different total system storage volumes.

As stated earlier, the yield in this study is defined as the average annual volume of water that can be supplied from the water supply system over a given planning period, subject to streamflow variability, operating rules and demand pattern, without violating the adopted level of service (or security criteria), defined by the supply reliability, worst restriction level and consecutive number of months of restrictions (VU and DSE 2005).

3.2. Handling of Input Variables

For SA, the input factors or variables must be sampled over a plausible range of values. This range can be absolute values, or denote a percentage change of their nominal values, depending on the type of data. A list of the 28 variables considered and their assigned ranges used in this case study can be seen in Table 1. See VU and DSE (2005) for more detail relating to these variables. Typical types of data relevant to this case study are as follows:

Time series data – Time series data used (i.e. streamflow, rainfall and evaporation) are based on measurements and therefore a percentage of the observations was considered to define the range so as to reflect the possible errors. For each variable, a single percentage, randomly sampled from the range is used to change all data points in the time series. However, such changes to time series may not be appropriate and can cause issues with any correlations that exist between time series variables.

Other data, whose range is defined by a percentage change of the nominal value – These are initial storage volumes, supply reliability, storage volume - surface area relationship of reservoirs, upper and lower RRCs, base demand

curve, and relative position and percentage of demand restrictable for various intermediate stages of restriction. A single percentage randomly selected from a range is used for these parameters.

Ranges defined by absolute values – The ranges of some input variables are defined by absolute values rather than percentage changes. They are: consecutive number of restriction months, worst restriction level, and factors A and B in modelling reservoir evaporation. A single randomly selected value from the range is used for each parameter.

Multi-factored variables that sum to a certain value – Some data items (i.e. monthly temporal disaggregation factors (TDFs), TRC points, and climatic index variables (CLINX)), which add to a certain value need to be handled differently. They are handled through an algorithm which adjusts the individual factors, to approximately the same randomly selected percentage, so that their sum maintains the required property.

4. DESIGN OF EXPERIMENTS

The aim of this study was to apply three sensitivity analysis techniques, the Morris method, FAST, and Sobol', to determine the importance of each variable on the estimation of yield and evaluate their appropriateness for use on a urban water supply simulation model. Firstly the Morris method was used as a screening technique on all 28 variables with EFAST used to confirm the results. Several experiments of both techniques using different random seeds were performed to avoid any possible anomalies that could result from a particular sample selection. To ensure that the sensitivity indices have reached convergence, experiments of increasing number of simulations, or resolution, were performed. Holding the non-

Table 1. Description of the 28 Input Variables Used

Variable	Range	Variable	Range
Streamflow Time Series	±5% of historic data	Initial Volume of Reservoir A	25-100% of capacity
TDFs	±5% of nominal position	Initial Volume of Reservoir B	25-100% of capacity
CLINX Time Series	±5% of historic data	Stage 1 Percentage Restrictable	±5% of nominal position
Consecutive Restriction Months	6 – 18 months	Stage 2 Percentage Restrictable	±5% of nominal position
Worst Restriction Level	3 – 4	Stage 3 Percentage Restrictable	±5% of nominal position
Supply Reliability	80% – 98%	Stage 4 Percentage Restrictable	±5% of nominal position
Rainfall Time Series	±5% of historic data	Upper RRC	±5% of nominal position
EVAP Time Series	±5% of historic data	Lower RRC	±5% of nominal position
Evaporation Factor A - Storage A	0 – 5	Base Demand	±5% of nominal position
Evaporation Factor A - Storage B	0 – 5	Stage 1 Relative Position	±5% of nominal position
Evaporation Factor B - Storage A	0.95 – 1.05	Stage 2 Relative Position	±5% of nominal position
Evaporation Factor B - Storage B	0.95 – 1.05	Stage 3 Relative Position	±5% of nominal position
Volume to Surface Area Relationship	±5% of nominal volumes	Target Curve	±5% of nominal position

influential variables at their nominal values/states, detailed FAST and Sobol' experiments were then used to provide an accurate estimate of the importance of the remaining variables.

5. RESULTS AND DISCUSSION

5.1. Morris and FAST Screening

Table 2 shows the highest 15 ranked results (considering the 28 variables listed in Table 1) in terms of the μ^* and S_i of the Morris Method and EFAST screening experiments, respectively. Typically the result of a Morris experiment is displayed on a $\mu - \sigma$ plane, but for convenience of comparison and space limitation they are presented in Table 2 in this paper.

Strong similarity exists between the μ^* and S_i ranks with only some lower influential variables' ranks differing. The quantitative values of μ^* and S_i cannot be directly compared as S_i is the percentage of the i -th variable's variance to the total variance, and μ^* is a measure of an expected output change due to a unit change in the input. However, the rankings confirm that the Morris method is efficient in screening and ranking a large number of input variables. Also noticeable from both sets of indices is the presence of possible interactions or non-uniform behaviour. This is seen from the Morris sensitivity index, σ , and from the difference between EFAST S_i and S_{Ti} measures.

5.2. FAST and Sobol'

Variables that displayed lower influence on the estimation of yield were eliminated and higher resolution FAST and Sobol' experiments were performed on the 10 most significant variables. Results presented in Table 3 show the sensitivity indices at the maximum number of model simulations for each sensitivity technique performed. At this point the sensitivity indices had satisfactorily converged or, in the case of the Sobol' experiment, the number of runs required in the next experiment was impracticable.

The results of the FAST and Sobol' sensitivity analyses indicate that the streamflow and reliability are the most important input variables in the estimation of yield, followed by the upper restriction curve, and the number of consecutive months in restriction. The remaining variables are relatively insignificant. The differences in results between the experiments are due to the difference between the techniques used, but all show reasonably similar results.

Comparing the S_i and S_{Ti} can indicate possible two and higher factor interaction effects. However, by doing so errors within the Sobol' indices were identified. For some variables (such as CLINX and rainfall) the total-order is less than the first-order index, which according to the definition of variance should not occur. This problem occurs

Table 2. Comparison results for the Morris and FAST screening experiments. Experiments consisted of 28 variables: the highest 15 variables are displayed.

	Morris Method – 30 Trajectories				EFAST – 19956 Runs		
	μ	μ^*	σ	Ranked μ^*	S_i	S_{Ti}	Ranked S_i
Streamflow time-series	6332	6332	930	1	0.6841	0.7135	1
TDFs	-427	460	280	10	0.0038	0.0073	10
CLINX	-748	748	271	6	0.0097	0.0217	6
Rainfall time-series	806	806	230	5	0.0102	0.0210	5
Evaporation time-series	-639	651	250	9	0.0055	0.0157	9
Evaporation Factor A Storage A	-658	673	224	8	0.0074	0.0197	8
Evaporation Factor A Storage B	-736	736	177	7	0.0084	0.0214	7
Evaporation Factor B Storage A	-292	292	233	12	0.0016	0.0120	11
Evaporation Factor B Storage B	-312	312	242	11	0.0014	0.0129	12
Volume to Surface area	179	179	187	14	0.0013	0.0201	13
Consecutive Restriction Months	877	877	1816	4	0.0219	0.0555	4
Reliability	-4024	4024	1109	2	0.2418	0.2706	2
Base RRC	-195	213	250	13	0.0011	0.0053	14
Upper RRC	-1145	1145	627	3	0.0289	0.0357	3
Percentage Restrictable 1	78	134	232	15	0.0003	0.0035	15

Table 3. Comparison results for the detailed FAST and Sobol' experiments.

SA technique	FAST	EFAST		Sobol'	
Simulations	10000	19930		45056	
	S_i	S_i	S_{Ti}	S_i	S_{Ti}
Streamflow	0.6243	0.6288	0.6451	0.6274	0.6367
CLINX	0.0097	0.0100	0.0162	0.0112	0.0107
Rainfall	0.0123	0.0118	0.0178	0.0135	0.0120
Evaporation	0.0016	0.0016	0.0070	0.0018	0.0027
Evaporation factor A - Storage A	0.0076	0.0083	0.0165	0.0100	0.0097
Evaporation factor A - Storage B	0.0080	0.0079	0.0139	0.0102	0.0094
Consecutive Restriction Months	0.0213	0.0228	0.0560	0.0251	0.0477
Reliability	0.2397	0.2472	0.2780	0.2481	0.2673
Upper RRC	0.0309	0.0307	0.0414	0.0341	0.0355
TDFs	0.0042	0.0047	0.0113	0.0054	0.0068

when the analytical indices of variables are close to zero (Saltelli *et al.* 2004). To resolve this issue, experiments with greater number of model simulations, and accuracy, should be performed.

Since only a few input variables were significant and dominated the variance of the output, further sensitivity analysis should be performed on all variables excluding the highly influential variables. This will allow a better understanding of the less influential variables and improve the overall knowledge of the model, its variables, and their behaviour.

6. CONCLUSION

The sensitivity analysis performed in this research indicates that the yield estimate of the hypothetical urban water supply system is most sensitive to streamflow, followed by reliability of supply. The upper restriction curve, and the number of consecutive restriction months also showed some influence on the output.

The three sensitivity analysis techniques that were applied to this case study performed well. The screening experiment of the Morris method provided a good overall understanding of the importance of each variable in the model and was confirmed by the EFAST screening experiment. In the more detailed study of the most influential variables, FAST and EFAST provided the most reliable first- and total-order sensitivity measures estimates, the results of which converged within a relatively limited number of model simulations.

7. REFERENCES

- Campolongo, F., and R. Braddock (1999), The use of graph theory in the sensitivity analysis of the model output; a second-order screening method, *Reliability Engineering and System Safety*, 64, 1-12.
- Campolongo, F., J. Cariboni, and A. Saltelli (2007), An effective screening design for sensitivity analysis of large models, *Environmental Modelling and Software*, 22, 1509-1518.
- Cukier, R.I., C.M. Fortuin, K.E. Shuler, A.G. Petschek, and J.H. Schaibly (1973), Study of the sensitivity of coupled reaction systems to uncertainties in rate coefficients. I. Theory, *Journal of Chemical Physics*, 59, 3873-3878.
- Morris, M.D. (1991), Factorial sampling plans for preliminary computational experiments, *Technometrics*, 33(2), 161-174.
- Perera, B.J.C. and B. James (2003), A generalised water supply simulation computer software package – REALM. *Hydrology Journal, Institute of Engineers, India*, 26(1-2),67-83.
- Perera, B.J.C., B. James and M.D.U. Kularanthna (2005), Computer software tool REALM for sustainable water allocation and management, *Journal of Environmental Management*, 77(4), 291-300.
- Saltelli, A., S. Tarantola, and K.P.-S. Chan (1999), A quantitative model-independent method for global sensitivity analysis of model output, *Technometrics*, 41(1), 39-56.
- Saltelli, A., What is sensitivity analysis?, in. *Sensitivity Analysis*. Eds. A. Saltelli, K. Chan and E.M. Scott, Wiley, 65-80, 2000.
- Saltelli, A., S. Tarantola, F. Campolongo, and M. Ratto (2004), *Sensitivity Analysis in Practice*, Wiley, New York.
- Sobol', I.M. (1993), Sensitivity Analysis for Nonlinear Mathematical Models, *Mathematical Modeling & Computational Experiment*, 1, 407-414.
- Victoria University and Department of Sustainability and Environment (VU and DSE), REALM: Getting Started Manual, 2005, [www.dse.vic.gov.au/vro/water].