Using an SVG simulation tool to design an urban stormwater harvesting system for the City of Salisbury

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EXTENDED ABSTRACT

We describe development of a Scalable Vector Graphics (SVG) simulation tool for design and optimal management of urban stormwater harvesting systems. The system is modelled on the Helps Road Drain in the City of Salisbury, South Australia and consists of a stormwater supply channel in an urban catchment with a series of in-line storage dams. It is used as a temporary water storage for flood mitigation and control of environmental flows, and to capture, hold and harvest stormwater for supply to consumers or for aquifer storage and recovery (ASR). The purpose of the simulation is to analyse the operation of a typical urban stormwater harvesting system and to study the movement of water through a series of connected dams.

Our model uses a system of ordinary differential equations to describe the flows. The differential equations relate the rates of change in storage volumes to rates of inflow, rates of drainage outflow and rates of harvesting. We used experimental simulations to test various different configurations and to gain insight into the operation of the system. Each storage unit consists of a permanent storage component below the level of the outflow pipe or weir and a temporary storage component above this level. During “wet” periods water flow is controlled by filling of the temporary storage component above the outflow level. This water continually drains away and passes on to the next downstream storage. During “dry” periods the water level is often below the outflow level. At all times water can be harvested by pumping from the dams for direct supply to consumers or for aquifer storage. Our work can be used to determine effective design parameters for the various storage units and to simulate the operation of the system using appropriate management policies.

The simulation will provide water managers with an easy-to-use computer-based management tool. The model will incorporate generations of simulated rainfall and run-off scenarios to enable a realistic assessment of system capacity and performance and will allow managers to visualise limiting factors and assist in understanding the interdependence of different system components.

The simulation tool can be used to investigate the effectiveness of various water management policies by evaluating the Conditional Value-at-Risk (CVaR) for failure of the stormwater supply system. In this application CVaR is the expected volume of shortfall in supply to consumers given that the shortfall exceeds some significant threshold known as the Value-at-Risk (VaR). CVaR was introduced originally in financial applications (Rockafellar and Uryasev, 2000).

Simulations on configurations with identical dams found that a system with low storage capacity was unable to satisfy demand on a regular basis and that upstream dams were likely to overflow. The term overflow is used to describe water that escapes from the system and may cause flooding. Systems with greater storage capacity were better able to satisfy demand but the downstream dams were often empty.

The simulation includes a comparison mode in which identical systems with different initial states are subject to the same inflow patterns and the same consumer demands. The simulation shows that two such systems, one initially empty and the other initially full, eventually converge to the same state. The average time taken for a full system and an empty system to converge to the same state defines an intrinsic time-scale for each fixed configuration. We found that systems with smaller storage capacity converged more quickly and that systems with greater storage capacity took longer to converge. Systems with very large capacity and very long time-scales may be difficult to manage.

The simulations suggest that upstream dams should be designed primarily for flood mitigation with large temporary storage capacities and high outflow rates and that downstream dams should be smaller and designed primarily for permanent storage. A system of this type can be more easily managed and should allow evenly spread demand with less overflow upstream and less drying out downstream. Although this is a prototype simulation modelled on a particular system the model could be adapted and applied to other stormwater harvesting systems, both locally and nationally.

1 INTRODUCTION

The purpose of this paper is to describe a new Scalable Vector Graphics (SVG) simulation tool that has been used to study the behaviour of a typical urban stormwater harvesting scheme. The system consists of a stormwater stream and a series of in-line storage dams each with their own separate inflows and outflows. The dams are used to control the flow of urban stormwater in order to minimise flooding and maintain environmental flows and also to capture stormwater for direct supply to local industry. Excess water captured during "wet" periods can be stored in an underground aquifer and extracted later for use during "dry" periods.

The simulation has been developed to model operation of the Helps Road Drain and to optimise design of the individual storage units. The Helps Road Drain is located in the City of Salisbury, in the northern Adelaide region of South Australia. The new simulation will provide water managers with an easy-to-use computer-based management tool that provides a realistic assessment of system capacity and performance for various storage configurations by simulating a representative range of operating scenarios. The tool could be used to investigate the effectiveness of various water management policies and system configurations by evaluating the Conditional Value-at-Risk (CVaR) of significant shortfall in contracted water supply to local industry and by determining key control parameters such as the intrinsic system time-scale.

The mathematical literature on storage dams was developed largely from the work of Moran (1954; 1955). More recently, Piantadosi (2004) and Howlett *et al.* (2007) consider mathematical models for the management of urban stormwater in two connected dams (a capture and holding dam). They consider a practical pumping policy where they pump to fill the holding dam without allowing it to overflow. Prototype simulations of stormwater management systems were developed for Parafield and Mawson Lakes, in the City of Salisbury.

The scarcity of water has made it necessary to search for alternative sources of this limited resource. Adelaide currently depends on the River Murray for much of its water supply but many scientists warn this supply may soon become insufficient or even unsuitable. Stormwater is a valuable resource that can be used to reduce the demand for water from the River Murray. There are already several large users of reclaimed water in the City of Salisbury and the council is keen to extend the scope of their stormwater operations. It is important to note the simulation program is flexible enough to be adapted to virtually any system of dams and is thus a valuable tool.

Future work will include a stochastic dynamic programming (SDP) formulation of the system of dams with CVaR criterion. SDP aims to find the optimal policy from a given group of policies, based on specific performance criteria. Archibald *et al.* (2006) propose a new method to determine an effective operating policy for a multi-reservoir system that uses SDP but is practical for systems with many reservoirs.

2 SYSTEM DESCRIPTION

The Helps Road Drain forms part of an Integrated Water Cycle Management Plan for the City of Salisbury. The system consists of a series of five in-line storage dams each serving as a temporary water storage for flood mitigation and control of environmental flows and also to capture and hold stormwater for harvesting and subsequent supply to local users.

Stormwater run-off from the surrounding urban catchment is directed into the first dam, Edinburgh Parks North. Once the water in the dam reaches the desired permanent storage capacity water spills over a weir and flows downstream through an open channel to the next dam, Edinburgh Parks South. When the second dam reaches the desired permanent storage level water once again spills over a weir and flows downstream to the next storage dam. This process of continual inflow and outflow either over a weir or through an outflow pipe is repeated until the water eventually flows out of the fifth and final dam. Any such water is lost from the system and eventually flows out to sea. For large inflow rates the water level in the individual dams may rise temporarily above the level of the weir or outflow pipe until the inflow rate subsides. Thus each dam contains a temporary storage capacity above the normal outflow level. In cases of extreme inflow when water levels become too high we assume water overflows from the dam and escapes from the system. This water cannot be recovered and may cause flooding. Figure 1 shows a diagram of the system.

Each dam is capable of supplying water to consumers in order to satisfy demand. Such water is pumped from the dam into a balance tank, and then directed to consumers through a pipeline. Excess water in the balance tank can be pumped into an aquifer storage and recovery (ASR) system. When the water in the balance tank is insufficient to meet contracted supply, stored water can be recovered from the aquifer to assist in satisfying demand. There is an environmental constraint on the ASR system in that the volume of water withdrawn from the aquifer cannot exceed 80% of the volume previously injected, thus ensuring that underground water storages are adequately protected.

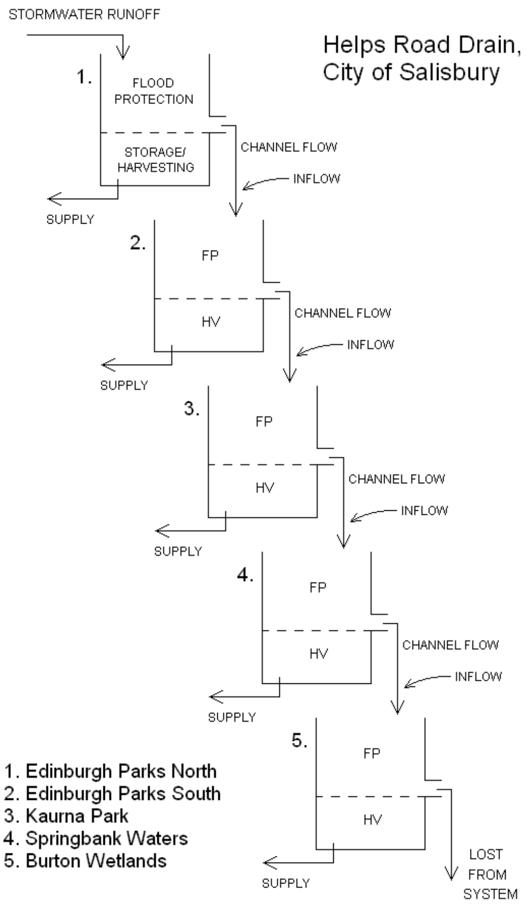

Figure 1. Diagram of the Helps Road Drain urban stormwater harvesting system

3 MATHEMATICAL MODEL

A system of differential equations is used to describe the operation of the Helps Road system utilising the basic principle of Torricelli's law. See Thomas *et al.* (2007) for further details. Let $V_i(t)$ denote the volume of water in dam i at time t. The rate of change of volume is given by

$$\frac{dV_i}{dt} = \alpha_i(t) - \beta_i(t) - q_i(t), \qquad (1)$$

where $\alpha_i(t)$ is the rate of inflow, $\beta_i(t)$ is the rate of outflow due to drainage over the weir or through an outflow pipe (see Figure 5), and $q_i(t)$ is the pumping outflow rate. If $h_i(t)$ denotes the level of water in the dam and w_i is the outflow level then the draining rate is given by

$$\beta_i(t) = k_i\,[h_i(t) - w_i]^{\frac{1}{2}}$$

if $h_i(t) \geq w_i$, and $\beta_i = 0$ otherwise for outflow through a pipe. The outflow over a weir is given by

$$\beta_i(t) = k_i'\,[h_i(t) - w_i]^{\frac{3}{2}}$$

if $h_i(t) \geq w_i$, and $\beta_i = 0$ otherwise, where k_i and k_i' are constants. A brief explanation is given in the Appendix. The inflow to the first dam is $\alpha_1(t) = r(t)$ where $r(t)$ is determined by local rainfall and run-off. The inflow to dam i at time t is given by $\alpha_i(t) = \beta_{i-1}(t - \tau_i)$ where τ_i is the time taken for water to flow downstream from dam $(i-1)$ to dam i, for $i > 1$.

4 SIMULATION FEATURES

The simulation tool has been created in the Scalable Vector Graphics (SVG) programming language. SVG is a web standard for producing two-dimensional graphics (see <http://www.w3.org/Graphics/SVG>) and can be viewed in a web-browser. The components

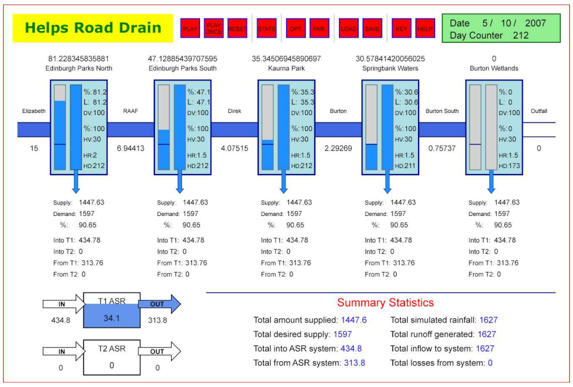

Figure 2. Main simulation screen

of the stormwater harvesting system are drawn as two-dimensional images whose attributes, such as size, shape, and colouring, are modified by ECMA-Script functions acting on them. The differential equations described in Section 3 are solved at 6 minute intervals using a fourth-order Runge-Kutta method, and the results for each day are displayed. The main simulation screen, shown in Figure 2, displays the level of water in each dam, the contents of the channel and the volume of aquifer storage at each simulated stage, and the volumes of stormwater extracted from each dam.

4.1 Risk assessment

The effectiveness of various system configurations under typical rainfall scenarios is calculated using Conditional Value-at-Risk (CVaR). CVaR is a coherent measure of risk associated with a stochastic process. It was originally introduced by Rockafellar and Uryasev (2000) as a measure of risk in management of share portfolios and was defined as the expected value of monetary loss given that the actual loss exceeds some unacceptable Value-at-Risk (VaR) threshold. In our case we set the VaR as an unacceptable shortfall in daily supply to consumers and calculate CVaR as the expected shortfall given that the threshold has been exceeded.

The user sets the unacceptable VaR shortfall threshold and then uses the simulation to determine the probability p, that the actual shortfall exceeds the VaR threshold. The simulation is also used to find the expected value CVaR of the shortfall given that the threshold is exceeded. Thus p is estimated by the proportion of days on which the shortfall exceeds the unacceptable threshold and CVaR is the average shortfall on those occasions. These calculations, with corresponding histograms, are displayed in the simulation and may be used for risk assessment and planning to meet future demands.

4.2 Comparison mode

The simulation may be run in two modes: the Main Simulation mode in which all components of the stormwater harvesting system are displayed on the screen (see Figure 2); and the Comparison mode, which compares two identical systems with the same inputs and demands but each with different initial volumes of water in the dams (see Figure 3). The

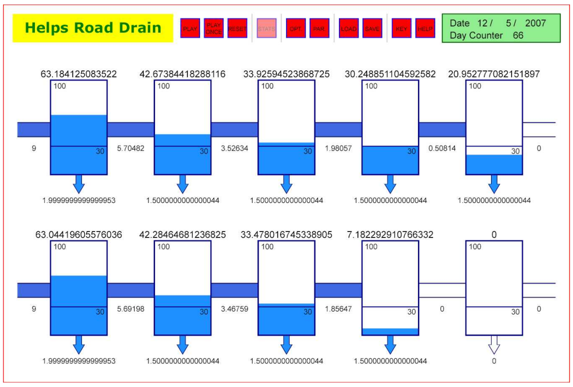

Figure 3. Comparison mode screen

main purpose of the Comparison mode output is to demonstrate that the two systems converge after a finite number of steps to the same state and to determine by repeated simulation the intrinsic time-scale for this convergence. That is, a system that is empty in the beginning will eventually reach the same state as a system that started full, when subjected to the same operating conditions. The average time taken for this convergence is a characteristic property of the system.

5 CHARACTERISTIC SYSTEM BEHAVIOUR

The characteristic behaviour of a system of in-line storage dams depends on the relative levels of expected inflow and expected outflow and on the storage capacities of the dams. If expected inflow exceeds expected outflow the system will gradually fill. If expected outflow exceeds expected inflow the system will gradually empty. As the capacity of the system increases the time taken to fill or empty the system also increases.

5.1 The intrinsic time-scale

For given levels of expected inflow-outflow we will show that the storage capacity of the system determines an intrinsic time-scale that increases as the storage capacity increases. The time-scale is a measure of how long the system takes to return to normal after a drought or a flood. Thus it is a measure of the capacity for the system to recover from an extreme event. The practical problem that we address is the compromise between a system with very large storage that takes a very long time to fill but will then be able to satisfy demand over a prolonged "dry" period and an alternative system with smaller storage that can be filled more quickly but which then has a limited capacity to satisfy demand during a "dry" period. To define the time scale we consider two identical systems E and F with the same inflows and the same outflows but with different initial states. Indeed we imagine that system E is initially empty and system F is initially full. As long as each individual dam has a non-zero probability of filling or emptying we argue that the two systems must reach the same state after a finite number of steps. The average time taken to do this is defined as the intrinsic system time-scale. We call this process *stochastic convergence* and we will measure the *rate of convergence* by the reciprocal of the time-scale.

We consider corresponding dams D_E and D_F. Assume that all upstream dams have reached the same state on day N but that dams D_E and D_F have not. Let W denote the capacity of dams D_E and D_F and denote the respective contents on day $n \geq N$ by $V_{E,n}$ and $V_{F,n}$ where $V_{E,N} < V_{F,N}$. On day n let f_n denote the total inflow volume and let d_n denote the demand and suppose that the respective net inflow volumes, outflow drainage volumes, and outflow pumping volumes are $\alpha_{E,n}$ and $\alpha_{F,n}$, $\beta_{E,n}$ and $\beta_{F,n}$ and $q_{E,n}$ and $q_{F,n}$. We assume that all variables take non-negative integer values. It is relatively easy to see that $\alpha_{E,n} \geq \alpha_{F,n}$, $\beta_{E,n} \leq \beta_{F,n}$ and $q_{E,n} \leq q_{F,n}$ and hence

$$V_{F,n+1} - V_{E,n+1} \leq V_{F,n} - V_{E,n}.$$

However it is important to note that

- $\alpha_{E,n} > \alpha_{F,n}$ when $V_{F,n} + f_n > W$; and
- $q_{E,n} < q_{F,n}$ when $V_{E,n} - d_n < 0$

and hence

$$V_{F,n+1} - V_{E,n+1} < V_{F,n} - V_{E,n}$$

on any day when one or the other of these two extreme events occurs. That is the inflow to system E is greater when system F overflows and the outflow from system F is greater when system E fails to meet demand. Thus after a finite number of extreme events the difference $V_{F,n} - V_{E,n}$ must decrease to zero. As long as the probability of extreme events is positive we can argue that this will happen with probability one after a finite number of steps. Note that we may also have $\beta_{F,n} > \beta_{E,n}$. This will increase the rate of convergence even though we have not used this in our argument. We can now apply the same argument to the next downstream dam.

The rate of stochastic convergence is therefore proportional to the rate of occurrence of extreme events. For a balanced system where expected inflow equals expected outflow the number of extreme events will decrease as the storage capacity increases. In simple terms we expect the system to gradually fill during "wet" periods when inflow exceeds outflow and to gradually empty during "dry" periods when outflow exceeds inflow. As storage capacity increases the "wet" and "dry" events need to become more extreme before the system will fill or empty. Thus the chance that these events occur also decreases.

5.2 The balance between expected inflow and expected outflow

In most real systems the occurrence of extreme events means that some water will be lost to overflow during "wet" events and that some demand will not be satisfied during "dry" events. To ensure a strict stochastic balance where expected inflow equals expected outflow it is necessary to have some knowledge of expected overflow and expected unmet demand. For a given level of expected total inflow and expected demand these quantities depend on the capacity of the system. Simulation can help us to determine sensible design parameters. In the first instance we note that storage can be used to mollify irregular behaviour. In simple terms we can use a storage capacity of 10 units and an inflow sequence $\{10, 0, 10, 0, 10, \ldots\}$ to generate an outflow sequence $\{5, 5, 5, 5, 5, \ldots\}$ and a storage sequence $\{5, 0, 5, 0, 5, \ldots\}$. Thus we expect the behaviour of our storage dams to become more regular as we move downstream. In a system where expected inflow exceeds expected outflow the system is likely to converge to a stochastic limit where all dams are full and where the upstream dams will often overflow. In a system where expected outflow exceeds expected inflow the system will converge to a stochastic limit where the downstream dams are often empty and the upstream dams are far less likely to overflow.

5.3 Design of sensible systems

The simulation was used as a design tool in the following way. We considered a sequence of identical dams each with the same outflow levels and each with the same capacity. For a given level of inflow and a given level of demand we adjusted the capacity of the dams to control the level of overflow and the levels of unmet demand. In the first instance we showed that by increasing the capacities we could reduce the level of overflow and the level of unmet demand but in so doing the intrinsic time-scale of the system was increased. We observe that for a high capacity system with a very long time-scale it is difficult to determine a precise stochastic balance and if, for example, the actual outflow exceeds the actual inflow for a sustained period of time it may then be very difficult to refill the system. The Murray-Darling system is a typical large capacity system which has slowly emptied over a long time period. In our simulation the system of identical dams was quite difficult to balance in that the first few dams tended to overflow if demand was too low and the last few dams tended to empty if demand was too high. The best way to balance this system with a reasonable intrinsic time-scale was to meet most demand by pumping from the first dam. In order to extract similar volumes from each dam, which might be desirable in practice, it was much better to design the dams differently with the upstream dams designed primarily for flood mitigation and the downstream dams designed primarily for storage. Thus the upstream dams need to be larger and to have larger drainage capacity.

6 SIMULATION EXAMPLES

Consider a series of five dams with the differential equations from (1), each with drainage outflow through a pipe, a one day delay between each of the dams ($\tau_i = 1$, for $i > 1$), and $A_i(h) = 1$, $k_i = 1$, $w_i = 30$, $q_1 = 2$, $q_i = 1.5$ for $i > 1$, and $V_i(0) = w_i$. The inflow $r(t)$ and the demand are integers from the $U(0, 16)$ uniform distribution.

6.1 Risk of supply shortfall

The system was simulated many times over a time horizon of 500 days with varying storage capacities, and their effectiveness in satisfying contracted supply to consumers was evaluated. The VaR was set at 2 units of daily shortfall, a typical value that might be considered unacceptable in practice, and the CVaR was calculated. A summary of the results is displayed in Table 1. In a realistic system the units would be in megalitres per day.

The results show that the examples with the greater

Table 1. Chance of exceeding threshold (VaR=2) and expected shortfall (CVaR)

Storage (5 dams)	Chance of exceeding VaR	Expected shortfall CVar when VaR exceeded
250	0.34	5.63
350	0.16	5.05
500	0.13	4.92

storage have a lower probability of exceeding the VaR of 2 units daily supply shortfall, and also a lower CVaR. For the case of a 500 unit total capacity the chance of more than 2 units shortfall is approximately 0.13 and the average value of shortfall on these occasions was 4.92 units. For the lesser capacities the system is clearly unable to satisfy demand.

6.2 Intrinsic system time-scales

Multiple simulations were also run in Comparison mode, counting the number of days taken for the empty system and the full system to converge to the same state. The results are displayed in Table 2.

Table 2. Intrinsic time-scales

Capacity (5 dams)	Time-scale (days)
250	126.6
350	203.3
500	301

When the total storage capacity was set at 500 units, the average time for convergence was 301 days. The limiting behaviour was characterised by regular emptiness of the final dam. There was very little wastage of water due to overflow in this system but the rate of stochastic convergence was quite slow. There were limited downstream flows and the potential for environmental damage was high. When the storage capacity was set at 350 units the time-scale was significantly reduced to 203 days. However the limiting behaviour now showed persistent emptiness in the final dam and also that the first dam was often full and overflowed regularly. The configuration with the smallest storage demonstrated rapid convergence but the limiting behaviour was undesirable. The first dam was full most of the time and overflowed frequently and the final two dams were usually empty. The system converges to the characteristic state more quickly but "wastes" a large amount of water to overflow and regularly fails to satisfy demand. These results illustrate important aspects of the relationship between storage capacity and intrinsic time-scales.

The task is to design sensible storage configurations that are easy to control and will satisfy other criteria, such as contracted water supply and reliable environmental flows. On the basis of the above observations we designed a new system that would seldom be full or empty and would regularly meet contracted demand. The first 2 dams were designed primarily for flood mitigation with larger temporary storage capacity and smaller permanent storage capacity, the intermediate dams were smaller with a more even balance between temporary and permanent storage that ensured regular inflow and regular outflow while the final dam was designed to be full most of the time with a regular but small environmental outflow. The parameters for the system were the total storage levels $S = [100, 100, 50, 50, 50]$, the permanent storage levels $w = [40, 30, 30, 30, 30]$ and the harvesting rates $q = [4.5, 1.5, 1.5, 0.25, 0.25]$. The results are shown in Table 3.

Table 3. Chance of exceeding threshold (VaR=2), expected shortfall (CVaR) when VaR is exceeded and time-scales for new system

Chance of exceeding VaR	Expected shortfall CVaR	Time-scale (days)
0.11	4.52	349

Although the time-scale is too large the limiting state is characterised by water levels that hover around the weir heights. The dams are seldom empty or full, thus maintaining environmental flow requirements through creeks and wetlands, and demand is regularly satisfied.

6.3 Oscillations

We observe that levels in the upstream dams oscillate more than those of the downstream dams which show more regular level variations. See Figure 4. In this particular example it is clear that the storage capacity in each particular dam can be used to progressively mask the random component of the inflow. We observe, in this case, lengthy periods with no stream flow during which time the downstream dams are regularly pumped dry.

7 CONCLUSION

The SVG simulation is a useful tool for the analysis of urban stormwater management systems. It allows the user to visualise the limiting factors and understand the interdependence of different components within the system, and to evaluate the effectiveness of a system configuration by calculating the CVaR for shortfall in supply and the characteristic behaviour.

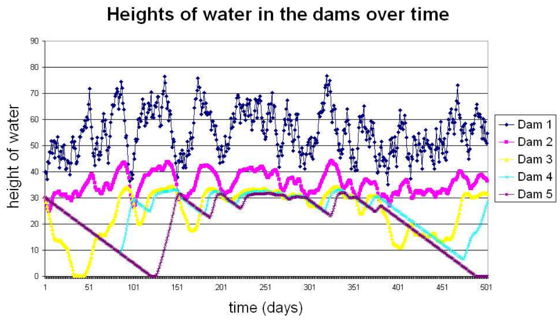

Figure 4. The levels of water in the dams over time tend to become smoother further downstream

We establish and explain the idea of an intrinsic time-scale and use the simulation to demonstrate the predicted relationship between the storage capacity and system time-scale. We showed that our observations of the simulation for various different configurations allowed us to improve the overall design. In particular we concluded that upstream dams should be used primarily for flood mitigation and supply and should have relatively large temporary storage capacities. Downstream dams should be used to maintain environmental flows.

Future work will include the incorporation of additional inflows and outflows; more realistic assessments of supply and demand; allowance for losses due to evaporation; and inclusion of environmental considerations, such as water flow through creeks and wetlands, in risk assessment. We will use the work by Archibald *et al.* (2006) with a CVaR criterion to investigate optimal operating policies.

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APPENDIX

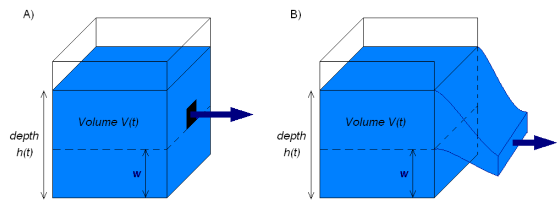

Figure 5. Alternative dam configurations

Outflow through a pipe

For the case of outflow through a pipe at height w with water level $h = w + y$ Torricelli's law (see Edwards and Penney (2000)) gives the speed v of the draining water by $v \propto \sqrt{y}$. Thus we obtain the differential equation

$$\frac{dV}{dt} = A(h)\frac{dh}{dt} = -k\,[h-w]^{\frac{1}{2}} \quad \text{if } h > w$$

and $dV/dt = 0$ otherwise, where $h = h(t)$ is the height of water in the tank at time t.

Outflow over a weir

Let y be the height of the water above a weir, then once again $v \propto \sqrt{y}$ and hence the total discharge rate Q for depth y is given by

$$Q = k'W \int v\,dy = \frac{2}{3}k'Wy^{\frac{3}{2}},$$

where W is the length of the weir crest. The differential equation is given by

$$\frac{dV}{dt} = A(h)\frac{dh}{dt} = -k(h-w)^{\frac{3}{2}} \quad \text{if } h > w$$

and $dV/dt = 0$ otherwise, where k is a constant (see Jain (2001)).